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RESEARCH ARTICLE

[illegible]** CLOSED SETS INBITOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce α**-closed sets in bitopological spaces. Properties of these sets are investigated and seven new bitopological spaces namely, (i,j)- Tα**, (i,j)- αgTα**,(i,j)- gsTα**, (i,j) –gTα**, (i,j)- gspTα**,(i,j)- gpTα**,(i,j)- gprTα** are introduced.

INTRODUCTION

A triple (X, τ_i ,τ_j) where X is a non-empty set and τ_i and τ_j are topologies in X is called a bitopological space and Kelly**[7]** initiated the study of such spaces. Levine **[10]** introduced the class of generalized closed sets, a super class of closed sets in 1970. In 1985,Fukutake**[3]** introduced the concepts of g-closed sets in bitopological spaces. Veerakumar**[18]** introduced and studied the concepts of g*-closed sets and g*-continuity in topological spaces. Sheik John and Sundaram**[15]** introduced and studied the concepts of g*-closed sets in bitopological spaces in 2002.Pauline Mary Helen, *et.al***[14]** introduced g**-closed sets in topological spaces in 2012.In this paper we introduce the concepts of (i,j)-α**-closed sets, (i,j)- Tα** spaces, (i,j)- αgTα** spaces,(i,j)- gsTα** spaces, (i,j) - gTα** spaces, (i,j)- gspTα** spaces,(i,j)- gpTα** spaces,(i,j)- gprTα** spaces in bitopological spaces and investigate some of their properties.

Preliminaries

Definition

A subset A of a topological space (X,τ) is said to be

1. a pre-open set**[11]** if A⊆int(cl(A)) and a pre-closed set if cl(int(A)) ⊆A
2. a semi-open set **[9]** if A⊆ cl(int(A)) and a semi-closed set if int(cl(A)) ⊆A
3. a regular open set**[11]** if A=int(cl(A))
4. a generalized closed set**[10]** (briefly g-closed) if cl(A) ⊆ U whenever A ⊆ U and U is open in (X,τ).
5. an α-open set **[12]** if A ⊆int(cl(int(A))) and an α-closed set if cl(int(cl(A))) ⊆ A
6. a semi-preopen set**[1]** if A ⊆ cl(int(cl(A))) and a semi – preclosed set if int(cl(int(A))) ⊆ A.
7. an α*-closed set **[19]** if cl(A) ⊆ U whenever A ⊆ U and U is α-open in (X,τ).

If A is a subset of X with topology τ,then the closure of A is denoted by τ-cl(A) or cl(A),the interior of A is denoted by τ-int(A) or int(A) and the complement of A in X is denoted by A^c.

For a subset A of(X, τ_i , τ_j), τ_j-cl(A)(resp. τ_i -int(A)) denotes the closure (resp.interior)of A with respect to the topology τ_j.

Definition

A subset A of a topological space (X, τ_i , τ_j) is called

1. (i,j) –g-closed**[3]** if τ_j-cl(A) ⊆ U whenever A⊆ U and U is open in τ_i .
2. (i,j) –rg-closed **[13]** if τ_j-cl(A) ⊆ U whenever A⊆ U and U is regular open in τ_i .
3. (i,j) –gpr-closed **[5]** if τ_j-pcl(A) ⊆ U whenever A⊆ U and U is regular open in τ_i .
4. (i,j) –ωg-closed **[4]** if τ_j-cl(τ_i –int(A)) ⊆ U whenever A⊆ U and U is open in τ_i .
5. (i,j) –ω-closed **[6]** if τ_j-cl(A) ⊆ U whenever A ⊆U and U is semi open in τ_i .
6. (i,j)-gs-closed**[17]** if τ_j-scl(A) ⊆ U whenever A⊆ U and U is open in τ_i .
7. (i,j)-gsp-closed**[2]** if τ_j-spcl(A) ⊆ U whenever A⊆ U and U is open in τ_i .
8. (i,j)-αg-closed**[17]** if τ_j-αcl(A) ⊆ U whenever A⊆ U and U is open in τ_i .
9. (i,j)- gα-closed**[8]**if τ_j-αcl(A) ⊆ U whenever A⊆ U and U is α-open in τ_i .
10. (i,j) – g*-closed **[15]**if τ_j-cl(A) ⊆ U whenever A ⊆U and U is g-open in τ_i .
11. (i,j) – α*-closed**[20]** if τ_j-cl(A) ⊆ U whenever A ⊆U and U is α-open in τ_i .

Definition

A bitopological space (X, τ_i , τ_j) is called

1. an (i,j)-$T_{1/2}$space**[3]** if every (i,j)-g-closed set is τ_j-closed.
2. an (i,j)- $T_b$space **[17]** if every (i,j)-gs-closed set is τ_j-closed.

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3. an (i,j)- $T_d$space **[17]** if every (i,j)-gs-closed set is (i,j)-g-closed.
4. an (i,j)- $_\alpha T_d$space **[3]** if every (i,j)-αg-closed set is (i,j)-g-closed.
5. an (i,j)- $_\alpha T_b$space **[17]** if every (i,j)-αg-closed set is τ_j –closed.

(i, j) - α closed sets**

Definition

A subset A of a topological space (X, τ_i , τ_j) is said to be an (i, j)- α** closed set if τ_j - cl(A) ⊆ U whenever A⊆U and U is α* open in τ_i.

We denote the family of all (i, j)- α** closed sets in(X, τ_i, τ_j) by α**C(i, j).

Remark

By setting $\tau_i = \tau_j$ in definition (3.1), an (i, j)- α** closed set is α** closed set.

Proposition

Every τ_j – closed subset of (X, τ_i, τ_j) is (i, j) - α** closed.
The converse of the above proposition is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, c}, X}, τ_2 = {φ, {a}, X}. The set A = {b} is (1, 2) - α** closed but not τ_2- closed.

Proposition

If A is (i, j) - α** closed and τ_i - α* open, then A is τ_j - closed.

Corollary

If A is (i, j) - α** closed and τ_i - α* open, then A is τ_j - α closed.

Corollary

If A is (i, j) - α** closed and τ_i - α* open, then A is τ_j - α* closed.

Theorem

Every (i, j) - α** closed set is (i, j) – g closed.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, b}, X}, τ_2 = {φ, {a}, X}.The set A = {a, c} is (1, 2) - g closed but not (1, 2) - α** closed.

Theorem

Every (i, j) - α** closed set is (i, j) – rg closed.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {a}, X}, τ_2 = {φ, {a}, {a, b}, X}. The set A = {a} is (1, 2) - rg closed but not (1, 2) - α** closed.

Theorem

Every (i, j) - α** closed set is (i, j) – gpr closed.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, b}, X}, τ_2 = {φ, {a}, X}. The set A = {b} is (1, 2) - gpr closed but not (1, 2) - α** closed.

Theorem

Every (i, j) - α** closed set is (i, j) – ωg closed.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, b}, X}, τ_2 = {φ, {a}, X}. The set A = {a} is (1, 2) - ωg closed but not (1, 2) - α** closed.

Theorem

Every (i, j) - α** closed set is (i, j) – gs closed.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, b}, X}, τ_2 = {φ, {a}, X}. The set A = {b} is (1, 2) - gs closed but not (1, 2) - α** closed.

Theorem

Every (i, j) - α** closed set is (i, j) – gp closed.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, b}, X}, τ_2 = {φ, {a}, X}. The set A = {b} is (1, 2) - gp closed but not (1, 2) - α** closed.

Theorem

Every (i, j) - α** closed set is (i, j) – αg closed.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, b}, X}, τ_2 = {φ, {a}, X}. The set A = {a, c} is (1, 2) - αg closed but not (1, 2) - α** closed.

Theorem

Every (i, j) - α** closed set is (i, j) – gsp closed.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, c}, X}, τ_2 = {φ, {a}, X}. The set A = {c} is (1, 2) - gsp closed but not (1, 2) - α** closed.

Remark

(i, j) - α** closedness is independent of (i, j) – gα closedness.

Example

Let X = {a, b, c}, τ_1 = {φ, {a}, X}, τ_2 = {φ, {a}, {a, b}, X}. The set A = {a, b} is (1, 2) – α** closed but not (1, 2) - gα closed.

Example

Let X = {a, b, c}, τ_1 = {φ,{a, b}, X}, τ_2 = {φ, {a}, X}. The set A = {b} is (1, 2) - gα closed but not (1, 2) - α** closed.

Remark

(i, j) - α** closedness is independent of (i, j) – ω closedness.

Example

Let X = {a, b, c}, τ_1 = {φ, {a}, {b}, {a, b}, X}, τ_2 = {φ, {a}, {a, b}, X}. The set A = {a, c} is (1, 2) - α** closed but not (1, 2) - ω closed.

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, b}, X}, τ_2 = {φ, {a}, X}. The set A = {a, c} is (1, 2) - ω closed but not (1, 2) - α** closed.

Theorem

If A,B Є α**C(i, j), then (AUB) Є α**C(i, j)

Remark

The intersection of two (i, j) - α** closed sets need not be (i, j) - α** closed.

Example

Let X = {a, b, c}, τ_1 = {φ, {a}, X}, τ_2 = {φ, {a}, {a, b}, X}. Let A = {a, b} and B = {a, c}. The sets A and B are (1, 2) - α** closed but A∩B = {a} is not (1, 2) - α** closed.

Remark

α**C(1, 2) is generally not equal to α**C(2, 1)

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, c}, X}, τ_2 = {φ, {a}, X}. The set A = {c} ∉ α**C(1, 2) but A = {c} Є α**C(2, 1). Therefore, α**C(2, 1) ≠ α**C(1, 2).

Theorem

If A is (i, j) - α** closed, then τ_j – cl(A)\ A contains no non-empty τ_i – α* - closed set.

The converse of the above theorem is not true as shown in the following example

Example

Let X = {a, b, c}, τ_1 = {φ, {b}, {c}, {b, c}, {a, c}, X}, τ_2 = {φ, {a}, {b, c}, X}. The set A = {b} is not (1, 2) - α** closed. τ_2 – cl(A)\ A = {b, c}\{b} = {c} which is not τ_1- α – closed. Therefore, τ_2 – cl(A)\ A contains no non-empty τ_1- α – closed set and A = {b} is not (1, 2) – α** closed.

Theorem

If A is (i, j) - α** closed in (X, τ_i, τ_j), then A is τ_j – closed if and only if τ_j – cl(A)\ A is τ_i – α* - closed.

Theorem

If A is an (i, j) - α** closed set of (X, τ_i, τ_j) such that A ⊆ B ⊆τ_j – cl(A), then B is also an (i, j) - α** closed set of (X, τ_i, τ_j)

Theorem

For each element 'x' of (X, τ_i, τ_j), {x} is either τ_i – α* - closed or X – {x} is (i, j) - α** closed

Theorem

Every (i, j) - g* closed set is (i, j) - α** closed.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {a}, X}, τ_2 = {φ, {a}, {a, b}, X}. The set A = {b} is (1, 2) - α** closed but not (1, 2) - g* closed.

Remark

(i, j) - α** closedness is independent of (i, j) - α* closedness.

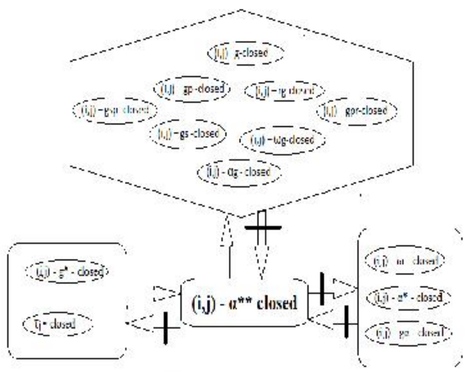
Example

Let X = {a, b, c}, τ_1 = {φ, {a}, X}, τ_2 = {φ, {a}, {a, b}, X}. The set A = {b} is (1, 2) - α** closed but not (1, 2) - α* closed.

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, b}, X}, τ_2 = {φ, {a}, X}. The set A = {a, c} is (1, 2) – α* closed but not (1, 2) - α** closed.

The following figure gives the results we have proved.



Here A ⇒ B represents A implies B and A ⇏ B represents A does not imply B

Applications of (i , j) - α closed sets**

As applications of (i, j) - α** closed sets, we introduce seven new bitopological spaces, namely, (i, j) - T_α** space, (i, j) - $_{\alpha g}$ T_α** space, (i, j) - $_{gs}$ T_α** space, (i, j) - $_g$ T_α** space, (i, j) - $_{gsp}$ T_α** space, (i, j) - $_{gp}$ T_α** space and (i, j) - $_{gpr}$ T_α** space.

We introduce the following definitions.

Definition

A bitopological space (X, τ_i, τ_j) is said to be an (i, j) - T_α** space, if every (i, j) - α** closed set is τ_j – closed.

Definition

A bitopological space (X, τ_i, τ_j) is said to be an (i, j) - $_{\alpha g}$ T_α** space, if every(i, j) – αg closed set is (i, j) - α** closed.

Definition

A bitopological space (X, τ_i, τ_j) is said to be an (i, j) - $_{gs}T_{\alpha}^{**}$ space, if every(i, j) – gs closed set is (i, j) - α^{**} closed.

Definition

A bitopological space (X, τ_i, τ_j) is said to be an (i, j) - $_{g}T_{\alpha}^{**}$ space, if every(i, j) – g closed set is (i, j) - α^{**} closed.

Definition

A bitopological space (X, τ_i, τ_j) is said to be an (i, j) - $_{gsp}T_{\alpha}^{**}$ space, if every(i, j) –gsp closed set is (i, j) - α^{**} closed.

Definition

A bitopological space (X, τ_i, τ_j) is said to be an (i, j) - $_{gp}T_{\alpha}^{**}$ space, if every(i, j) – gp closed set is (i, j) - α^{**} closed.

Definition

A bitopological space (X, τ_i, τ_j) is said to be an (i, j) - $_{gpr}T_{\alpha}^{**}$ space, if every(i, j) – gpr closed set is (i, j) - α^{**} closed.

Theorem

Every (i, j) - $T_{1/2}$ space is an (i, j) - T_{α}^{**} space.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, b}, X}, τ_2 = {φ, {a}, X}. Here all the (1, 2) - α^{**} closed sets are τ_2-closed. ∴ (X, τ_1, τ_2) is an (1, 2) - T_{α}^{**} space. The set A = {a, c} is (1, 2) –g closed but not τ_2- closed. Hence (X, τ_1, τ_2) is not a (1, 2)- $T_{1/2}$ space.

Theorem

Every (i, j) - T_b space is an (i, j) - T_{α}^{**} space.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, b}, X}, τ_2 = {φ, {a}, X}. Here all the (1, 2) - α^{**} closed sets are τ_2-closed. ∴ (X, τ_1, τ_2) is an (1, 2) - T_{α}^{**} space. The set A = {b} is (1, 2) –gs closed but not τ_2- closed. Hence (X, τ_1, τ_2) is not a (1, 2)- T_b space.

Theorem

A space which is both (i, j) - $_{\alpha}T_d$ and (i, j) - $T_{1/2}$ is an (i, j) - T_{α}^{**} space.

Theorem

Every (i, j) - $_{\alpha}T_b$ space is an (i, j) - T_{α}^{**} space.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, b}, X}, τ_2 = {φ, {a}, X}. Here all the (1, 2) - α^{**} closed sets are τ_2-closed. ∴ (X, τ_1, τ_2) is an (1, 2) - T_{α}^{**} space. The set A = {b} is (1, 2) –αg closed but not τ_2- closed. Hence (X, τ_1, τ_2) is not a (1, 2)- $_{\alpha}T_b$ space.

Theorem

Every (i, j) - T_b space is an (i, j) - $_{gs}T_{\alpha}^{**}$ space.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {a}, X}, τ_2 = {φ, {a}, {a, b}, X}. Here every (1,2) – gs closed set is (1, 2) - α^{**} closed. ∴ (X, τ_1, τ_2) is a (1, 2) - $_{gs}T_{\alpha}^{**}$ space. The set A = {b} is (1, 2) –gs closed but not τ_2- closed. Hence (X, τ_1, τ_2) is not a (1, 2)-T_b space.

Theorem

Every (i, j) - $_{\alpha}T_b$ space is an (i, j) - $_{\alpha g}T_{\alpha}^{**}$ space.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {a}, X}, τ_2 = {φ, {a}, {a, b}, X}. Here every (1,2) – αg closed set is (1, 2) - α^{**} closed. ∴ (X, τ_1, τ_2) is a (1, 2) - $_{\alpha g}T_{\alpha}^{**}$ space. The set A = {b} is (1, 2) – αg closed but not τ_2- closed. Hence (X, τ_1, τ_2) is not a (1, 2)- $_{\alpha}T_b$ space.

Theorem

Every (i, j) - $T_{1/2}$ space is an (i, j) - $_{g}T_{\alpha}^{**}$ space.

The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {a}, X}, τ_2 = {φ, {a}, {a, b}, X}. Here every (1, 2) – g closed set is (1, 2) - α** closed. ∴ (X, τ_1, τ_2) is a (1, 2) - $_{g}T_{\alpha}^{**}$ space. The set A = {b} is (1, 2) – g closed but not τ_2- closed. Hence (X, τ_1, τ_2) is not a (1, 2)- $T_{1/2}$ space.

Theorem

A space is both (i, j) - $_{g}T_{\alpha}^{**}$ space and (i, j) - T_{α}^{**} space if and only if it is an (i, j) - $T_{1/2}$ space.

Theorem

A space (X, τ_i, τ_j) which is both (i, j) - $_{gs}T_{\alpha}^{**}$ space and (i, j) - T_{α}^{**} space is an (i, j) - T_b space.

Theorem

A space (X, τ_i, τ_j) which is both (i, j) - $_{\alpha g}T_{\alpha}^{**}$ space and (i, j) - T_{α}^{**} space is an (i, j) - $_{\alpha}T_b$ space.

Theorem

Every (i, j) - $_{\alpha g}T_{\alpha}^{**}$ space is an (i, j) - $_{g}T_{\alpha}^{**}$ space.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, c}, X}, τ_2 = {φ, {a}, X}. Here every (1, 2) – g closed set is (1, 2) - α^{**} closed. ∴ (X, τ_1, τ_2) is a (1, 2) - $_{g}T_{\alpha}^{**}$ space. The set A = {c} is (1, 2) – αg closed but not (1, 2) - α^{**} closed. Hence (X, τ_1, τ_2) is not a (1, 2)-$_{\alpha g}T_{\alpha}^{**}$ space.

Theorem

Every (i, j) - $_{gs}T_{\alpha}^{**}$ space is an (i, j) - $_{g}T_{\alpha}^{**}$ space.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, c}, X}, τ_2 = {φ, {a}, X}. Here every (1, 2) – g closed set is (1, 2) - α^{**} closed. ∴ (X, τ_1, τ_2) is a (1, 2) - $_{g}T_{\alpha}^{**}$ space. The set A = {c} is (1, 2) – gs closed but not (1, 2) - α^{**} closed. Hence (X, τ_1, τ_2) is not a (1, 2)-$_{gs}T_{\alpha}^{**}$ space.

Theorem

Every (i, j) - $_{gp}T_{\alpha}^{**}$ space is an (i, j) - $_{g}T_{\alpha}^{**}$ space.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, c}, X}, τ_2 = {φ, {a}, X}. Here every (1, 2) – g closed set is (1, 2) - α^{**} closed. ∴ (X, τ_1, τ_2) is a (1, 2) - $_{g}T_{\alpha}^{**}$ space. The set A = {c} is (1, 2) – gp closed but not (1, 2) - α^{**} closed. Hence (X, τ_1, τ_2) is not a (1, 2)-$_{gp}T_{\alpha}^{**}$ space.

Theorem

Every (i, j) - $_{gsp}T_{\alpha}^{**}$ space is an (i, j) - $_{g}T_{\alpha}^{**}$ space.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, c}, X}, τ_2 = {φ, {a}, X}. Here every (1, 2) – g closed set is (1, 2) - α^{**} closed. ∴ (X, τ_1, τ_2) is a (1, 2) - $_{g}T_{\alpha}^{**}$ space. The set A = {c} is (1, 2) – gsp closed but not (1, 2) - α^{**} closed. Hence (X, τ_1, τ_2) is not a (1, 2)-$_{gsp}T_{\alpha}^{**}$ space.

Theorem

Every (i, j) - $_{gpr}T_{\alpha}^{**}$ space is an (i, j) - $_{g}T_{\alpha}^{**}$ space.
The converse of the above theorem is not true as shown in the following example.

Example

Let X = {a, b, c}, τ_1 = {φ, {c}, {a, c}, X}, τ_2 = {φ, {a}, X}. Here every (1, 2) – g closed set is (1, 2) - α^{**} closed. ∴ (X, τ_1, τ_2) is a (1, 2) - $_{g}T_{\alpha}^{**}$ space. The set A = {a} is (1, 2) – gpr closed but not (1, 2) - α^{**} closed. Hence (X, τ_1, τ_2) is not a (1, 2)-$_{gpr}T_{\alpha}^{**}$ space.

Theorem

A space which is both (i, j) - T_d and (i, j) - $T_{1/2}$ is an (i, j) - $_{gs}T_{\alpha}^{**}$ space.

Theorem

A space which is both (i, j) - T_d and (i, j) - $T_{1/2}$ is an (i, j) - T_{α}^{**} space.

Theorem

Every (i, j) - $_{gs}T_{\alpha}^{**}$ space is an (i, j) - T_d space.

Theorem

A space is both (i, j) - T_d space and (i, j) - $_{g}T_{\alpha}^{**}$ space if and only if it is an (i, j) - $_{gs}T_{\alpha}^{**}$ space.

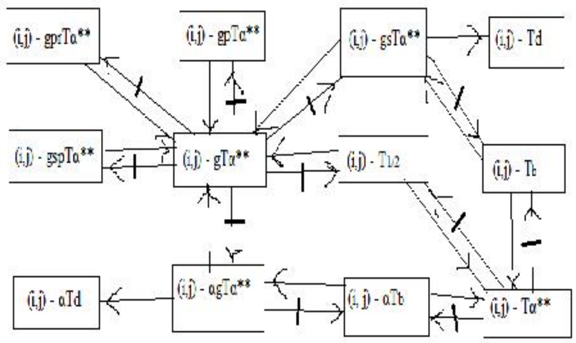
Theorem

Every (i, j) - $_{\alpha g}T_{\alpha}^{**}$ space is an (i, j) - $_{\alpha}T_d$ space.

Theorem

A space is both (i, j) - $_{\alpha}T_d$ space and (i, j) - $_{g}T_{\alpha}^{**}$ space if and only if it is an (i, j) - $_{\alpha g}T_{\alpha}^{**}$ space.

The following figure gives the results we have proved.



A → B represents A implies B and A ↛ B represents A does not imply B

References

1. Andrijevic. D, Semi-pre open sets, Mat.vesnik, 38(1)(1986),24-32.
2. Dontchev.J, On generalizing semi - preopensets,Mem.Fac.Sci.Kochi.Univ.Ser.A.Math.,16 (1995),35-48.
3. Fukutake.T, Bull, Fukuoka Univ.Ed. Part III,35(1985),19-28.
4. Fukutake. T,P. Sundaran and M. Sheik John, Bull, Fukuoka Univ. Ed. Part III,51(2002),1-9.
5. Fukutake. T,P. Sundaran and M. SheikJohn,ω-closed sets,ω-open sets and ω-continuity in bitopological spaces Bull, Fukuoka Univ.Ed. Part III,51(2002),1-9.
6. Fukutake. T,P. Sundaran and N. Nagaveni, Bull, Fukuoka Univ. Ed. Part III,48(1999),33-40.
7. Kelley.J.C,proc,London.Math,Sci,13(1963),71-89.
8. Khedr, F.H and Hanan S. AlSaddi, On pair wise semi generalized closed sets, JKAU; Sci, Vol 21 No.2, PP:269 – 295(2009 A.D/1430 AH); DOI 10.4197 Sci. 21-27
9. Levine. N, Generalized closed sets in topology,Rend,Circ.Math.Palermo,19(2)(1970)89-96
10. Levine. N, Semi-open sets and Semi continuity in topological spaces, Amer. Math. Monthly, 70(1963),36-41
11. Mashhour. A.S, M.E.Abd El-Mon sef and S.N.El-Deeb, On Pre-continuous and weak-pre continuous

mappings, proc. Math and Phys.Soc.Egypt,53(1982),47-53.
12. Njastad.O, On some classes of nearly open sets, Pacific J.Math.,15(1965),961-970.
13. Palaniappan.N and K.C.Rao,Regular generalized closed sets,KyungpookMath.J,33(2)(1993),211-219.
14. Pauline Mary Helen. M, Ponnuthai Selvarani, Veronicavijayan,g**-closed sets in topological spaces,IJMA,3(5)(2012),1-15.
15. Sheik John.M and P. Sundaram,g*-closed sets in bitopological spaces, Indian J.Pure,Appl.Math,35(1)(2004),71-80.
16. Stone. M,Trans. Amer.Math.Soc,41(1937),374-481.
17. Tantawy.O.A.El and H.M.Abu Donia, Generalized
18. separation Axioms in bitopological spaces. The Arabian Jl for science and Engg.Vol.30.No.1A(2005),117-129.
19. Veerakumar. M.K.R.S, Mem. Fac.Sci. Kochi. Univ(Math),21(2000),1-19.
20. Veronica Vijayan, Abarna.A, α* closed sets in bitopological spaces, IJCA, Issue 3, Volume 4(July-August 2013), ISSN: 2250-1797, PP : 30-39
21. Veronica Vijayan, Priya.F.,α*-closed sets in topological spaces, IJCA, Issue 3, Volume 4(July-August 2013), ISSN: 2250-1797, PP : 49-60

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