RELATIONAL STRUCTURE OF S-FUZZY SOFT SUBGROUPS

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ABSTRACT

In this paper, soft subgroup structure under fuzzy techniques over s-norm has been discussed. By using s-norm of S, we characterize some basic properties of S-fuzzy soft subgroups and Cartesian product of normal subgroups. Also, we define the relational concept of S-fuzzy soft subgroups and prove some basic properties.

INTRODUCTION

Soft set is a parameterized general mathematical tool which deals with a collection of approximate descriptions of objects. Each approximate description has two parts, a predicate and an approximate value set. In classical mathematics, a mathematical model of an object is constructed and defines the notion of exact solution of this model. The origin of soft set theory could be traced from the work of Pawlak [7] in 1993 entitled hard and soft set in Proceeding of the international E Work shop on rough sets and discovery at Banff. His notion of soft sets is a unified view of classical, rough and fuzzy Sets. In order to solidify the theory of soft set, P.K. Maji et al., [5] in 2002, defined some basic terms of the theory such as equality of two soft sets, subsets and super set of a soft set, complement of a soft set, null soft set, and absolute soft set with examples. This motivated D. Molodtsov’s work [6] in 1999 titled soft set Theory first results. The notion of a fuzzy subset of a set is due to Lotfi Zadeh [8]. At present this concept has been applied to many mathematical branches, such as group, functional analysis, probability theory, topology, and so on. The notion of fuzzy subgroup was introduced by A. Rosenfeld et al [3], [5] in his pioneering paper. In fact many basic properties in group theory are found to be carried over to fuzzy groups. In 1979 Anthony and Sherwood [2] redefined a fuzzy subgroup of a group using the concept of triangular norm (t-norm, for short).

In this paper, S-fuzzy soft subgroup structure has been analyzed and discussed some related properties.

Section-2 Preliminaries

Definition 2.1: Let $G_1$ and $G_2$ be two arbitrary groups with a multiplication binary operations and identities $e_1$, $e_2$ respectively. A fuzzy subset of $G_1 \times G_2$ is defined as a function from $G_1 \times G_2$ into $[0,1]$. The set of all fuzzy subsets of $G_1 \times G_2$ is called the $[0,1]$-power set of $G_1 \times G_2$ and is denoted by $[0,1]^{G_1 \times G_2}$.

Definition 2.2: By an s-norm $S$, we mean a function $S: [0,1] \times [0,1] \rightarrow [0,1]$ satisfying the following conditions;

(S1) $S(x, 0) = x$  
(S2) $S(x, y)$ is continuous for all $x, y \in [0,1]$. 
(S3) $S(x,y) = S(y,x)$ for all $x, y \in [0,1]$. 
(S4) $S(x, S(y,z)) = S(x,y) + S(y,z)$ for all $x, y, z \in [0,1]$.

Proposition 2.3: For an s-norm $S$, then the following statements holds $S(x,y) \geq \sup \{x, y\}$, for all $x, y \in [0,1]$. We say that $S$ is idempotent if for all $x \in [0,1]$, $S(x,x) = x$.

Example 2.4: The basic s-norms are:

- $Sm(x,y) = \sup \{x, y\}$
- $Sb(x,y) = \inf \{0, x+y - 1\}$ and
- $Sp(x,y) = xy$, which are called standard union, bounded sum, and algebraic product respectively.
**Definition 2.5**: A soft set $f_A$ over $U$ is defined as $f_A: E \rightarrow P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. 

Clearly, a soft set is not a set. For illustration, Molodtsov consider several examples in [6].

**Definition 2.6**: Let A be a fuzzy soft subset of a group $G_1 \times G_2$. Then A is called a fuzzy soft subgroup of $G_1 \times G_2$ under a s-norm $S$, (S-fuzzy soft subgroup) iff for all $(a,b), (c,d) \in G_1 \times G_2$

$S((a,b), (c,d)) \leq S((a,b), (c,d))$. 

**Lemma 3.3**: Let A be a fuzzy soft subset of a finite group $G_1 \times G_2$ and S be idempotent. If A satisfies condition (SFSG1) of Definition 2.1, then $A \in SFS(G_1 \times G_2)$.

**Proof**: Let $(a,b) \in G_1 \times G_2, (a,b) \neq (e_1, e_2)$.

Since $G_1 \times G_2$ is finite, $(a,b)$ has finite order, say $n > 1$.

So $(a,b)^n = (e_1, e_2)$ and $(a,b)^{n-1} = (a,b)$.

Now by using (SFSG1) repeatedly, we have that $A((a,b)^n) = (A((a,b)^{n-1}) = (A(a,b)^{n-2}, (a,b)) \leq S(A((a,b)^{n-1}), (a,b))$

$\leq S(A(a,b), (a,b), \ldots, (a,b))(n$-times) $= A(a,b)$.

**Lemma 3.4**: Let A be an S-fuzzy subgroup of $G_1 \times G_2$. If S be an idempotent. Then for all $(x,y) \in G_1 \times G_2$, and $n \geq 1$.

1. $A(x,y)^n \leq A((x,y)^n)$
2. $A(x,y) \leq A((x,y)^n)$
3. $A(x,y)^n \leq A((x,y)^n)$

**Proof**: Let $(a,b) \in G_1 \times G_2$ and $n \geq 1$.

1. $A(x,y)^n \leq A((x,y)^n)$
2. $A(x,y) \leq A((x,y)^n)$
3. $A(x,y)^n \leq A((x,y)^n)$

**Proposition 3.5**: Let A be a fuzzy soft subset of a group $G_1 \times G_2$ and (a,b) be a fuzzy soft subgroup. If S is idempotent, then $A \in SFS(G_1 \times G_2)$ if and only if $A(a,b) = (a,b)$. 

**Proof**: Suppose that $A(a,b) = (a,b)$, for all $(a,b) \in G_1 \times G_2$.

Then by letting $(a,b) = (c,d)$, we get that $(a,b) = (a,b)$. Conversely, suppose that $A(a,b) = (a,b)$. By lemma 3.5, we have

$A(a,b) \leq A((a,b))^n$, $A(a,b) \leq A((a,b))^n$, $A(a,b) \leq A((a,b))^n$.

**Example 3.6**: Let $Z_2 = \{e, a\}$ and A be a fuzzy soft set in $Z_2 \times Z_2$. Then $A(e,a)^n = A(e,a)$.

**Definition 7.1**: Let $\phi$ be a mapping from $G_1 \times G_2$ into $H_1 \times H_2$. A $\in \{0,1\}^{G_1 \times G_2}$ and $\alpha \in \{0,1\}^{H_1 \times H_2}$. By (D.S Mallik) $\phi(A) \in \{0,1\}^{H_1 \times H_2}$ and $\phi(A) \in \{0,1\}^{G_1 \times G_2}$, defined for all $(c,d) \in H_1 \times H_2$, $\phi(A, (c,d)) = \inf \{A(a,b), (a,b) \in G_1 \times G_2, (a,b) = (c,d)\}$ if $\phi^{-1}(c,d)$ is empty. Also for all $(a,b) \in G_1 \times G_2$, $\phi^{-1}(a,b) = \phi(a,b)$.

**Lemma 3.8**: Let $A \in SFS(G_1 \times G_2)$ and $H_1 \times H_2$ be a group. Suppose that $\phi$ is aepimorphism of $G_1 \times G_2$ into $H_1 \times H_2$. Then $\phi(A) \in SFS(H_1 \times H_2)$. 

$(A(0,0) = 0.5, A(0,1) = 0.5, A(1,0) = 0.5, A(1,1) = 0.5, A(2,0) = 0.5, A(2,1) = 0.5, A(0,0) = 0.6, A(0,1) = 0.6, A(1,0) = 0.6, A(1,1) = 0.6, A(2,0) = 0.6, A(2,1) = 0.6, A(0,0) = 0.7, A(0,1) = 0.7, A(1,0) = 0.7, A(1,1) = 0.7, A(2,0) = 0.7, A(2,1) = 0.7, A(0,0) = 0.8, A(0,1) = 0.8, A(1,0) = 0.8, A(1,1) = 0.8, A(2,0) = 0.8, A(2,1) = 0.8)$. 

respectively. If $S(a,b) = S((a,b), (c,d)) = \inf \{0, a+b-1\}$, for all $(a,b) \in Z_2 \times Z_2$, then $A(e,e), A(a,a), A(a,e) \leq S((a,b), (c,d))$. 

$(A(0,0) = 0.5, A(0,1) = 0.5, A(1,0) = 0.5, A(1,1) = 0.5, A(2,0) = 0.5, A(2,1) = 0.5, A(0,0) = 0.6, A(0,1) = 0.6, A(1,0) = 0.6, A(1,1) = 0.6, A(2,0) = 0.6, A(2,1) = 0.6, A(0,0) = 0.7, A(0,1) = 0.7, A(1,0) = 0.7, A(1,1) = 0.7, A(2,0) = 0.7, A(2,1) = 0.7, A(0,0) = 0.8, A(0,1) = 0.8, A(1,0) = 0.8, A(1,1) = 0.8, A(2,0) = 0.8, A(2,1) = 0.8)$. 

$33435 | Page$
Proof: Let \((a_1, \alpha_1), (b_1, \beta_1) \in H_1 \times H_2\) and \((a_2, c_2)(d) \in G_1 \times G_2\). If \((a_1, \alpha_2) \in G_1 \times G_2\) or \((\beta_1, \beta_2) \in \Phi(G_1 \times G_2)\), then \(\Phi(A)(a_1, \alpha_2) = \Phi(A)(\beta_1, \beta_2) = 0 \leq \Phi(A)((a_1, \alpha_2)(\beta_1, \beta_2))\).

Suppose \((a_1, \alpha_2) = \Phi(a, b)\) and \((\beta_1, \beta_2) = \Phi(c, d)\), then
\[
\Phi(A)(\alpha_2, \beta_2) = \inf \{ (A(a, b)(c, d) / (\alpha_1, \alpha_2) = \Phi(a, b) \text{and} (\beta_1, \beta_2) = \Phi(c, d) \}
\leq \inf \{ S(A(a, b), A(c, d) / (\alpha_1, \alpha_2) = \Phi(a, b) \text{and} (\beta_1, \beta_2) = \Phi(c, d) \}
= S(\inf \{ (A(a, b) / (\alpha_1, \alpha_2) = \Phi(a, b), \inf \{ A(c, d) / (\beta_1, \beta_2) = \Phi(c, d) \}
= S(\Phi(A)(\alpha_1, \alpha_2), \Phi(A)(\beta_1, \beta_2)).
\]
Also since \(A \in SFS(G_1 \times G_2)\), we have
\[
\Phi(A)(\alpha_2, \alpha_2^{-1}) = \Phi(A)(\alpha_1, \alpha_2).
\]

Lemma 3.9: Let \(H_1 \times H_2\) be a group and \(\alpha \in SFS(H_1 \times H_2)\). If \(\Phi\) be a epimorphism of \(G_1 \times G_2\) into \(H_1 \times H_2\), then \(\Phi^{-1}(A) \in SFS(G_1 \times G_2)\).

Proof: Let \((a, b), (c, d) \in G_1 \times G_2\). Then \(\Phi^{-1}(\alpha) ((a, b)(c, d)) = \alpha (\Phi(a, b) \Phi(c, d)) \leq S(\alpha(a, b), \alpha(c, d)) = S(\Phi^{-1}(a, b), \Phi^{-1}(c, d))\). The proof is completed.

CONCLUSION

Usually the mathematical model is too complicated and the exact solution is not easily obtained. So, the notion of approximate solution is introduced and the solution is calculated. By using s-norm of \(S\), we characterize some basic properties of \(S\)-fuzzy soft subgroups and cartesian product of normal subgroups. Also, we define the relational concept of \(S\)-fuzzy soft subgroups and proved some elementary aspects.

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