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Research Article

ESTIMATION OF PARAMETER AND RELIABILITY FUNCTION OF EXPONENTIATED INVERTED WEIBULL DISTRIBUTION USING CLASSICAL AND BAYESIAN APPROACH

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ABSTRACT

The exponentiated inverted Weibull distribution (EIWD) is a generalization of the exponentiated inverted exponential distribution as well as the inverted Weibull distribution and exponentiated exponential family is a generalization of Gamma and Weibull distribution so EIWD has wide applicability in reliability theory. In this paper we derived classical estimator i.e. maximum likelihood estimator and Bayes estimator of unknown parameter and reliability function of Exponentiated Inverted Weibull Distribution using Precautionay loss function and De-groot loss function.

INTRODUCTION

The two parameter Exponentiated Inverted Weibull Distribution (EIWD) has been proposed by Flaih *et al*. (2012) and is a generalization to the Inverted Weibull Distribution and exponentiated inverted exponential distribution. The probability density function of EIWD with two shape parameters θ and β is given by

$$f(x,\theta,\beta) = \theta\,\beta\,x^{-(\beta+1)}\,e^{-\theta x^{-\beta}}\ ;\ x > 0,\ \theta > 0,\ \beta > 0 \quad (1)$$

EIWD represents the standard inverted Weibull distribution for $\theta = 1$, and exponentiated inverted exponential distribution for $\beta = 1$.

The cumulative distribution function (c. d. f) of EIWD is given by

$$F_\theta(x) = \left(e^{-x^{-\beta}}\right)^{\theta};\ \ x,\ \theta,\ \beta > 0.$$

The reliability and failure rate function of EIWD are given respectively, by

$$R(x) = 1 - \left(e^{-x^{-\beta}}\right)^{\theta};\ x > 0,\ \theta,\ \beta > 0 \quad (2)$$

$$h(x) = \frac{\theta\,\beta\,x^{-(\beta+1)}}{e^{\theta x^{-\beta}} - 1}\ ;\ x > 0,\ \theta,\ \beta > 0 \quad (3)$$

Many characteristics of Exponentiated Weibull Distribution has been found in the literature. Soland (1968) discusses the Bayesian analysis of the Weibull Process. Mudholka *et.al* (1995) introduced the exponentiated weibull distribution as a generalization of the standard weibull distribution and as a suitable model for bus motor failure time data. Mudholkar and Huston (1996) applied the exponentiated weibull distribution to the flood data with some properties. Ahmad *et al* (2015) discusses the Bayesian estimator of shape parameter of exponentiated inverted weibull distribution under different loss function namely square error loss function, entropy loss function and precautionary loss function. Aljouharah (2013) estimates the parameters of EIWD under type-II censoring. Bayesian estimation approach has received great attention by the researchers and they had proved that Bayes estimate perform better than classical estimators. Singh *et al* (2011) provided Bayes estimate of parameters and reliability function of exponentiated exponential distribution under general entropy loss function. Shawky and Bakoban (2008) proposed Bayesian and Non-Bayesian method of estimation and derived estimates of parameter of exponentiated gamma distribution. Gupta and

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Gupta (2017) studied Bayes and E-Bayes estimate of unknown shape parameter of exponentiated inverted weibull distribution using conjugate prior under three different loss functions.

In this paper, Bayes and classical estimators i.e. maximum likelihood estimator have been obtained for the unknown shape parameter and reliability function of exponentiated inverted Weibull distribution when a complete sample is available from EIWD. Bayes estimates are obtained under Precautionary and De- groot loss function. This was done with respect to non-informative prior distribution for the shape parameter.

Maximum Likelihood Estimation of Parameter, Reliability Function and Failure Rate Function

Let X_1, X_2 ,X_3…..be a sequence of random variables from exponentiated inverted weibull distribution, whose density function is given by (1), then the likelihood function is given by

$$L(\underline{x},\theta,\beta)=\prod_{i=1}^{n} f(x_i,\theta,\beta)$$

$$=(\theta\beta)^n * \prod_{i=1}^{n} x_i^{-(\beta+1)} * \prod_{i=1}^{n} e^{-x_i^{-\theta\beta}}$$

$$=\prod_{i=1}^{n} x_i^{-(\beta+1)} * (\theta\beta)^n * e^{-\sum x_i^{-\theta\beta}}$$

Log of likelihood function is given by

$$\text{Log } L = n\log\theta\beta + \log\prod_{i=1}^{n} x_i^{-(\beta+1)} + \log\prod_{i=1}^{n} e^{-x_i^{-\theta\beta}}$$

$$= n\log\theta + n\log\beta - (\beta+1)\sum_{i=1}^{n}\log x_i + \theta\sum_{i=1}^{n} x_i^{-\beta}$$

On solving $\frac{d}{d\theta}\text{Log } L = 0$, we get MLE of θ given by,

$$\hat{\theta}_{Mle} = \frac{n}{\sum_{i=1}^{n} x_i^{-\beta}}. \tag{4}$$

On using the invariance property of Maximum likelihood estimators, $\hat{R}_{Mle}$, the Mle of reliability function R(x) may be obtained by replacing θ by $\hat{\theta}_{Mle}$ in (2) i.e.,

$$\hat{R}_{Mle} = 1-\left(e^{-x^{-\beta}}\right)^{\hat{\theta}_{Mle}} = 1-e^{\frac{-x^{-\beta}\, n}{\sum_{i=1}^{n} x_i^{-\beta}}} \tag{5}$$

The Mle of failure rate function is also obtained by replacing θ by $\hat{\theta}_{Mle}$ in (3) i.e.,

$$\hat{h}_{Mle} = \frac{\hat{\theta}_{Mle}\,\beta\, x^{-(\beta+1)}}{e^{\hat{\theta}_{Mle} x^{-\beta}}-1} \tag{6}$$

And the Mle of cumulative failure rate $H(x) = -\log R(x)$ is given by

$$\hat{H}_{Mle} = -\log \hat{R}_{Mle}.$$

Bayesian Estimation

In this section, Bayes estimators of the unknown shape parameter of EIWD are obtained by using non - informative prior under two different loss functions. Let a random sample of size n be drawn from EIWD with pdf given by (1), then the likelihood function is given by:-

$$L(\underline{x},\theta,\beta) = (\theta\beta)^n \prod_{i=1}^{n} x_i^{-(\beta+1)} e^{-\theta\sum_{i=1}^{n} x_i^{-\beta}} \tag{7}$$

Here, we are using the non-informative prior i.e. Jeffrey prior distribution which is given by

$g(\theta)\ \alpha\ \sqrt{I(\theta)}$; where $I(\theta)$ is a Fisher's (information) matrix given by $-E\left[\frac{\partial^2}{\partial\theta^2}\log L(x/\theta)\right]$.

Here, $-E\left[\frac{\partial^2}{\partial\theta^2}\log L(x/\theta)\right] = \frac{n}{\theta^2}$

$$\Rightarrow g(\theta)\ \alpha\ \frac{\sqrt{n}}{\theta}$$

$$\Rightarrow g(\theta)\ \alpha\ \frac{1}{\theta} \tag{8}$$

On combining (7) and (8), and using Bayes theorem, the posterior density of θ given $\underline{x}$ is given by

$$P(\theta/\underline{x})\ \alpha\ L(\underline{x},\theta,\beta) * g(\theta)$$

$$P(\theta/\underline{x})\ \alpha\ (\theta\beta)^n \prod_{i=1}^{n} x_i^{-(\beta+1)} e^{-\theta\sum_{i=1}^{n} x_i^{-\beta}} * \frac{1}{\theta}$$

$$= k\,\theta^{n-1} e^{-\theta\sum_{i=1}^{n} x_i^{-\beta}}$$

where k is independent of θ.

$$k^{-1} = \int_0^\infty \theta^{n-1} e^{-\theta\sum_{i=1}^{n} x_i^{-\beta}} d\theta$$

$$\Rightarrow k = \frac{\Gamma n}{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^n}$$

Thus, Posterior density is given by

$$P(\theta/\underline{x}) = \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^n}{\Gamma n}\theta^{n-1} e^{-\theta\left[\sum_{i=1}^{n} x_i^{-\beta}\right]};\ \theta>0,\ c,\ r>0 \tag{9}$$

Bayes estimate of θ under Precautionary loss function

The Precautionary loss function is given by

$$L(\hat{\theta},\theta) = \frac{(\hat{\theta}-\theta)^2}{\hat{\theta}}$$

The Bayes estimator under Precautionary loss function is obtained by solving the equation

$\hat{\theta} = \left[E\left(\theta^2/\underline{x}\right)\right]^{\frac{1}{2}}$. This loss function is useful when under estimation is more serious than over estimation.

$$\text{Here, } E\left(\theta^2/\underline{x}\right) = \int_0^\infty \theta^2 * P(\theta/\underline{x})\, d\theta$$

$$= \int_0^\infty \theta^2 * \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^n}{\Gamma n}\theta^{n-1} e^{-\theta\left[\sum_{i=1}^{n} x_i^{-\beta}\right]} d\theta$$

$$= \frac{n(n+1)}{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^2}.$$

Hence, Bayes estimate of θ using Precautionary loss function is given by

$$\hat{\theta}_P = \frac{\sqrt{n(n+1)}}{\sum_{i=1}^{n} x_i^{-\beta}} \tag{10}$$

Bayes Estimate of θ Under De-Groot Loss Function

If $\hat{\theta}$ is an estimator of θ then the De-groot loss function is given by $(\theta,\hat{\theta}) = \frac{\theta-\hat{\theta}}{\hat{\theta}}$. For more details about De-groot loss function, one may refer Degroot (1970), optimal statistical decision.

The Bayes estimate using De-groot loss function is given by

$$\hat{\theta}_{DG} = \frac{E\left(\theta^2/\underline{x}\right)}{E\left(\theta/\underline{x}\right)}$$

Now, $E\left(\theta/\underline{x}\right) = \int_0^\infty \theta * P\left(\theta/\underline{x}\right) d\theta$

$$= \int_0^\infty \theta * \frac{\left(\sum_{i=1}^n x_i^{-\beta}\right)^n}{\Gamma n} \theta^{n-1} e^{-\theta\left[\sum_{i=1}^n x_i^{-\beta}\right]} d\theta$$

$$= \frac{n}{\sum_{i=1}^n x_i^{-\beta}}$$

Also, $E\left(\theta^2/\underline{x}\right) = \frac{n\,(n+1)}{(\sum_{i=1}^n x_i^{-\beta})^2}$.

Hence, Bayes estimate of θ using De-groot loss function is given by

$$\hat{\theta}_{DG} = \frac{\frac{n\,(n+1)}{(\sum_{i=1}^n x_i^{-\beta})^2}}{\frac{n}{\sum_{i=1}^n x_i^{-\beta}}}$$

$$= \frac{n+1}{\sum_{i=1}^n x_i^{-\beta}} \tag{11}$$

Bayesian Estimation of Reliability Function

Consider the Reliability R = R(x) is a parameter itself, and on replacing θ in terms of R from (2), we obtain the posterior density of R as

$$P\left(R/\underline{x}\right) = \frac{\left(\sum_{i=1}^n x_i^{-\beta}\right)^n}{\Gamma n} \left(\frac{\log(1-R)}{-x^{-\beta}}\right)^{n-1} * e^{\frac{\log(1-R)}{x^{-\beta}}\left[\sum_{i=1}^n x_i^{-\beta}\right]} * \frac{1}{x^{-\beta}(1-R)},\ 0<R<1. \tag{12}$$

Bayes estimate of Reliability function under Precautionary loss function

The Bayes estimator of reliability function under Precautionary loss function is obtained by solving the equation

$$E\left(R^2/\underline{x}\right) = \int_0^1 R^2 * P\left(R/\underline{x}\right) * dR$$

$$= \int_0^1 R^2 * \frac{\left(\sum_{i=1}^n x_i^{-\beta}\right)^n}{\Gamma n} \left(\frac{\log(1-R)}{-x^{-\beta}}\right)^{n-1} * e^{\frac{\log(1-R)}{x^{-\beta}}\left[\sum_{i=1}^n x_i^{-\beta}\right]} * \frac{1}{x^{-\beta}(1-R)} * dR$$

On solving this integral, we get

$$E\left(R^2/\underline{x}\right) = \frac{(n-1)*(n-2)\ldots\ldots 3*2*\left(\sum_{i=1}^n x_i^{-\beta}\right)^n}{\Gamma(n)*(-1)^{n-2}} * \left\{\frac{1}{\left(\sum_{i=1}^n x_i^{-\beta}\right)^n} + \frac{1}{\left(2x^{-\beta}+\sum_{i=1}^n x_i^{-\beta}\right)^n} - \frac{2}{\left(x^{-\beta}+\sum_{i=1}^n x_i^{-\beta}\right)^n}\right\} \tag{13}$$

Hence, Bayes estimate under Precautionary loss function is given by

$$\hat{R}_P = \sqrt{\frac{(n-1)*(n-2)\ldots\ldots 3*2*\left(\sum_{i=1}^n x_i^{-\beta}\right)^n}{\Gamma(n)*(-1)^{n-2}} * \left\{\frac{1}{\left(\sum_{i=1}^n x_i^{-\beta}\right)^n} + \frac{1}{\left(2x^{-\beta}+\sum_{i=1}^n x_i^{-\beta}\right)^n} - \frac{2}{\left(2x^{-\beta}+\sum_{i=1}^n x_i^{-\beta}\right)^n}\right\}} \tag{14}$$

Bayes estimate of Reliability function under Degroot loss function

The Bayes estimate of reliability function using De-groot loss function is given by

$$\hat{R}_{DG} = \frac{E\left(R^2/\underline{x}\right)}{E\left(R/\underline{x}\right)}$$

$$E\left(R/\underline{x}\right) = \int_0^1 R^2 * P\left(R/\underline{x}\right) * dR$$

$$= \int_0^1 R * \frac{\left(\sum_{i=1}^n x_i^{-\beta}\right)^n}{\Gamma n} \left(\frac{\log(1-R)}{-x^{-\beta}}\right)^{n-1} * e^{\frac{\log(1-R)}{x^{-\beta}}\left[\sum_{i=1}^n x_i^{-\beta}\right]} * \frac{1}{x^{-\beta}(1-R)} * dR$$

$$= \frac{(n-1)*(n-2)\ldots\ldots 3*2*\left(\sum_{i=1}^n x_i^{-\beta}\right)^n}{\Gamma(n)*(-1)^{n-2}} * \left\{\frac{1}{\left(\sum_{i=1}^n x_i^{-\beta}\right)^n} - \frac{1}{\left(x^{-\beta}+\sum_{i=1}^n x_i^{-\beta}\right)^n}\right\} \tag{15}$$

On utilizing, (13) and (15), Bayes estimate of reliability function under De-groot loss function is given by

$$\hat{R}_{DG} = \frac{\left\{\frac{1}{\left(\sum_{i=1}^n x_i^{-\beta}\right)^n} + \frac{1}{\left(2x^{-\beta}+\sum_{i=1}^n x_i^{-\beta}\right)^n} - \frac{2}{\left(x^{-\beta}+\sum_{i=1}^n x_i^{-\beta}\right)^n}\right\}}{\left\{\frac{1}{\left(\sum_{i=1}^n x_i^{-\beta}\right)^n} - \frac{1}{\left(x^{-\beta}+\sum_{i=1}^n x_i^{-\beta}\right)^n}\right\}} \tag{16}$$

CONCLUSION

In this paper, we have obtained the classical and Bayes estimators for the unknown parameter and reliability function of Exponentiated inverted weibull distribution using two different loss functions for complete sample.

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