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Research Article

COMPARATIVE STUDY ON BOOLEAN ALGEBRA, C-ALGEBRA AND PRE A*-ALGEBRA

Vijayabarathi S and Baskaran D

SCSVMV University, Enathur, Kanchipuram

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ABSTRACT

Generally algebra is the study of algebraic structure. In this paper Boolean algebra, C-algebra and Pre A*-algebra have been compared.

INTRODUCTION

In 1850 Boole introduced Boolean algebras. A*-algebra is analogous to Ada (Algebra of disjoint alternatives) $(A,\wedge,\vee,(-)^{\sim},(-)_{®},0,1,2)$. A*-algebra is denoted by $(A,\wedge,\vee,*,(-)^{\sim},(-)_{®},0,1,2)$ where $\wedge,\vee,*$ are binary operations and $(-)^{\sim},(-)_{®}$ are unary operations. P.Koteswara Rao[6] has proved that A*-algebra is generated by Boolean algebra and Boolean algebra over the matrices forms an A*-algebra. C-algebra, A*-algebra are regular extension of Boolean logic to three truth values where the third truth value is undefined truth value. Pre A*-algebra [10] is a reduct of A*-algebra which is analogous to C-algebra[5].

Preliminary

Boolean Algebra[1] $(B,\vee,\wedge,',0,1)$

Definition1.1.1: A non-empty set B together with two binary operations $\vee,\wedge$ and one unary operation $'$ and two distinct elements 0 and 1 satisfying the following axioms for all $a,b,c\in B$ is called Boolean Algebra.

1. $a\vee b\in B$ $\qquad a\wedge b\in B$
2. $a\vee b=b\vee a$ $\qquad a\wedge b=b\wedge a$
3. $a\vee(b\vee c)=(a\vee b)\vee c$ $\qquad a\wedge(b\wedge c)=(a\wedge b)\wedge c$
4. $a\vee 0=a$ $\qquad a\wedge 1=a$
5. $a\vee a'=1$ $\qquad a\wedge a'=0$

where a' is complement of a.

6. $a\vee(b\wedge c)=(a\vee b)\wedge(a\vee c)$ $\qquad a\wedge(b\vee c)=(a\wedge b)\vee(a\wedge c)$
7. $a\wedge(a\vee b)=a\vee(a\wedge b)=a$

A Boolean algebra is a lattice which contains a least element and a greatest element and which is both complemented and distributive.

Boolean lattice is an algebra with two binary operations + and · and one unary operation‘. Boolean lattice so considered are called Boolean algebras.

Every set is a partial ordered set. We can see application of lattices more in other science like chemistry and life science course. The complement of each element in Boolean algebra B is unique. It satisfies Idempotent, commutative, associative, absorption, distributive, complement laws. Here every maximal ideal is a prime ideal and vice versa.

Claud shanon’s contribution is more in the application of Boolean algebra in circuit theory. Circuit theory is a major part in Electrical and Electronics Engineering, Electronics and Communication Engineering and day to day practical life.

C-algebra

C – algebra is defined by three truth values T, F and U. The third truth value is undefined. Here also every set is a poset. C-algebra has 7 postulates

*Corresponding author: **Vijayabarathi S**
SCSVMV University, Enathur, Kanchipuram

Definition: An algebra of type (2.2.1) with binary operations $\wedge, \vee$, and unary operation $'$ satisfying the following identities:

(i) $x'' = x$

(ii) $(x \wedge y)' = x' \vee y'$

(iii) $x \wedge (y \wedge z) = (x \wedge y) \wedge z$

(iv) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$

(v) $(x \vee y) \wedge z = (x \wedge z) \vee (x' \wedge y \wedge z)$

(vi) $x \vee (x \wedge y) = x$

(vii) $(x \wedge y) \vee (y \wedge x) = (y \wedge x) \vee (x \wedge y)$

The three element algebra C-{T,F,U} with the operations given by the following tables is a C-algebra.

$\wedge$	T	F	U	$\vee$	T	F	U	x	x'
T	T	F	U	T	T	T	T	T	F
F	F	F	F	F	T	F	U	F	T
U	U	U	U	U	U	U	U	U	U

Note

1. The identities (i), (ii) imply that the variety of C-algebras satisfies all the dual elements of the identities (i), (vi)
2. $\wedge$ and $\vee$ are not commutative in C-algebra
3. The ordinary distributive law of $\wedge$ over $\vee$ fails in C-algebra.
4. Every Boolean algebra is a C-algebra.

Pre A*-algebra

Definition: An algebra $(A, \wedge, \vee, (-)^{\sim})$ where A is non-empty set with 1, $\wedge$, $\vee$ are binary operations and $(-)^{\sim}$ is a unary operation satisfying

(a) $x^{\sim\sim} = x \ \forall x \in A$

(b) $x \wedge x = x, \quad \forall x \in A$

(c) $x \wedge y = y \wedge x, \quad \forall x, y \in A$

(d) $(x \wedge y)^{\sim} = x^{\sim} \vee y^{\sim}, \forall x, y \in A$

(e) $x \wedge (y \wedge z) = (x \wedge y) \wedge z, \quad \forall x, y, z \in A$

(f) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z), \quad \forall x, y, z \in A$

(g) $x \wedge y = x \wedge (x^{\sim} \vee y), \forall x, y \in A$.

is called a Pre A*-algebra.

Example

3 = {0, 1, 2} with operations $\wedge, \vee, (-)^{\sim}$ defined below is a Pre A*-algebra.

ė	0	1	2	Ę	0	1	2	x	$x^{\sim}$
0	0	0	2	0	0	1	2	0	1
1	0	1	2	1	1	1	2	1	0
2	2	2	2	2	2	2	2	2	2

Note

1. $(\mathbf{2}, \wedge, \vee, (-)^{\sim})$ is a Boolean algebra. So every Boolean algebra is a Pre A* algebra
2. The identities 1.1(a) and 1.1(d) imply that the varieties of Pre A*-algebras satisfies all the dual statements of 1.1

Comparison

Boolean algebra depends on two element logic. C-algebra and our Pre A*-algebra are regular extensions of Boolean logic to 3 truth values, the third truth value stands for an undefined truth value. Application of Boolean algebra is more in switching theory.

Boolean algebra is the centre of C – algebra. C- algebra has absorption law. Right distributive $\wedge$ over $\vee$ fails here. But it satisfies the distributive law of $\vee$ over $\wedge$. C- algebra does not satisfy commutative law. Here every maximal ideal is a prime ideal. Boolean algebra has commutative and distributive law but these two laws are not in C – algebra particularly right distributive. Application of Boolean algebra is more in switching theory but that much is not for C-algebra and research is going on Boolean algebra is the centre of Pre A*-algebra. Pre A* - algebra has seven laws. They are involution Idempotent, Commutative, distributive, associative, complement, Demorgan's laws. It does not satisfy absorption law. Pre A* - algebra becomes a lattice [10] when it satisfies absorption law. In Pre A* - algebra every maximal ideal is a prime ideal but there are prime ideals which are not maximal [8].

Pre A* - algebra is a distributive but in C- algebra ordinary right distributive law $\wedge$ over $\vee$ fails. But it has left distributive law $\vee$ over $\wedge$ C – algebra has absorption law. Pre A* - algebra is Commutative but C- algebra is not commutative. Pre A* - algebra instead of absorption law it has $x \wedge y = x \wedge (x^{\sim} \vee y), \forall x, y \in A,$ which is not in C- algebra and Boolean algebra.

CONCLUSION

Boolean Algebra is the centre of Pre A* - algebra and C-algebra.

Commutative law is in Boolean Algebra and Pre A*-algebra but not in C- algebra.

Absorption law is in Boolean Algebra and C-algebra but not in Pre A*-algebra.

complemented distributive Boolean algebra is a lattice but other two algebras are proved to be Lattices under certain conditions. Application of Boolean algebra is more in switching circuit theory. Whereas application of Pre A* - algebra and C-algebra in this field is not more. Pre A*-algebra and C-algebra are extension of Boolean algebra. Cayley theorem is proved for the Boolean Algebra, centre of C-algebra and Pre A*-algebra.

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