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A STUDY OF LACUNARY INTERPOLATION OF DEGREE FIVE AND SIX BY G-SPLINES

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ABSTRACT

In this paper, we construct a special kind of g-splines, which are the solution of (0, 1, 3, 4)-interpolation problem : Let Δ: $0 = x_0 < x_1 < \ldots < x_{n-1} < x_n = 1$ be a partition of the interval I=[0,1] with $X_{k+1} - x_k = h_k$, $k = o(1)n - 1$ and a set of real numbers $\{f_k^{(q)}\}$, k=0(1)n where q = 0,1,3,4. Now we solve the problem and proves convergence theorems that satisty the theory of best approximation using spline polynomials of degree 5&6.

INTRODUCTION

Spline interpolation method, as applied to the solution of differential equation employ some from approximating function such as polynomials to approximate the solution by evaluating the function for sufficient number of points in the domain of the solution. Spline functions are a good tool for the numerical approximation of functions on one hand and they suggest new challenging and rewarding problem's on the other hand. Piecewise linear functions as well as step functions have along been important theoretical and practical tools for approximation of function. Lacunary interpolation by splines appears whenever observation gives scattered or irregular information about a function and its derivatives. The data in the problem of lacunary interpolation has also values of the functions and its derivatives but without Hermite conditions that only consecutive derivative is used at each node. Spline function are arise in many problems of mathematical Physics such as viscoelasticity, hydrodynamics, electromagnetic theory, mixed boundary problems in mathematical physics, biology and Engineering.

Th Fawzy ([3] [4]) constructed special kinds of lacunary quintic g-splines and proved that for functions $f \in C^{(4)}$ the method converges faster that investigated by A.K. Verma[1] and for functions $f \in C^{(5)}$ the order of approximation is the same as the best order of approximation using quintic g-splines. Saxena and Tripathi [7] have studied splines methods for solving the (0,1,3) interpolation problem. They have used spline interpolants of degree six for functions $\mathbf{f} \in \boldsymbol{C}^{(6)}$ to solved the problem. R.S.Misra and K.K. Mathur [2] solved lacunary interpolation by splines (0;0 2,3) and(0;0,2,4) cases. During the past twentieth both theories of splines and experiences with their use in numerical analysis have undergone a considerable degree of development. According to Fawzy [3] the interest in spline function is due to the fact that spline function are a good tool for the numerical approximation of functions. The collection of polynomials that form the curve of polynomials that form the curve is collectively referred to as "the spline". The traditional and constrained cubic splines are few different groups of the same family. The group of traditional cubic splines can furthermore be divided into sub group natural, parabolic, runout, cubic run-out and damped cubic splines. The natural cubic spline is by far the most popular and widely used version of the cubic splines family. Spline functions are used in many areas such as interpolation, data fitting, numerical solution of ordinary partial differential equation and also numerical solution of integral equations Lacunary interpolation by splines appears function about a function and its derivatives but without Hermite condition in which consecutive derivatives are used at each nodes, Several researchers have studied the use of spline to solve such interpolation [5 ,8 , 9 ,10 , 11] One uses polynomial for approximation because they can be evaluated. cubic spline interpolation is the most common piecewise polynomial method and is referred as "piecewise" since a unique polynomial is fitted between each pair of data points.

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In addition to the paper mentioned above dealing with best interpolation on approximation by splines there were also few papers that deal with constructive properties of space of splines interpolation. In my earlier work [6] [12] [13] [14] some kinds of lacunary interpolation by g-splines have been investigated. In this paper we will continue to discuss the problem.

This paper is organized as follows- In Section 2, we construct a lacunary interpolation (0,1,3, 4) through g-spline of degree six. In section 3, we construct almost quintic spline interpolant – (0, 1, 3, 4). we establish the convergence behavior of Interpolatory polynomials for $\mathbf{f} \in \boldsymbol{C}^{(5)}$ and $\mathbf{f} \in \boldsymbol{C}^{(6)}$ here we also define a Lemma and theorems about spline method converges faster than the earlier investigations.

spline interpolant (0, 1,3, 4) for $f \in C^{(6)}(I)$

LetΔ: $0 = x_0 < x_1 < \ldots < x_{n-1} < x_n = 1$
be a partition of the interval I = [0,1] with $X_{k+1} - x_k = h_k$, $k = o(1)n-1$.

And $s_{2,\Delta}$ be a piecewise polynomial of degree ≤ 6,which is a solution of (0,1,3,4)- interpolation for functions$f \in C^{(6)}[x_0, x_n]$ in the form :

(2.1) $s_{2,\Delta}(x) \equiv s_{2,k}(x) = \Sigma_{j=0}^{6} \frac{s_{k,j}^{(2)}}{j!}(x-x_k)^j, x_k \leq x \leq x_{k+1}$ for $k = 0(1)n-1$,

Where $s_{k,j}^{(2)}$, s are explicitly given below in terms of the prescribed data $\{f_k^{(j)}\}$, j = 0,1,3,4; K = 0(1)n

In particular, for j = 0,1,3,4

(2.2) $s_{k,j}^{(2)} = f_k^{(j)}$, k = 0(1) n-1 and for j = 2, 5, 6, we have

(2.3) $s_{k,2}^{(2)} = \frac{2}{h^3} [f_{k+1} - f_k - hf_k^{(1)} - \frac{h^3}{3!} f_k^{(3)} - \frac{h^4}{4!} f_k^{(4)} - \frac{h^5}{5!} s_{k,5}^{(2)} - \frac{h^6}{6!} s_{k,6}^{(2)}]$

and

(2.4) $s_k^{(5)} = \frac{114}{23} \frac{1}{h^5} [\{f_{k+1} - f_k - hf_k^{(1)} - \frac{h^3}{3!} f_k^{(3)} - \frac{h^4}{4!} f_k^{(4)}\} - \frac{h}{2}\left\{f_{k+1}^{(1)} - f_k^{(1)} - \frac{h^2}{2!} f_k^{(3)} - \frac{h^3}{3!} f_k^{(4)}\right\} + h^{(3)}\left\{f_{k+1}^{(3)} - f_k^{(3)} - hf_k^{(4)}\right\} - \frac{236}{6!} h^{(4)} (f_{k+1}^{(4)} - f_k^{(4)})]$

(2.5) $s_k^{(6)} = -\frac{288}{23} \frac{1}{h^6} [\{f_{k+1} - f_k - hf_k^{(1)} - \frac{h^3}{3!} f_k^{(3)} - \frac{h^4}{4!} f_k^{(4)}\} - \frac{h}{2}\left\{f_{k+1}^{(1)} - f_k^{(1)} - \frac{h^2}{2!} f_k^{(3)} - \frac{h^3}{3!} f_k^{(4)}\right\} + h^{(3)}\left\{f_{k+1}^{(3)} - f_k^{(3)} - hf_k^{(4)}\right\} - \frac{117}{2.5!} h^{(4)} (f_{k+1}^{(4)} - f_k^{(4)}\}]$

The coefficients $s_{k,j}^{(2)}$, j = 2,5,6 have been determined by the conditions :

That

$D_L^{(p)} s_{2,k}(x_{k+1}) = D_R^{(p)} S_{2,k+1}(x_{k+1})$, p = 0, 1, 3, 4; k = 0(1) n −1
Thus

$s_{2,\Delta} \epsilon C^{(0.1,3,4)}[I] = \{f : f^{(p)} \in C(I)\}$, p = 0, 1, 3, 4 }.
Is a unique piecewise polynomial of degree six satisfying interpolatory conditions (2.2).

Lemma 2.1

If f$\in C^{(6)}$ [I], then owing to (2.3) – (2.5) and using Taylor ' s expansion, we have

(2.6) $\left|s_{k,j}^{(2)} - f_k^{(j)}\right| \leq C_{k,j}^{(2)} h^{6-j} \omega(f^{(6)}; h)$, j = 2, 5, 6; k = 0(1)n − 1

Where the constant $C_{k,j}^{(2)}$ are given by :
$C_{k,6}^{(2)} = \frac{357}{115}$, $C_{k,5}^{(2)} = \frac{433}{920}$ and $C_{k,2}^{(2)} = \frac{303}{18400}$.

Proof

For j = 2, 5 and 6, Using Taylor's expansion from (2.1)-(2.5), we have

(2.7) $\left|s_{k,6}^{(2)} f_k^{(6)}\right| \leq \frac{357}{115} \omega(f^{(6)}; h)$,
(2.8) $\left|s_{k,5}^{(2)} - f_k^{(5)}\right| \leq \frac{433}{920} h\omega(f^{(6)}; h)$,
(2.9) $\left|s_{k,2}^{(2)} - f_k^{(2)}\right| \leq \frac{303}{18400} h^{(4)} \omega(f^{(6)}; h)$,
$C_{k,2}^{(2)} = \frac{303}{18400}$, $C_{k,5}^{(2)} = \frac{433}{920}$, $C_{k,6}^{(2)} = \frac{357}{115}$.

Theorem 2.1

Let $f \in C^{(6)}$ $(\boldsymbol{I})$ and $S_{2,\Delta} \in C^{(0,1,3,4)}$ [I] be the unique spline interpolant (0, 1,3, 4) given in (2.1) - (2.5),
Then

(2.10) $| D^{(j)} (f\text{-}S_{2,\Delta}) ||_{L_\infty}[x_k, x_{k+1}] \leq c_{2,k}^{j} h^{6-j} \omega(f^{(6)}, h)$, j =0(1) 6; k=0 (1) n-1

Where the constants $\mathbf{c}_{2,k}^{(j)}$'s are given by

$c_{2,k}^{(0)} = \frac{303}{18400}$, $c_{2,k}^{(1)} = \frac{6839}{110400}$, $c_{2,k}^{(2)} = \frac{12379}{55200}$, $c_{2,k}^{(3)} = \frac{831}{1104}$, $c_{2,k}^{(4)} = \frac{1861}{920}$ $c_{2,k}^{(5)} = \frac{3289}{920}$, $c_{2,k}^{(4)} = \frac{357}{115}$.

Proof

For k = 0(1)n-1 , j = 0(1)6
$| f(x) - S_{2,\Delta} | \leq | f(x) - S_k(x) |$
$\leq \Sigma_{j=0}^{5} \frac{|f^{(j)}(xk) - s_k^{(j)}| h^{(j)}}{j!} + \frac{|f^{(6)}(\delta k) - s_k^{(6)}| h^{(6)}}{6!}$

Where $x_k < S_k < x_{k+1}$ Using Lemma 2.1 and the definition of the modulus of continuity of $\boldsymbol{f}^{(6)}$**(x),** we obtain

(2.11) $|f_{(x)} - f_k(x)| \leq \frac{303}{18400} h^{(6)} \omega(f^{(6)}; h)$, (2.12) $\left|f^{(1)}(x) - S_k^{(1)}(x)\right| \le \frac{6839}{110400} h^{(5)} \omega\left(f^{(6)}; h\right),$

(2.13) $\left|f^{(2)}(x) - S_k^{(2)}(x)\right| \le \frac{12379}{55200} h^{(4)} \omega\left(f^{(6)}; h\right),$

(2.14) $\left|f^{(3)}(x) - S_k^{(3)}(x)\right| \le \frac{831}{1104} h^{(3)} \omega\left(f^{(6)}; h\right),$

(2.15) $\left|f^{(4)}(x) - S_k^{(4)}(x)\right| \le \frac{1861}{920} h^{(2)} \omega\left(f^{(6)}; h\right),$

(2.16) $\left|f^{(5)}(x) - S_k^{(5)}(x)\right| \le \frac{3289}{920} h \,\omega\left(f^{(6)}; h\right),$

(2.17) $\left|f^{(6)}(x) - S_k^{(6)}(x)\right| \le \frac{357}{115} \omega\left(f^{(6)}; h\right),$

Using (2.11)-(2.17), completes the Proof of the Theorem 2.1

Almost Quintic Spline Interpolant (0,1, 3, 4) * for f $\in C^{(5)}$ (I).

Almost quintic-spline interpolant (0,1,3, 4)* is a piecewise polynomial of degree 5 in each subinterval except in the last one, where it is a polynomial of degree 6. In this case, we have

(3.1) $S_{2,\Delta}^{*}(x) = S_{2,k}^{*}(x) = \sum_{j=0}^{5} \frac{S_{k,j}^{*(2)}}{j!}(x - x_k)^j$, $x_k \le x \le x_{k+1}$, $k = 0(1)n-2$

$=\sum_{j=0}^{6} \frac{S_{n-1,j}^{*(2)}}{j!}(x - x_{n-1})^j$, $x_{n-1} \le x \le x_n$, $k = n-1$

The coefficients $S_{k,j}^{*(2)}$ are explicitly given in terms of the data.
In particular , for K=O(1) n-1, we prescribe

(3.2) $S_{k,j}^{*(2)} = f_k^{(j)}$, j = 0,1, 3, 4 .

For k = 0(1)n-2 and j = 2, 5, $S_{k,j}^{*(2)}$ are given by
K=0(1)n-2, j=2,5

(3.3) $S_{k,2}^{*(2)} = \frac{20}{h^2} [\{f_{k+1} - f_k - \mathrm{h}f_k^{(1)} - \frac{h^3}{3!}f_k^{(3)} - \frac{h^4}{4!}f_k^{(4)}\} - \frac{h^2}{20} \{f_{k+1}^{(2)} - \mathrm{h}f_k^{(3)} - \frac{h^2}{2!}f_k^{(4)}\}]$

and

(3.4) $S_{k,5}^{*(2)} = \frac{40}{3} \frac{1}{h^5} [\{f_{k+1} - f_k - \mathrm{h}f_k^{(1)} - \frac{h^3}{3!}f_k^{(3)} - \frac{h^4}{4!}f_k^{(4)}\} - \frac{h^2}{2} \{f_{k+1}^{(2)} - \mathrm{h}f_k^{(3)} - \frac{h^2}{2!}f_k^{(4)}\}]$

K=n-1, j=2,5,6

(3.5) $S_{n-1,2}^{*(2)} = \frac{1}{h^2} [f_n - f_{n-1} - \mathrm{h}f_{n-1}^{(1)} - \frac{h^3}{3!}f_{n-1}^{(3)} - \frac{h^4}{4!}f_{n-1}^{(4)} - \frac{h^5}{5!} S_{n-1,5}^{*(2)} - \frac{h^6}{6!} S_{n-1,6}^{*(2)}]$

(3.6) $S_{n-1,5}^{*(2)} = \frac{114}{23} \frac{1}{h^5} [\{f_n - f_{n-1} - \mathrm{h}f_{n-1}^{(1)} - \frac{h^3}{3!}f_{n-1}^{(3)} - \frac{h^4}{4!}f_{n-1}^{(4)}\} - \frac{\mathrm{h}}{2} \left\{f_n^{(1)} - f_{n-1}^{(1)} - \frac{h^2}{2!}f_{n-1}^{(3)} - \frac{h^3}{3!}f_{n-1}^{(4)}\right\} + h^3\left\{f_n^{(3)} - f_{n-1}^{(3)} - \mathrm{h}f_{n-1}^{(4)}\right\} + \frac{236}{6!} h^4\{f_n^{(4)} - f_{n-1}^{(4)}\}]$

(3.7) $S_{n-1,6}^{*(2)} = \frac{-288}{23} \frac{1}{h^6} [\{f_n - f_{n-1} - \mathrm{h}f_{n-1}^{(1)} - \frac{h^3}{3!}f_{n-1}^{(3)} - \frac{h^4}{4!}f_{n-1}^{(4)}\} - \frac{\mathrm{h}}{2} \left\{f_n^{(1)} - f_{n-1}^{(1)} - \frac{h^2}{2!}f_{n-1}^{(3)} - \frac{h^3}{3!}f_{n-1}^{(4)}\right\} + h^3\left\{f_n^{(3)} - f_{n-1}^{(3)} - \mathrm{h}f_{n-1}^{(4)}\right\} - \frac{117}{2.5!} h^4\{f_n^{(4)} - f_{n-1}^{(4)}\}]$

Here, (3.3) and (3.4) are obtained from the condition.

(3.8) $S_{2,\Delta}^{*} \in C^{(1)}$ [I] ,

While (3.5)-(3.7) are determined from interpolatory conditions (3.2) for k = n-1 in (3.1).

Analogous to (2.6) for $\in C^{(5)}$ [I] , one can establish

(3.9) $| S_{k,j}^{*(2)} - f_k^{(j)}| \le C_{k,j}^{*(2)} h^{5-j} \omega\,(f^{(5)}, \mathrm{h})$,

Where the constants $C_{k,j}^{*(2)}$ are given by

$C_{k,j}^{*(2)} = \begin{cases} \frac{1}{54}, & j = 2 \\ \frac{10}{9}, & j = 5 \end{cases}$; k=0(1)n-2

$C_{k,j}^{*(2)} = \begin{cases} \frac{122}{17\ 25}, & j = 2 \\ \frac{336}{115}, & j = 5 \end{cases}$; for k=n-1

Finally, similar to theorem 2.1 , we have

Theorem 3.1

Let $f \in C^{(5)}$ [I] and $S_{2,\Delta}^{*}$ be the unique almost quintic spline interpolant (0, 1, 3, 4)* , given by (3.1) , then

(3.10) $\left\|D^{(j)}(f - S_{2,\Delta}^{*})\right\| L_\infty[x_k, x_{k+1}] \le C_{2,k}^{*(j)} h^{5-j} \omega\left(f^{(5)}, h\right),$

Where the constants $C_{2,k}^{*(j)}$ are given by

Table 1

	$C_{2,k}^{*(0)}$	$C_{2,k}^{*(1)}$	$C_{2,k}^{*(2)}$	$C_{2,k}^{*(3)}$	$C_{2,k}^{*(4)}$	$C_{2,k}^{*(5)}$
k=0(1)n-2	$\frac{1}{54}$	$\frac{7}{108}$	$\frac{1}{54}$	$\frac{5}{9}$	$\frac{10}{9}$	$\frac{10}{9}$
K=n-1	$\frac{46299}{476100}$	$\frac{32269}{158700}$	$\frac{23851}{39675}$	$\frac{183}{115}$	$\frac{366}{115}$	$\frac{366}{115}$

CONCLUSION

In this paper, we have studied the existence and uniqueness of the lacunary g-splines 0f degree six and almost quintic spline interpolant - (0,1,3,4).Also we conclude that this new technique which we have used in the proving of two important theorems which are solutions of (0,1,3,4)- interpolation and obtained their local approximations with functions belonging to $C^{(5)}$(I) and $C^{(6)}$(I). Our methods are of lower degree having better convergence property than the earlier investigations.

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