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RESEARCH ARTICLE

A LACUNARY INTERPOLATION WITH SPLINES OF DEGREE SIX

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ABSTRACT

In this paper, we consider a new technique of spline methods is used for (0, 3, 5)- lacunary interpolation by splines with functions belonging to $C^{(6)}(I)$ and Error bond, using piecewise polynomials with certain specific properties. Our methods show better convergence property than the earlier investigations.

INTRODUCTION

The collection of polynomials that form the curve of polynomials that form the curve is collectively referred to as " the spline". The traditional and constrained cubic splines are few different groups of the same family. The group of traditional cubic splines can furthermore be divided into sub group natural, parabolic, runout, cubic run-out and damped cubic splines. The natural cubic spline is by far the most popular and widely used version of the cubic splines family. Spline functions are used in many areas such as interpolation, datafitting, numerical solution of ordinary partial differential equation and also numerical solution of integral equations Lacunary interpolation by splines appears function about a function and its derivatives but without Hermite condition in which consecutive derivatives are used at each nodes, Several researchers have studied the use of spline to solve such interpolation [5 ,8 , 9 ,10 , 11] One uses polynomial for approximation because they can be evaluated. cubic spline interpolation is the most common piecewise polynomial method and is referred as "piecewise" since a unique polynomial is fitted between each pair of data points.

In recent, years, splines functions have arise in many problems of mathematical Physics, such as for solving differential Equations in hydrodynamics viscoelasticity, electromagnetic theory, mixed boundary problems in mathematical physics biology and Engineering.The spline interpolation is based on the following principle: The interpolation interval is divided into small subintervals. The polynomial coefficients are chosen to satisfy certain conditions. A "spline" was a common drafting tool a flexible rod, that was used to help draw smooth curves connecting widely spaced points. Spline interpolation method, as applied to the solution of differential equation employ some from approximating function such as polynomials to approximate the solution by evaluating the function for sufficient number of points in the domain of the solution.

Th Fawzy ([3] [4]) constructed special kinds of lacunary quintic g-splines and proved that for functions $f \in C^{(4)}$ the method converges faster that investigated by A.K. Verma[1] and for functions $f \in C^{(5)}$ the order of approximation is the same as the best order of approximation using quintic g-splines. Saxena and Tripathi [7] have studied splines methods for solving the (0,1,3) interpolation problem. They have used spline interpolants of degree six for functions $\underline{f} \in \underline{C^{(6)}}$ to solved the problem ..R.S.Misra and K.K. Mathur [2] solved lacunary interpolation by splines (0;0 2,3) and(0;0,2,4) cases. During the past twentieth both theories of splines and experiences with their use in numerical analysis have undergone a considerable degree of development. According to Fawzy [3] the interest in spline function is due to the fact that spline function are a good tool for the numerical approximation of functions.

In addition to the paper mentioned above dealing with best interpolation on approximation by splines there were also few papers that deal with constructive properties of space of splines interpolation. In my earlier work [6] [12] [13] some kinds of

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lacunary interpolation by g-splines have been investigated. In this paper we will continue to discuss the problem.

This paper is organized as follows- In Section 2, we construct a spline function of degree six which interpolates the lacunary data (0, 3, 5) In section 3 we establish the Error bond for interpolatary polynomials for $\mathbf{f} \in \boldsymbol{C}^{(6)}$ here we also define a Lemma and theorems about spline functions, by using some specific conditions , the method converges faster than the earlier investigations .

Construction Of The Spline Interpolant (0, 3, 5) FOR f ∈ $\boldsymbol{C}^{(6)}$ [I]

Let

$$\Delta:\ 0 = x_0 < x_1 < \ldots < x_{n-1} < x_n = 1$$

be a partition of the interval I = [0,1] with $X_{k+1} - x_k = h_k$, $k = o(1)n - 1$.

and

$S_{2,\Delta}$ be a piecewise polynomial of degree ≤ 6, which is a solution of (0, 3, 5)- interpolation for functions $f \in C^{(6)}[x_0, x_n]$ is given by :

(2.1) $S_{2,\Delta}(x) = s_{2,k}(x) = \Sigma_{J=0}^{6} \frac{s_{k,j}^{(2)}}{j!}(x - x_k)^j, \quad x_k \le x \le x_{k+1}$, $k = O(1)n - 1$,

Where $s_{k,j}^{(2)}$'s are explicitly given below in terms of the prescribed data

$\{f_k^{(j)}\}$, j = 0,3,5 ; k = 0(1)n.

In particular for k=0(1)n-1

(2.2) $s_{k,j}^{(2)} = f_k^{(j)}$, j = 0,3,5 for j = 1, 2, 4 ,6,

we have

(2.3) $s_{k,2}^{(2)} = \frac{2}{h^2} [\{f_{xk+1} - f_{xk} - \frac{h^3}{3!} f_{xk}^{(3)} - \frac{h^5}{5!} f_{xk}^{(5)}\} - h\left\{f_{(xk+1)}^{(1)} - \frac{h^2}{2!} f_{(xk)}^{(3)} - \frac{h^4}{4!} f_{xk}^{(5)}\right\} + \boldsymbol{h}^{(2)}\left\{f_{(xk+1)}^{(2)} - hf_{(xk)}^{(3)} - \frac{h^3}{3!} f_{(xk)}^{(5)}\right\} - \frac{3}{8} h^{(3)}\left\{f_{(xk+1)}^{(3)} - f_{(xk)}^{(3)} - \frac{h^2}{2!} f_{(xk)}^{(5)}\right\} + \frac{20}{6!} h^{(6)} S_{k,6}^{(2)}]$

(2.4) $s_{k,1}^{(2)} = \frac{1}{h} [\{f_{(xk+1)} - f_{(xk)} - \frac{h^3}{3!} f_{(xk)}^{(3)} - \frac{h^5}{5!} f_{(xk)}^{(5)} - \frac{h^2}{2!} S_{k,2}^{(2)} - \frac{h^4}{4!} s_{k,4}^{(2)} - \frac{h^6}{6!} S_{k,6}^{(2)}]$

(2.5) $S_{k,4}^{(2)} = \frac{24}{h^4} [\{f_{(xk+1)} - f_{(xk)} - \frac{h^3}{3!} f_{(xk)}^{(3)} - \frac{h^5}{5!} f_{(xk)}^{(5)}\} - h\left\{f_{(xk+1)}^{(1)} - \frac{h^2}{2!} f_{(xk)}^{(2)} - \frac{h^4}{4!} f_{(xk)}^{(5)}\right\} + \frac{h^2}{2} \{f_{(xk+1)}^{(2)} - hf_{(xk)}^{(2)} - \frac{h^3}{3!} f_{(xk)}^{(5)} - \frac{h^3}{12} \{f_{(xk+1)}^{(3)} - f_{(xk)}^{(3)} - \frac{h^2}{2!} f_{(xk)}^{(5)}\}]$

(2.6) $S_{k,6}^{(2)} = \frac{144}{h^6} [\ h\ \{ f_{(xk+1)}^{(1)} - \frac{h^2}{2!} f_{(xk)}^{(3)} - \frac{h^4}{4!} f_{(xk)}^{(5)} \} - \left\{f_{(xk+1)} - f_{(xk)} - \frac{h^3}{3!} f_{(xk)}^{(3)} - \frac{h^5}{5!} f_{(xk)}^{(5)}\right\} - \frac{h^2}{2}\left\{f_{(xk+1)}^{(2)} - hf_{(xk)}^{(3)} - \frac{h^3}{3!} f_{(xk)}^{(5)}\right\} + \frac{h^3}{8} \{f_{(xk+1)}^{(3)} - f_{(xk)}^{(3)} - \frac{h^2}{2!} f_{(xk)}^{(5)}\}]$

Error Bonds For Interpolatary Polynomials

Suppose $\mathbf{f} \in \boldsymbol{C}^{(6)}$ **[I],** then by the Tayor expansions, we establish the following lemma by using the modulus of continuity $\omega(f^{(6)}; h)$.

Lemma 3.1

For j = 1, 2, 4 and 6, we have

$$\left|S_{k,j}^{(2)} - f_k^{(j)}\right| \le C_{k,j}^{(2)} h^{6-j}\ \omega(f^{(6)}; h),$$

J = 1, 2, 4 and 6.
K = 0(1)n-1

Where the constants $C_{k,j}^{(2)}$ are given by:

$C_{k,1}^{(2)} = \frac{169}{225}$, $C_{k,2}^{(2)} = \frac{23}{72}$, $C_{k,4}^{(2)} = \frac{8}{15}$, $C_{k,6}^{(2)} = \frac{21}{5}$

Proof

For j = 1, 2, 4 and 6, Using Taylor's expansion from (2.1)-(2.6), we have

(3.1) $\left|S_{k,1}^{(2)} - f_k^{(1)}\right| \le \frac{169}{225} h^5\ \omega(f^{(6)}; h)$,

(3.2) $\left|S_{k,2}^{(2)} - f_k^{(2)}\right| \le \frac{23}{72} h^4\ \omega(f^{(6)}, h)$,

(3.3) $\left|S_{k,4}^{(2)} - f_k^{(4)}\right| \le \frac{8}{15} h^2\ \omega(f^{(6)}; h)$,

and

(3.4) $\left|S_{k,6}^{(2)} - f_k^{(6)}\right| \le \frac{21}{5}\ \omega(f^{(6)}; h)$,

This completes the Proof of the Lemma 3.1

Theorem 3.1

Let f □ $C^{(6)}$ $(\boldsymbol{I})$ and $S_{2,\Delta}$ □ $C^{(0,3,5)}$ [I] be the unique spline interpolant (0, 3, 5) given in (2.1) - (2.5),
Then

(3.5) $\| D^{(j)} (f - S_{2,\Delta}) \|_{L_\infty [x_k, x_{k+1}]} \le c_{2,k}^{j} h^{6-j} \omega(f^{(6)}, h)$, j =0(1) 6; k=0 (1) n-1

Where the constants $c_{2,k}^{(j)}$'s are given by:

$c_{2,k}^{(0)} = \frac{169}{180}$, $c_{2,k}^{(1)} = \frac{43}{36}$, $c_{2,k}^{(2)} = \frac{137}{180}$, $c_{2,k}^{(3)} = \frac{37}{30}$, $c_{2,k}^{(4)} = \frac{79}{30}$, $c_{2,k}^{(5)} = c_{2,k}^{(6)} = \frac{21}{5}$.

Proof

For k = 0(1)n-1 , j = 0(1)6

$$| f(x) - S_{2,\Delta} | \le | f(x) - S_k(x) |$$

$$\le \Sigma_{J=0}^{5} \frac{|f^{(j)}(xk) - s_k^{(j)}|\, h^{(j)}}{j!} + \frac{|f^{(6)}(\delta k) - s_k^{(6)}|\, h^{(6)}}{6!}$$

Where $x_k < S_k < x_{k+1}$ Using Lemma 3.1 and the definition of the modulus of continuity of $f^{(6)}$**(x),** we obtain the required result**.**

CONCLUSION

In this paper, we have studied the existence and uniqueness of (0 ,3 , 5) of degree six and Error bond for interpolatory polynomials for functions belonging to $C^{(6)}$(I). Our methods are having better convergence property Also we conclude that this new technique we used in proving of the Lemma and one important theorem of spline function is far more better than the earlier investigations.

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