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Anupama Gupta

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n-NORMAL AND N-BINORMAL COMPOSITE CONVOLUTION OPERATORS WITH WEIGHT

Anupama Gupta

Department of Mathematics Government College for Women, Gandhi Nagar, Jammu, J&K, India

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ABSTRACT

This paper gives us some of the Characterizations of Binormal, Hyponormal, Quasinormal Composite Convolution Operators with Weight. The Conditions for n-Normal and n-Binormal Composite Convolution Operators with weight have been investigated. The Criterions for Composite Convolution Operators with Weight Minus Identity to be Isometric and Unitary are also presented.

INTRODUCTION

Let $L^p(\mu)$ denotes the collection of all measurable functions $f : X \to R$ (or C) such that $(\int_X |f(x)|^p \, d\mu)^{1/p} < \infty$. The space $L^p(X, S, \mu)$ is a Banach space under the norm defined by $\| f \|_p = (\int_X |f|^p \, d\mu)^{1/p}$. If p = 2, then $L^2(\mu)$ is Hilbert space and it is a space of square-integrable functions of complex numbers. By $B(L^2(\mu))$, we denote the Banach space of all bounded linear operators from $L^2(\mu)$ into itself. An operator $T \in B(H)$ is called normal if $T^* T = T T^*$, n-normal if $T^* T^n = T^n T^*$, T is binormal if $T^* T$ commutes with $T T^*$ and n-binormal if $T^* T^n$ commutes with $T^n T^*$. Again, an operator $T \in B(H)$ is known as hyponormal if $T^* T \geq T T^*$ and T is quasinormal if T commutes with T^*T. Let (X, Ω, μ) be a σ-finite measure space and $\phi: X \to X$ be a non-singular measurable transformation ($\mu(E) = 0 \Rightarrow \mu\phi^{-1}(E) = 0$). Then a composition transformation , for $1 \leq p < \infty$, $C_\phi : L^p(\mu) \to L^p(\mu)$ is defined by $C_\phi f = fo\phi$, for every $f \in L^p(\mu)$. In case C_ϕ is continuous, we call it a composition operator induced by ϕ. It is easy to see that C_ϕ is a bounded operator if and only if $\frac{d\mu\phi^{-1}}{d\mu} = f_o$, the Radon-Nikodym derivative of the measure $\mu\phi^{-1}$ with respect to the measure μ, is essentially bounded. For more detail about composition operator and weighted composition operators, we refer to Singh and Manhas [9], Campbell [6] and Takagi [5]. For each $f \in L^p(\mu)$, $1 \leq p < \infty$, there exists a unique $\phi^{-1}(\Omega)$ measurable function E(f) such that

$$\int g f \, d\mu = \int g E(f) \, d\mu,$$

Corresponding author:* **Anupama Gupta
Department of Mathematics Government College for Women, Gandhi Nagar, Jammu, J&K, India

for every $\phi^{-1}(\Omega)$ measurable function g for which left integral exists. The function E(f) is called conditional expectation of f with respect to the sub- algebra $\phi^{-1}(\Omega)$. For more properties of the expectation operator, we refer to Parthasarthy [7]. Given f, g $\in L^2(R)$, then convolution of f and g, $f_* g$ can be defined by

$$f_*g(x) = \int g(x-y)f(y)\,d\mu(y),$$

where g is fixed, k(x,y) = g(x-y) is a convolution kernel and the integral operator defined by

$$Wf(x) = \int k(x-y)f(y)\,d\mu(y)$$

is known as Convolution operator. Suppose $\phi : [0,1] \rightarrow [0,1]$ is a measurable transformation, then

$$W_\phi f(x) = \int k(x-y)f(\phi(y))d\mu(y)$$
$$= \int k_\phi(x-y)\, f(y)d\mu(y)$$

is known as composite convolution operator induced by pair (k,ϕ), where

$$k_\phi(x-y) = E(f_o(y)k(x-y)\,\phi^{-1}(y)).$$

Suppose $u : X \rightarrow \mathbb{C}$ is a measurable function. Then the bounded operator $W_{u,\phi}$ defined by

$$W_{u,\phi} f(x) = \int u(x)k(x-y)f(\phi(y))d\mu(y)$$
$$= \int k_{u,\phi}(x-y)\, f(y)d\mu(y),$$

is known as composite convolution operators with weight, where

$$k_{u,\phi}(x-y) = u(x)E(k(x-\phi^{-1}(y))\, f_o(y)) = u(x)E(k_x(\phi^{-1}(y))\, f_o(y)).$$

The composite convolution operators with weight are a class of operators which consists of composite convolution operators and multiplication operators. For literature related to the convolution operators we refer to Stepanov ([11],[12]), Bloom and Kerman [9], Halmos and Sunder [7]. Lybic's [13] conjecture was introduced by Whitley [14] and generalized it to Volterra composition operators on $L^p[0,1]$. Gupta and Komal ([1], [2]), Gupta [3] also studied composite integral operators and composite convolution operators. In this paper the characterizations of quasinormal, binormal and hyponormal composite convolution operators with weight are explored. The criterions for n-normal and n-binormal composite convolution operator with weight are also studied. The conditions for isometry and unitary composite convolution operator with weight minus identity operator are also characterized.

Quasinormal, Binormal and Hyponormal Composite Convolution Operators with Weight

In this section the characterizations of quasinormal, binormal and hyponormal composite convolution operators with weight have been investigated.

Theorem: Let $W_{u,\phi} \in B(L^2(\mu))$. Then $W_{u,\phi}$ is quasinormal if and only if

$$\int_X \int_X \int_X k^*_{u,\phi}(x-y)\, k_{u,\phi}(y-z)\, k_{u,\phi}(z-t)\, d\mu(y)d\mu(z)d\mu(t) = \int_X \int_X \int_X k_{u,\phi}(x-y)\, k^*_{u,\phi}(y-z)\, k_{u,\phi}(z-t)d\mu(y)d\mu(z\, d\mu(t).$$

Proof: Suppose $W_{u,\phi}$ is quasinormal. Assume any measurable rectangle $E\times F$ of finite measure. Then, we have

$$\langle W^*_{u,\phi} W_{u,\phi} W_{u,\phi}\, \chi_E , \chi_F \rangle = \int_X \int_X \iint_{E\times F} k^*_{u,\phi}(x-y)\; k_{u,\phi}(y - z)\; k_{u,\phi}(z-t)\, d\mu(t)\, d\mu(z)\, d\mu(y)\, d\mu(x)$$

and

$$\langle W_{u,\phi} W^*_{u,\phi} W_{u,\phi}\, \chi_E , \chi_F \rangle = \int_X \int_X \iint_{E\times F} k_{u,\phi}(x-y)\, k^*_{u,\phi}(y-z)\, k_{u,\phi}(z-t)d\mu(t)d\mu(z)d\mu(y)d\mu(x).$$

Thus, it follows that

$$\int_X k^*_{u,\phi}(x-y)\, k_{u,\phi}(y-z)\, k_{u,\phi}(z-t)d\mu(t)d\mu(z)d\mu(y)$$

$$= \int_X k_{u,\phi}(x-y)\, k^*_{u,\phi}(y-z)\, k_{u,\phi}(y-z)d\mu(t)d\mu(z)d\mu(y).$$

Conversely, if the condition is true then it is obvious that $W_{u,\phi}$ is a quasinormal operator.
Thus the proof of the theorem is complete.

Theorem: Let $W_{u,\phi} \in B(L^2(\mu))$. Then $W_{u,\phi}$ is hyponormal if and only if

$$\int_X k^*_{u,\phi}(x-y)\, k_{u,\phi}(y-z)d\mu(z)d\mu(y) \geq \int_X k_{u,\phi}(x-y)\, k^*_{u,\phi}(y-z)d\mu(z)d\mu(y).$$

Proof: Firstly, suppose that the given condition is true then it is obvious that $W_{u,\phi}$ is a hyponormal operator.
Conversely, if $W_{u,\phi}$ is hyponormal. Then for any measurable rectangle $E\times F$ of finite measure, we have

$$\langle W^*_{u,\phi} W_{u,\phi}\, \chi_E, \chi_F\rangle = \int_X \iint k^*_{u,\phi}(x-y)\, k_{u,\phi}(y-z)\chi_E(z)\, \chi_F(x)d\mu(z)d\mu(y)d\mu(x)$$

$$= \int_X \iint_{E\times F} k^*_{u,\phi}(x-y)\, k_{u,\phi}(y-z)d\mu(z)d\mu(y)d\mu(x)$$

and $$\langle W_{u,\phi} W^*_{u,\phi}\, \chi_E, \chi_F\rangle = \int_X \iint k_{u,\phi}(x-y)\, k^*_{u,\phi}(y-z)\chi_E(z)\, \chi_F(x)d\mu(z)d\mu(y)d\mu(x)$$

$$= \int_X \iint_{E\times F} k_{u,\phi}(x-y)\, k^*_{u,\phi}(y-z)d\mu(z)d\mu(y)d\mu(x).$$

Thus, it follows that

$$\int_X k^*_{u,\phi}(x-y)\, k_{u,\phi}(y-z)d\mu(z)d\mu(y) \geq \int_X k_{u,\phi}(x-y)\, k^*_{u,\phi}(y-z)d\mu(z)d\mu(y).$$

Thus the proof of the theorem is complete.

Theorem: Let $W_{u,\phi} \in B(L^2(\mu))$. Then $W_{u,\phi}$ is binormal if and only if

$$\iiint k^*_{u,\phi}(x-y)\, k_{u,\phi}(y-z)\, k_{u,\phi}(z-t)\, k^*_{u,\phi}(t-p)\; d\mu(y)\, d\mu(z)\, d\mu(t)$$

$$= \iiint k_{u,\phi}(x-y)\, k^*_{u,\phi}(y-z)\, k^*_{u,\phi}(x-y)k_{u,\phi}(y-z)\, d\mu(y)\, d\mu(z)\, d\mu(t).$$

Proof: Firstly, suppose the condition is true. For $f, g \in L^2(\mu)$, we have

$$\langle W^*_{u,\phi} W_{u,\phi} W_{u,\phi} W^*_{u,\phi} f, g\rangle = \iint [(k^*_{u,\phi}\, x-y)(W_{u,\phi} W_{u,\phi} W^*_{u,\phi} f)(y)\, d\mu(y)]\, \bar{g}(x)d\mu(x)$$

$$= \iint k^*_{u,\phi}(x-y)(\int k_{u,\phi}(y-z)\, (W_{u,\phi} W^*_{u,\phi} f)(z)d\mu(z))d\mu(y)\, \bar{g}(x)d\mu(x)$$

$$= \iiint (k^*_{u,\phi}\, x-y)\, k_{u,\phi}(y-z)\, (\int k_{u,\phi}(z-t)\, (W^*_{u,\phi} f)(t)\, d\mu(t))\, d\mu(z)d\mu(y)\; \bar{g}(x)d\mu(x)$$

$$= \iiiint k^*_{u,\phi}(x-y)\, k_{u,\phi}(y-z)\, k_{u,\phi}(z-t) \int k^*_{u,\phi}(t-p)f(p)d\mu(p)d\mu(t)d\mu(z)\, d\mu(y)\, \bar{g}(x)d\mu(x).$$

$$= \iiiiint k^*_{u,\phi}(x-y)\, k_{u,\phi}(y-z)\, k_{u,\phi}(z-t)\, k^*_{u,\phi}(t-p)f(p)d\mu(p))d\mu(t)d\mu(z)\, d\mu(y)\, \bar{g}(x)d\mu(x).$$

$$= \iiiiiint k^*_{u,\phi}(x-y)\, k_{u,\phi}(y-z)\, k_{u,\phi}(z-t)\, k^*_{u,\phi}(t-p)d\mu(y)d\mu(z)d\mu(t)f(p)d\mu(p)\, \bar{g}(x)d\mu(x) \quad (1)$$

and $\langle W_{u,\phi} W^{*}_{u,\phi} W^{*}_{u,\phi} W_{u,\phi} f, g\rangle = \int W_{u,\phi} W^{*}_{u,\phi} W^{*}_{u,\phi} W_{u,\phi} f(x) \bar{g}(x)d\mu(x)$

$$= \iint k_{u,\phi}(x-y) (I^{*}_{k,\phi} I^{*}_{k,\phi} I_{k,\phi} f)(y) d\mu(y)) \bar{g}(x)d\mu(x)$$

$$= \iiiint\!\!\int k_{u,\phi}(x-y) k^{*}_{u,\phi}(y-z) k^{*}_{u,\phi}(z-t) k_{u,\phi}(t-p) f(p) d\mu(p)) d\mu(t) d\mu(z) d\mu(y) \bar{g}(x)d\mu(x)$$

$$= \iiiint\!\!\int k_{u,\phi}(x-y) k^{*}_{u,\phi}(y-z) k^{*}_{u,\phi}(z-t) k_{u,\phi}(t-p) d\mu(y)d\mu(z)d\mu(t) f(p) \bar{g}(x)d\mu(x). \qquad (2)$$

It follows from the equations (1) and (2) that $W_{u,\phi}$ is binormal.

Conversely, suppose $W_{u,\phi}$ is binormal. Take $f = \chi_E$ and $g = \chi_F$, we see that from (1) and (2)

$$\int \int_E \int_F k^{*}_{u,\phi}(x-y) k_{u,\phi}(y-z) k_{u,\phi}(z-t) k^{*}_{u,\phi}(t-p) d\mu(y) d\mu(z) d\mu(t)$$

$$= \int \int_E \int_F k_{u,\phi}(x-y) k^{*}_{u,\phi}(y-z) k^{*}_{u,\phi}(z-t) k_{u,\phi}(t-p)d\mu(y) d\mu(z) d\mu(t)$$

for all $E, F \in S\times S$. Hence the required condition holds.

n- normal and n-binormal Composite Convolution Operators with Weights

The necessary and sufficient conditions for n-normal and n-binormal composite convolution operators with weight have been derived in this section.

Theorem: Let $W_{u,\phi} \in B(L^2(\mu))$. Then $W_{u,\phi}$ is n-normal if and only if

$$\int k^{*}_{\phi}(x-y) k^{n}_{\phi}(y-z) d\mu(y) = \int k^{n}_{\phi}(x-y) k^{*}_{\phi}(y-z) d\mu(y)$$

Proof: Firstly, suppose $W_{u,\phi}$ is n- normal. Then for any measurable rectangle $E \times F$ of finite measure, we have

$$\langle W^{*}_{u,\phi} W^{n}_{u,\phi} \chi_E, \chi_F \rangle = \int W^{*}_{u,\phi} W^{n}_{u,\phi} \chi_E(x) \chi_F(x)d\mu(x)$$

$$= \iiint k^{*}_{u,\phi}(x-y) k^{n}_{u,\phi}(y-z) \chi_E(y) \chi_F(x) d\mu(z)d\mu(y) d\mu(x)$$

$$= \iint_{E\times F} \int k^{*}_{u,\phi}(x-y) k^{n}_{u,\phi}(y-z) d\mu(y) d(\mu\times\mu)$$

and similarly

$$\langle W^{n}_{u,\phi} W^{*}_{u,\phi} \chi_E, \chi_F \rangle = \iint_{E\times F} \int k^{n}_{u,\phi}(x-y) k^{*}_{u,\phi}(y-z) d\mu(y) d(\mu \times \mu)$$

Hence, the condition follows.

Conversely, if the condition is true, then $W_{u,\phi}$ is n- normal as the proof is straight forward.

Theorem: Let $W_{u,\phi} \in B(L^2(\mu))$. Then $W_{u,\phi}$ is n-binormal if and only if

$$\iiint k^{*}_{u,\phi}(x-y) k^{n}_{u,\phi}(y-z) k^{n}_{u,\phi}(z-t) k^{*}_{u,\phi}(t-p) d\mu(y) d\mu(z) d\mu(t)$$

$$= \iiint k^{n}_{u,\phi}(x-y) k^{*}_{u,\phi}(y-z) k^{*}_{u,\phi}(z-t) k^{n}_{u,\phi}(t-p) d\mu(y)d\mu(z) d\mu(t).$$

Proof: Firstly, suppose the condition is true. For $f, g \in L^2(\mu)$, we have

$$\langle W^{*}_{u,\phi} W^{n}_{u,\phi} W^{n}_{k,\phi} W^{*}_{u,\phi} f, g\rangle = \int (W^{*}_{u,\phi} W^{n}_{u,\phi} W^{n}_{u,\phi} I^{*}_{k,\phi} f)(x) \bar{g}(x) d\mu(x)$$

$$= \iint [k^*_{u,\phi}(x-y)(W^n_{u,\phi} W^n_{u,\phi} I^*_{k,\phi} f)(y)\, d\mu(y)]\, \bar{g}(x) d\mu(x)$$
$$= \iint k^*_{u,\phi}(x-y)(\int k^n_{u,\phi}(y-z)(W^n_{u,\phi} I^*_{k,\phi} f)(z) d\mu(z)) d\mu(y)\, \bar{g}(x) d\mu(x)$$
$$= \iiint k^*_{u,\phi}(x-y)\, k^n_{\phi}(y-z)\, (\int k^n_{\phi}(z-t)\, (I^*_{k,\phi} f)(t) d\mu(t)) d\mu(z) d\mu(y)\, \bar{g}(x) d\mu(x)$$
$$= \iiiint k^*_{u,\phi}(x-y)\, k^n_{u,\phi}(y-z)\, k^n_{u,\phi}(z-t) \int k^*_{u,\phi}(t-p) f(p) d\mu(p) d\mu(t) d\mu(z) d\mu(y)\, \bar{g}(x) d\mu(x).$$
$$= \iiiiint k^*_{\phi}(x-y)\, k^n_{\phi}(y-z)\, k^n_{\phi}(z-t)\, k^*_{\phi}(t-p) f(p) d\mu(p)) d\mu(t) d\mu(z)\, d\mu(y)\, \bar{g}(x) d\mu(x).$$
$$= \iiiiint k^*_{u,\phi}(x-y)\, k^n_{u,\phi}(y-z)\, k^n_{u,\phi}(z-t)\, k^*_{u,\phi}(t-p)\, d\mu(y) d\mu(z) d\mu(t) f(p) d\mu(p)\, \bar{g}(x) d\mu(x) \quad (1)$$

and

$$\langle W_{u,\phi} W^*_{u,\phi} W^*_{u,\phi} W_{k,\phi} f, g \rangle = \int W_{u,\phi} W^*_{u,\phi} W^*_{u,\phi} W_{u,\phi} f(x)\, \bar{g}(x) d\mu(x)$$

$$= \iiiiint k^n_{u,\phi}(x-y)\, k^*_{u,\phi}(y-z)\, k^*_{u,\phi}(z-t)\, k^n_{u,\phi}(t-p)\, d\mu(y) d\mu(z) d\mu(t)\, f(p)\, \bar{g}(x) d\mu(x) \quad (2)$$

Hence, $W_{u,\phi}$ is n- binormal using equations (1) and (2).

Conversely, suppose $W_{u,\phi}$ is n-binormal. For $f = \chi_E$ and $g = \chi_F$, we get from (1) and (2)

$$\int \int_E \int_F k^*_{u,\phi}(x-y)\, k^n_{\phi}(y-z)\, k^n_{\phi}(z-t)\, k^*_{\phi}(t-p)\, d\mu(y)\, d\mu(z)\, d\mu(t)$$
$$= \int \int_E \int_F k^n_{u,\phi}(x-y)\, k^*_{u,\phi}(y-z)\, k^*_{u,\phi}(z-t)\, k^n_{u,\phi}(t-p) d\mu(y)\, d\mu(z)\, d\mu(t)$$

for all $E, F \in S\times S$. Hence the required condition holds.

Isometric and Unitary Composite Convolution Operators with Weight

In this section criterion for composite convolution operators with weight minus identity operator to be isometry and unitary are obtained for real valued kernel function $k_{u,\phi}$.

***Theorem**: Let $W_{u,\phi} \in B(L^2(\mu))$ and $k_{u,\phi}$ be real valued function. Then $W_{u,\phi} - I$ is an isometry on $L^2(\mu)$ if and only if $\int_X k_{u,\phi}(z-x)\, k_{u,\phi}(z-y) dz = k_{u,\phi}(x-y) + k_{u,\phi}(y-x)$.

Proof: For $W_{u,\phi} \in B(L^2(\mu))$, we have

$$(W_{u,\phi} - I)^* \circ (W_{u,\phi} - I) = I$$
$$(W^*_{u,\phi} - I) \circ (W_{u,\phi} - I) = I$$
$$W^*_{u,\phi} \circ W_{u,\phi} - W^*_{u,\phi} - W_{u,\phi} + I = I$$

This implies that

$$W^*_{u,\phi} \circ W_{u,\phi} = W^*_{u,\phi} + W_{u,\phi} \quad (1)$$

Now, for $f \in L^2(\mu)$, we have

$$W^*_{u,\phi} \circ W_{u,\phi} f(x) = \int_X k_{u,\phi}(z-x)\, W_{u,\phi} f(z)\, d\mu(z)$$
$$= \int_X \int_X k_{u,\phi}(z-x)\, k_{u,\phi}(z-y)\, f((y))\, d\mu(y)\, d\mu(z) \quad (2)$$

(using $k^*_{u,\phi}$ (x- z) = $k_{u,\phi}$(z - x), $k_{u,\phi}$ is real valued function)

Also,

$$(W^*_{u,\phi} + W_{u,\phi}) f(x) = \int_X [k_{u,\phi}(y- x) + k_{u,\phi}(x-y)] f(y) d\mu(y) \quad (3)$$

Thus from the equations (1), (2) and (3), we have

$$\int_X k_{u,\phi}(z -x) k_{u,\phi}(z- y)dz = k_{u,\phi}(x- y) + k_{u,\phi}(y -x)$$

This proves the theorem.

***Theorem**:* Let $W_{u,\phi} \in B(L^2(\mu))$ and k be real valued function. Then the following statements are equivalent:

1. $W_{u,\phi}$ - I is an unitary.
2. $W^*_{u,\phi}$ - I is an unitary.
3. $\int_X k_{u,\phi}(z - x) k_{u,\phi}(z - y)dz = \int_X k_{u,\phi}(x- z) k_{u,\phi}(y - z)dz$

   $= k_{u,\phi}(x- y) + k_{u,\phi}(y - x).$

Proof: The proof follows from theorem 4.1, since to prove that $W_{u,\phi}$ - I is an unitary, it is enough to show that $W_{u,\phi}$ - I is a surjective isometry.

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