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F – INDEX OF GENERALIZED MYCIELSKIAN OF GRAPHS

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ABSTRACT

The F-index or forgotten topological index of a graph is defined as the sum of cubes of the vertex degrees of the graph. In this paper, we find F- index of generalized Mycielskian of graph and product graphs.

INTRODUCTION

For a graph G, the F-index or forgotten topological index is defined as the sum of cubes of the vertex degrees of the graph and it is denoted by F(G). That is, F(G) = $\sum_{v \in V(G)} d_G(v)^3$ where $d_G(v)$ denotes the degree of the vertices v in G. It can be easily shown that the above definition is equivalent to F(G) = $\sum_{uv \in E(G)} \left[d_G(u)^2 + d_G(v)^2 \right]$. This index was introduced in 1972, in the same paper where the first and second Zagreb indices were introduced to study the structure dependency of total electron in [10]. This index was reintroduced by Furtula B and Gutman I recently in 2015 [8]. After that, so many authors studied about this index.

De *et al*. gave some basic properties of this index and showed that this index can enhance the physico-chemical applicability of Zagreb index and found the F-index of different chemically interesting molecular graphs and nanostructures in [15]. Basavanagoud*et al*. computed the first Zagreb index, coindex and forgotten index of certain d-transformation graphs in [5]. Che*et al*. gave some lower and upper bounds of the forgotten index in terms of graphs irregularity, Zagreb indices, graph size, and maximum and minimum vertex degree in [21]. Akhter*et al.* determined the formulas for the F-index of four operation on graphs in [3]. Gao*et al.* gave the forgotten topological index of several widely used chemical structures which often appear in drug molecular graphs in [9]. Ghobadi *et al*. investigated first Zagreb index, F-index and F-coindex of the line graph of some chemical graphs using the subdivision concept in [11].

Jerline*et al*. gave the first and the second Zagreb indices of the generalized Mycielskian of a graph G, and the complements of $\mu_k(G)$, in terms of the order and size of the Graph G and obtained exact expressions for the first and the second Zagreb indices of the generalized Mycielskian of some graph operations in [1]. De *et al*. gave the Narumi-Katayama index of some derived graphs such as a Mycielski graph, subdivision graphs, double graph, extended double cover graph, thorn graph, subdivision vertex join and edge join graphs in [16]. Bindusree *et al*. obtained the First Zagreb index, First Zagreb Eccentricity index, Eccentric connectivity indices and polynomials and connective eccentricity index for the complement of Mycielski graphs in [7].

Mahdi *et al*. found the Gutman index of Mycielskian of graphs. They determined exact value of the Gutman index of the complement of arbitrary Mycielskian graphs in [14]. Behtoei*et al*. determined the degree distance of the complement of arbitrary Mycielskian graphs and also determined this graphical invariant for the Mycielskian of graphs with diameter two in

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[6]. Konchet *al*. determined the NK values of two derived graphs namely Mycielskigraph and thorn graph in [13].

In this work, we compute the F-index of Generalized Mycielskian of graphs and product graphs.

Preliminaries

We begin this section with some basic definitions. Let G be a simple undirected graph with n vertices and m edges. The complement of the graph G, denoted by $\bar{G}$, is the graph with the same vertices set as G, where any two distinct vertices are adjacent if and only if they are not adjacent in G. The degree of a vertex v of G is the number of edges adjacent to v and it is denoted by $d_G(v)$.

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple graphs. The **union** $\boldsymbol{G_1 \cup G_2}$ is the simple graph with $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$ is called the union of G_1 and G_2. When G_1 and G_2 are vertex disjoint, $G_1 \cup G_2$ is denoted by $\boldsymbol{G_1 + G_2}$ and it is called the **join** of the graphs G_1 and G_2. The **tensor product** $G_1 \times G_2$ is the simple graph with $V(G_1 \times G_2) = V_1 \times V_2$ as its vertex set and two vertices (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \times G_2$ if and only if, u_1 is adjacent to u_2 in G_1 and v_1 is adjacent to v_2 in G_2. Clearly, $|V(G_1 \times G_2)| = |V(G_1)||V(G_2| = n_1 n_2$ and $|E(G_1 \times G_2)| = 2|E(G_1)||E(G_2)| = 2m_1 m_2$. The **Cartesian product** $\boldsymbol{G_1 \square G_2}$ is the simple graph with $V(G_1 \times G_2) = V_1 \times V_2$ as its vertex set and two vertices (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \square G_2$ if and only if, either $u_1 = u_2$ and v_1 is adjacent to v_2 in G_2 or u_1 is adjacent to u_2 in G_2 and $v_1 = v_2$. Clearly $|V(G_1 \square G_2)| = |V(G_1)||V(G_2)| = n_1 n_2$ and $|E(G_1 G_2)| = |V(G_1)|E(G_2)| + |V(G_2)||E(G_1)| = n_1 m_2 + n_2 m_1$. The **lexicographic product** $\boldsymbol{G_1[G_2]}$ of two graphs G_1 and G_2 is the simple graph with $V(G_1[G_2]) = V_1 \times V_2$ as its vertex set and two vertices (u_1, v_1) and (u_2, v_2) are adjacent in $G_1[G_2]$ if and only if, either u_1 is adjacent to u_2 in G_1 or $u_1 = u_2$ and v_1 is adjacent to v_2 in G_2. Clearly $|V(G_1[G_2])| = |V(G_1)||V(G_2)| = n_1 n_2$ and $|E(G_1[G_2])| = |E(G_1)||V(G_2)|^2 + |E(G_2)||V(G_1)| = m_1 n_2^2 + m_2 n_1$. The **corona product** $\boldsymbol{G_1 \circ G_2}$ of twographs G_1 and G_2 is defined to be the graph obtained by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 and then joining the i^{th} copy of G_2, $i = 1,2,\ldots,|V(G_1)|$. Clearly $|V(G_1 \circ G_2)| = n_1(1 + n_2)$ and $|E(G_1 \circ G_2)| = m_1 + n_1 m_2 + n_1 n_2$. The **disjunction** $\boldsymbol{G_1 \vee G_2}$ of two graphs with $V(G_1 \vee G_2) = V_1 \times V_2$ as its vertex set and two vertices (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \vee G_2$ if and only if, either u_1 is adjacent to u_2 in G_1 or v_1 is adjacent to v_2 in G_2. Clearly $|V(G_1 \vee G_2)| = |V(G_1)||V(G_2)| = n_1 n_2$ and $|E(G_1 \vee G_2)| = |E(G_1)||V(G_2)|^2 + |E(G_2)||V(G_1)|^2 - 2|E(G_1)||E(G_2)| = m_1 n_2^2 + m_2 n_1^2 - 2m_1 m_2$.

Let G be a simple connected graph with n vertices and m edges. Let the n vertices of the given graph G be $v_1, v_2, \ldots, v_n$. For a graph $G = (V, E)$, the generalized Mycielskian, denoted by $\mu_k(G)$, of G is the graph whose vertex set is the disjoint union $V \cup \left(\bigcup_{i=1}^{k} V^i \right) \cup \{u\}$, where $V^i = \{x^i : x \in V\}$ is an independent set, $1 \le i \le k$, and edge set $E(\mu_k(G)) = E \cup_{i=1}^{k} \{y^{i-1}x^i; x^{i-1}y^i : xy \in E\}) \cup \{x^k u : x^k \in V^k\}$, where $x^0 = x$ and $y^0 = y$. By the definition of the generalized Mycielskian of G, we have the following observations.

- $|V(\mu_k(G))| = (k+1)n + 1$
- $|E(\mu_k(G))| = (2k+1)m + n$
- If $u^0v^0 \in E(G)$, then $u^0v^0, u^iv^{i+1}, u^{i+1}v^i \in E(\mu(G))$ for $0 \le i \le m-1$.
- $d_{\mu_k(G)}(v^i) = 2d_G(v); 0 \le i \le k-1$
- $d_{\mu_k(G)}(v^k) = d_G(v) + 1$ for all $v \in V(G)$
- $d_{\mu_k(G)}(u) = n$

For other notations in graph theory, please refer to [4].

Lemma 2.1[2] If G is a r-regular graph, then $M_1(G) = nr^2$

Lemma 2.2 If G is a r-regular graph, then $F(G) = nr^3$

Lemma 2.3 [17] If G is a simple connected graph with n vertices and m edges then

$$F(\mu_k(G)) = (8k+1)F(G) + 3M_1(G) + 6m + n(n^2+1)$$

Lemma 2.4 [18] If G is a simple connected graph with n vertices and m edges, then

$$F(\bar{G}) = n(n-1)^3 - 6m(n-1)^2 + 3(n-1)M_1(G) - F(G)$$

Lemma 2.5 If G is a simply connected graph with n vertices and m edges then

$$M_1(\bar{G}) = M_1(G) + (n-1)[n(n-1) - 4m].$$

F-index of Generalized Mycielskian Graph

In this section, we compute F-index of Generalized Mycielskian of graph complement and complement of Mycielskian graph.

Theorem 3.1 : IfG is a simple connected g*raph* with n vertices and m edges. Then

$$\begin{aligned} F(\mu_k(\bar{G})) = {} & (8k+1)n(n-1)^3 \\ & - 3(n-1)^2[2m(8k+1) - n] \\ & - F(G)(8k+1) \\ & +3M_1(G)[8kn - 8k + n] \\ & - 3(n-1)[4m - n] - 6m + n(n^2+1) \end{aligned}$$

Proof Let $\bar{n} = |V(\bar{G})| = n$ and $\bar{m} = |E(\bar{G})| = \frac{n(n-1)}{2} - m$.

By lemma 2.3, $F(\mu_k(\bar{G})) = (8k+1)F(\bar{G}) + 3M_1(\bar{G}) + 6\bar{m} + \bar{n}(\bar{n}^2 + 1)$

By lemma 2.4 and lemma 2.5.

$$\begin{aligned} & F(\mu_k(\bar{G})) \\ & = (8k+1)[n(n-1)^3 - 6m(n-1)^2 + 3(n-1)M_1(G) - F(G)] \\ & +3[M_1(G) + (n-1)[n(n-1) - 4m]] + 6\left[\frac{n(n-1)}{2} - m\right] \\ & + n(n^2+1) \\ & = (8k+1)n(n-1)^3 - (n-1)^2(6m(8k+1) - 3n) \\ & - F(G)(8k+1) \\ & +M_1(G)[3(n-1)(8k+1) + 3] \\ & - 12m(n-1) + 3n(n-1) - 6m \\ & + n(n^2+1) \\ & = (8k+1)n(n-1)^3 - (n-1)^2(6m(8k+1) - 3n) \\ & - F(G)(8k+1) \\ & +3M_1(G)[(n-1)(8k+1) + 1] \\ & - (n-1)[12m - 3n] - 6m + n(n^2+1) \end{aligned}$$

$$= (8k+1)n(n-1)^3 - 3(n-1)^2[2m(8k+1)-n] - F(G)(8k+1) + 3M_1(G)[8kn-8k+n] - 3(n-1)[4m-n] - 6m + n(n^2+1)$$

Corollary 3.2 If G is a r-regular graph then,

$$F(\mu_k(\bar{G})) = n[3r^2[8kn-8k+n] + (n-1)^3[r(8k+1)+1] - r^3(8k+1) - 3(n-1)[2r+1] - 3r + n^2 + 1]$$

Proof: Since G is r-regular, by the lemma 2.1,2.2 and by theorem 3.1

$$F(\mu_k(\bar{G})) = 3nr^2[8kn-8k+n] + (8k+1)n(n-1)^3 - (n-1)^2\left[6\frac{nr}{2}(8k+1) - 3n\right] - nr^3 (8k+1) - (n-1)\left[12\frac{nr}{2} - 3n\right] - 6\frac{nr}{2} + n(n^2+1)$$
$$= n[3r^2[8kn-8k+n] + (n-1)^3[r(8k+1)+1] - r^3(8k+1) - 3(n-1)[2r+1] - 3r + n^2 + 1]$$

Theorem 3.3 If G is a connected simple graph with n vertices and m edges. Then

$$F(\overline{\mu_k(G)}) = M_1(G)(12k^2n + 15kn + 3n - 3) + (8k+1)F(G) + n[(k+1)^4n^3 + (k+1)^3n^2 - 6(k+1)^2n(2mk+m-n) + 12m(k+1) + 3kn^2 + 3n] - 6m - n(n^2-1)$$

Proof Let $n^* = |V(\mu_k(G))| = (k+1)n + 1$ and $m^* = |E(\mu_k(G))| = (2k+1)m + n.$

By the lemma 2.3 & 2.4

$$F(\overline{\mu_k(G)}) = n^*(n^*-1)^3 - 6m^*(n^*-1)^2 + 3(n^*-1)M_1(\mu_k(G)) - F(\mu_k(G))$$
$$= [(k+1)n+1][(k+1)n+1-1]^3 - 6[(2k+1)m+n]$$
$$= [(k+1)n+1-1]^2 + 3[(k+1)+1-1][(4k+1)M_1(G) + 4m + n(n+1)]$$
$$-[(8k+1)F(G) + 3M_1(G) + 6m + n(n^2+1)]$$
$$= M_1(G)[12k^2n + 3kn + 12kn + 3n - 3] - (8k+1)F(G) - 6m - n(n^2+1)\, n[(k+1)^4 n^3 + (k+1)^3n^2 - (k+1)^2n(12mk + 6m - 6n) + 4m(3k+3) + 3kn^2 + 3n]$$
$$= M_1(G)(12k^2n + 15kn + 3n - 3) + (8k+1)F(G) + n[(k+1)^4n^3 + (k+1)^3n^2 - 6(k+1)^2n(2mk+m-n) + 12m(k+1) + 3kn^2 + 3n] - 6m - n(n^2-1)$$

Corollary 3.4. If G is a r-regular graph then,

$$F(\overline{\mu_k(G)}) = n[3r^2(4k^2n + 15kn - n - 3) - (8k+1)r^3 - [3n^2(k+1)^2(2k+1) + 6n(k+1) + 3] + (k+1)^4n^3 + (k+1)^3n^2 + (6(k+1)^2 + 3k - 1)n^2 + 3n - 1]$$

Proof Since G is a r-regular, by the lemma 2.1,2.2 and by the theorem 3.3,

$$F(\overline{\mu_k(G)}) = nr^2[12k^2n + 15kn + 3n - 3] - \left[(8k+1)nr^3 + 6\frac{nr}{2} + n(n^2+1)\right] + n[(k+1)^4n^3 + (k+1)^3n^2 - (k+1)^2n\left(12\frac{nr}{2}k + 6\frac{nr}{2} - 6n\right) + 4\frac{nr}{2}(3k+3) + 3k^2n + 3n]$$
$$= n[3r^2(4k^2n + 15kn - n - 3) - (8k+1)r^3 - [3n^2(k+1)^2(2k+1) + 6n(k+1) + 3] + (k+1)^4n^3 + (k+1)^3n^2 + (6(k+1)^2 + 3k - 1)n^2 + 3n - 1]$$

F-Index of Generalized Mycielskian of Graph Operations

Lemma4.1: [15] Let G_1 and G_2 be graphs. The F-index of union of G_1 and G_2 is given by

$F(G_1 \cup G_2) = F(G_1) + F(G_2).$

Lemma 4.2: Let G_1 and G_2 be graphs. The first Zagreb index of union of G_1 and G_2 is given by

$M_1(G_1 \cup G_2) = M_1(G_1) + M_1(G_2).$

Lemma4.3: [15] Let G_1 and G_2 be graphs. The F-index of tensor product of G_1 and G_2 is given by

$F(G_1 \times G_2) = F(G_1)F(G_2).$

Lemma 4.4: [19] Let G_1 and G_2 be graphs. The first Zagreb index of tensorproduct of G_1 and G_2 is given by

$M_1(G_1 \times G_2) = M_1(G_1)M_1(G_2).$

Lemma 4.5: [15] Let G_1 and G_2 be graphs. The F-index of Cartesian product of G_1 and G_2 isgiven by

$$F(G_1 \quad G_2) = n_2F(G_1) + n_1F(G_2) + 6m_2M_1(G_1) + 6m_1M_1(G_2).$$

Lemma 4.6: [12] Let G_1 and G_2 be graphs. The first Zagreb index of Cartesian product of G_1 and G_2 is given by

$$M_1(G_1 \square G_2) = n_1M_1(G_2) + n_2M_1(G_1) + 8m_1m_2.$$

Lemma 4.7: [15] Let G_1 and G_2 be graphs. The F-index of Lexicographicproduct of G_1 and G_2 is given by

$$F(G_1[G_2]) = n_2^4F(G_1) + n_1F(G_2) + 6n_2^2m_2M_1(G_1) + 6n_2m_1M_1(G_2).$$

Lemma 4.8: [12] Let G_1 and G_2 be graphs. The first Zagreb index of Lexicographic product of G_1 and G_2 is

given by $M_1(G_1[G_2]) = n_2^3M_1(G_1) + n_1M_1(G_2) + 8n_2m_1m_2.$

Lemma 4.9: [15] Let G_1 and G_2 be graphs. The F-index of corona product of G_1 and G_2 is given by

$$F(G_1 \circ G_2) = F(G_1) + n_1F(G_2) + 3n_2M_1(G_1) + 3n_1M_1(G_2) + 6n_2^2m_1 + 6n_1m_2 + n_1n_2(n_2^2+1).$$

Lemma 4.10: [20] Let G_1 and G_2 be graphs. The first Zagreb index of coronaproduct of G_1 and G_2 is given

by $M_1(G_1 \circ G_2) = M_1(G_1) + n_1M_1(G_2) + 4(n_2m_1 + n_1m_2) + n_1n_2(n_2+1).$

Lemma 4.11: [15] Let G_1 and G_2 be graphs. The F-index of disjunction product of G_1 and G_2 is given by

$$F(G_1 \vee G_2) = n_2^4 F(G_1) + n_1^4 F(G_2) - F(G_1)F(G_2) + 6n_1 n_2^2 m_2 M_1(G_1) + 6n_1^2 n_2 m_1 M_1(G_2) + 3n_2 F(G_1)M_1(G_2) + 3n_1 F(G_2)M_1(G_1) - 6n_2^2 m_2 F(G_1) - 6n_1^2 m_1 F(G_2) - 6n_1 n_2 M_1(G_1)M_1(G_2).$$

Lemma 4.12: [12] Let G_1 and G_2 be graphs. The first Zagreb index of disjunction product of G_1 and G_2 is given by

$$M_1(G_1 \vee G_2) = n_2(n_1n_2 - 4m_2)M_1(G_1) + n_1(n_1n_2 - 4m_1)M_1(G_2) + M_1(G_1)M_1(G_2) + n_1n_2m_1m_2.$$

Lemma 4.13: [15] Let G_1 and G_2 be graphs. The F-index of join of G_1 and G_2 is given by

$$F(G_1 + G_2) = F(G_1) + F(G_2) + 3n_2M_1(G_1) + 3n_1M_1(G_2) + 6n_2^2m_1 + 6n_1^2m_2 + n_1n_2^3 + n_2n_1^3.$$

Lemma 4.14: [12] Let G_1 and G_2 be graphs.The first Zagreb index of join of G_1 and G_2 is given by

$$M_1(G_1 + G_2) = M_1(G_1) + M_1(G_2) + n_1n_2[n_1 + n_2] + 4n_2m_1 + 4n_1m_2.$$

Theorem 4.15: Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple connected graphs such that $|V(G_1)| = n_1$, $|V(G_2)| = n_2$, $|E(G_1)| = m_1$ and $|E(G_2)| = m_2$. Then
$F(\mu_k(G_1 \cup G_2)) = F(\mu_k(G_1)) + F(\mu_k(G_2)) + 3n_1n_2(n_1 + n_2)$.

Proof: Clearly $n = |V(G_1 \cup G_2)| = n_1 + n_2$ and $m = |E(G_1 \cup G_2)| = m_1 + m_2$. By lemma 2.3, 4.1 and 4.2

$$F(\mu_k(G_1 \cup G_2)) = (8k+1)F(G_1 \cup G_2) + 3M_1(G_1 \cup G_2) + 6(m_1 + m_2) + (n_1 + n_2)((n_1 + n_2)^2 + 1)$$
$$= (8k+1)(F(G_1) + F(G_2)) + 3(M_1(G_1) + M_1(G_2)) + 6m_1 + 6m_2 + +(n_1 + n_2)(n_1^2 + n_2^2 + 2n_1n_2 + 1)$$
$$= (8k+1)F(G_1) + (8k+1)F(G_2) + 3M_1(G_1) + 3M_1(G_2) + 6m_1 + 6m_2 + n_1^3 + n_1n_2^2 + 2n_1^2n_2 + n_1 + n_1^2n_2 + n_2^3 + 2n_1n_2^2 + n_2$$
$$= (8k+1)F(G_1) + 3M_1(G_1) + 6m_1 + n_1^3 + n_1 + (8k+1)F(G_2) + 3M_1(G_2) + 6m_2 + n_2^3 + n_2 + 3n_1^2n_2 + 3n_1n_2^2$$
$$= (8k+1)F(G_1) + 3M_1(G_1) + 6m_1 + n_1(n_1^2 + 1) + (8k+1)F(G_2) + 3M_1(G_2) + 6m_2 + n_2(n_2^2 + 1) + 3n_1n_2(n_1 + n_2)$$
$$= F(\mu_k(G_1)) + F(\mu_k(G_2)) + 3n_1n_2(n_1 + n_2)$$

Theorem 4.16: Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple connected graphs such that $|V(G_1)| = n_1$, $|V(G_2)| = n_2$, $|E(G_1)| = m_1$ and $|E(G_2)| = m_2$. Then

$$F(\mu_k(G_1 \times G_2)) = (8k+1)F(G_1)F(G_2) + 3M_1(G_1)M_1(G_2) + 12m_1m_2 + n_1n_2(n_1^2n_2^2 + 1).$$

Proof: Clearly $n = |V(G_1 \times G_2)| = n_1n_2$ and $m = |E(G_1 \times G_2)| = 2m_1m_2$. By lemma 2.3, 4.3 and 4.4

$$F(\mu_k(G_1 \times G_2)) = (8k+1)F(G_1 \times G_2) + 3M_1(G_1 \times G_2) + 6(2m_1m_2) + (n_1n_2)((n_1n_2)^2 + 1)$$
$$= (8k+1)(F(G_1)F(G_2)) + 3(M_1(G_1)M_1(G_2)) + 12m_1m_2 + n_1n_2(n_1^2n_2^2 + 1)$$

Theorem 4.17: Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple connected graphs. Then

$$F(\mu_k(G_1 \square G_2)) = (8k+1)n_2F(G_1) + (8k+1)n_1F(G_2) + [(8k+1)6m_2 + 3n_2]M_1(G_1) + [(8k+1)6m_1 + 3n_1] + M_1(G_2) + 24m_1m_2 + 6(n_1m_2 + n_2m_1) + n_1n_2(n_1^2n_2^2 + 1).$$

Proof: Clearly $n = |V(G_1 \square G_2)| = n_1n_2$ and $m = |E(G_1 \square G_2)| = n_1m_2 + n_2m_1$. By lemma 2.3, 4.5 and 4.6

$$F(\mu_k(G_1 \square G_2)) = (8k+1)F(G_1 \square G_2) + 3M_1(G_1 \square G_2) + 6(n_1m_2 + n_2m_1) + n_1n_2(n_1^2n_2^2 + 1)$$
$$= (8k+1)\{n_2F(G_1) + n_1F(G_2) + 6m_2M_1(G_1) + 6m_1M_1(G_2)\} + 3\{n_1M_1(G_2) + n_2M_1(G_1) + 8m_1m_2\} + 6(n_1m_2 + n_2m_1) + n_1n_2(n_1^2n_2^2 + 1)$$
$$= (8k+1)n_2F(G_1) + (8k+1)n_1F(G_2) + [(8k+1)6m_2 + 3n_2]M_1(G_1) + [(8k+1)6m_1 + 3n_1]M_1(G_2) + 24m_1m_2 + 6(n_1m_2 + n_2m_1) + n_1n_2(n_1^2n_2^2 + 1)$$

Theorem 4.18 Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple connected graphs. Then

$$F(\mu_k(G_1[G_2]) = (8k+1)n_2^4F(G_1) + (8k+1)n_1F(G_2) + [(8k+1)6n_2^2m_2 + 3n_2^3]M_1(G_1) + [(8k+1)6n_2m_1 + 3n_1]M_1(G_2) + 24n_2m_1m_2 + 6(m_1n_2^2 + m_2n_1) + n_1n_2(n_1^2n_2^2 + 1)$$

Proof Clearly $n = |V(G_1[G_2])| = n_1n_2$ and $m = |E(G_1[G_2])| = m_1n_2^2 + m_2n_1$. By lemma 2.3, 4.7 and 4.8

$$F(\mu_k(G_1[G_2]) = (8k+1)F(G_1[G_2]) + 3M_1(G_1[G_2]) + 6(m_1n_2^2 + m_2n_1) + n_1n_2(n_1^2n_2^2 + 1)$$
$$= (8k+1)\{n_2^4F(G_1) + n_1F(G_2) + 6n_2^2m_2M_1(G_1) + 6n_2m_1M_1(G_2)\} + 3\{n_2^3M_1(G_1) + n_1M_1(G_2) + 8n_2m_1m_2\} + 6(m_1n_2^2 + m_2n_1) + n_1n_2(n_1^2n_2^2 + 1)$$
$$= (8k+1)n_2^4F(G_1) + (8k+1)n_1F(G_2) + [(8k+1)6n_2^2m_2 + 3n_2^3]M_1(G_1) + [(8k+1)6n_2m_1 + 3n_1]M_1(G_2) + 24n_2m_1m_2 + 6(m_1n_2^2 + m_2n_1) + n_1n_2(n_1^2n_2^2 + 1).$$

Theorem 4.19: Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple connected graphs. Then

$$F(\mu_k(G_1 \circ G_2)) = (8k+1)F(G_1) + n_1(8k+1)F(G_2) + [(8k+1)n_2 + 1]3M_1(G_1) + [(8k+1) + 1] 3n_1M_1(G_2) + 6(8k+1)n_2^2m_1 + 24(2k+1)n_1m_2 + (8k+1)n_1n_2^3 + (8k+11)n_1n_2 + 12n_2m_1 + 6m_1 + n_1(3n_2^2 + 1) + n_1^3[1 + 2n_2 + 3n_2^3 + n_2].$$

Proof Clearly $n = |V(G_1 \circ G_2)| = n_1(1 + n_2)$ and $m = |E(G_1 \circ G_2)| = m_1 + n_1m_2 + n_1n_2$. By lemma 2.3, 4.9 and 4.10

$$F(\mu_k(G_1 \circ G_2)) = (8k+1)F(G_1 \circ G_2) + 3M_1(G_1 \circ G_2) + 6[m_1 + n_1m_2 + n_1n_2] + [n_1(1 + n_2)][n_1^2(1 + n_2)^2 + 1]$$
$$= (8k+1)[F(G_1) + n_1F(G_2) + 3n_2M_1(G_1) + 3n_1M_1(G_2) + 6n_2^2m_1 + 6n_1m_2 + n_1n_2(n_2^2 + 1)] + 3\{M_1(G_1) + n_1M_1(G_2) + 4(n_2m_1 + n_1m_2) + n_1n_2(n_2 + 1)\} + 6[m_1 + n_1m_2 + n_1n_2] + [n_1(1 + n_2)][n_1^2(1 + n_2)^2 + 1]$$

$= (8k+1)F(G_1) + n_1(8k+1)F(G_2)$
$+ [(8k+1)n_2+1]3M_1(G_1)$
$+[(8k+1)+1]3n_1M_1(G_2) + S_1$

where $S_1 = 6(8k+1)n_2^2m_1 + 6(8k+1)n_1m_2 +$
$(8k+1)n_1n_2(n_2^2+1) + 12(n_2m_1+n_1m_2)$
$+3n_1n_2(n_2+1) + 6[m_1+n_1m_2+n_1n_2]$
$+ [n_1+n_1n_2][n_1^2(1+n_2)^2+1]$
$= 6(8k+1)n_2^2m_1 + 6(8k+1)n_1m_2 + (8k+1)n_1n_2^3$
$+ (8k+1)n_1n_2 + 12n_2m_1$
$+12n_1m_2 + 3n_1n_2^2 + 3n_1n_2 + 6m_1 + 6n_1m_2 + 6n_1n_2$
$+ [n_1+n_1n_2]$
$[n_1^2(1+2n_2+n_2^2)+1]$
$= 6(8k+1)n_2^2m_1 + 6(8k+1)n_1m_2 + (8k+1)n_1n_2^3$
$+ (8k+10)n_1n_2 + 12n_2m_1$
$+12n_1m_2 + 3n_1n_2^2 + 6m_1 + 6n_1m_2$
$+ [n_1+n_1n_2][n_1^2+2n_2n_1^2+n_1^2n_2^2+1]$
$= 6(8k+1)n_2^2m_1 + 24(2k+1)n_1m_2 + (8k+1)n_1n_2^3$
$+ (8k+10)n_1n_2 + 12n_2m_1$
$+3n_1n_2^2 + 6m_1 + n_1^3 + n_1^3n_2 + 2n_1^3n_2 + 2n_1^3n_2^2 + n_1^3n_2^2$
$+ n_1^3n_2^3 + n_1 + n_1n_2$
$= 6(8k+1)n_2^2m_1 + 24(2k+1)n_1m_2 + (8k+1)n_1n_2^3$
$+ (8k+11)n_1n_2 + 12n_2m_1$
$+3n_1n_2^2 + 6m_1 + n_1^3 + n_1^3n_2 + 2n_1^3n_2 + 2n_1^3n_2^2 + n_1^3n_2^2$
$+ n_1^3n_2^3 + n_1$
$= 6(8k+1)n_2^2m_1 + 24(2k+1)n_1m_2 + (8k+1)n_1n_2^3$
$+ (8k+11)n_1n_2 + 12n_2m_1$
$+6m_1 + n_1(3n_2^2+1) + n_1^3[1+3n_2+3n_2^3]$

$F(\mu_k(G_1 \circ G_2)) = (8k+1)F(G_1) + n_1(8k+1)F(G_2)$
$+ [(8k+1)n_2+1]3M_1(G_1) + [(8k+1)$
$+1]$
$3n_1M_1(G_2) + 6(8k+1)n_2^2m_1$
$+ 24(2k+1)n_1m_2 + (8k+1)n_1n_2^3$
$+ (8k+11)n_1n_2$
$+12n_2m_1 + 6m_1 + n_1(3n_2^2+1) + n_1^3[1+2n_2+3n_2^3+n_2]$

Theorem4.20 Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple connected graphs. Then

$F(\mu_k(G_1 \vee G_2)) = (8k+1)(n_2^2-6m_2)n_2^2F(G_1)$
$+ (8k+1)(n_1^2-6m_1)n_1^2F(G_2)$
$- (8k+1)F(G_1)F(G_2)$
$+(2n_1n_2^2m_2 + n_1n_2^2 - 4n_2m_2)(8k+1)3M_1(G_1)$
$+ (2n_2n_1^2m_1 + n_2n_1^2$
$- 4n_1m_1)(8k+1)3M_1(G_2)$
$+(8k+1)3n_2F(G_1)M_1(G_2) + (8k+1)3n_1F(G_2)M_1(G_1)$
$- (8k+1)6n_1n_2M_1(G_1)M_1(G_2)$
$+3n_1n_2m_1m_2 + 6(m_1n_2^2 + m_2n_1^2 - 2m_1m_2)$
$+ n_1n_2(n_1^2n_2^2+1).$

Proof Clearly $n = |V(G_1 \vee G_2)| = n_1n_2$ and $m = |E(G_1 \vee G_2)| = m_1n_2^2 + m_2n_1^2 - 2m_1m_2$. By lemma 2.3, 4.11 and 4.12

$F(\mu_k(G_1 \vee G_2)) = (8k+1)F(G_1 \vee G_2) + 3M_1(G_1 \vee G_2)$
$+ 6(m_1n_2^2 + m_2n_1^2 - 2m_1m_2) + n_1n_2(n_1^2n_2^2$
$+1)$
$= (8k+1)[n_2^4F(G_1) + n_1^4F(G_2) - F(G_1)F(G_2)$
$+ 6n_1n_2^2m_2M_1(G_1) + 6n_1^2n_2m_1M_1(G_2)$
$+3n_2F(G_1)M_1(G_2) + 3n_1F(G_2)M_1(G_1) - 6n_2^2m_2F(G_1)$
$- 6n_1^2m_1F(G_2)$
$-6n_1n_2M_1(G_1)M_1(G_2)] + 3\{n_2(n_1n_2 - 4m_2)M_1(G_1)$
$+ n_1(n_1n_2 - 4m_1)M_1(G_2)$
$+M_1(G_1)M_1(G_2) + n_1n_2m_1m_2\}$
$+ 6(m_1n_2^2 + m_2n_1^2 - 2m_1m_2)$
$+ +n_1n_2(n_1^2n_2^2+1)$
$= (8k+1)n_2^4F(G_1) + (8k+1)n_1^4F(G_2) - (8k$
$+1)F(G_1)F(G_2) + (8k+1)6n_1n_2^2m_2$
$M_1(G_1) + (8k+1)6n_1^2n_2m_1M_1(G_2)$
$+ (8k+1)3n_2F(G_1)M_1(G_2) + (8k+1)$
$3n_1F(G_2)M_1(G_1) - (8k+1)6n_2^2m_2F(G_1) - (8k$
$+1)6n_1^2m_1F(G_2) - (8k+1)6n_1n_2$
$M_1(G_1)M_1(G_2) + 3n_1n_2^2M_1(G_1) - 12n_2m_2M_1(G_1)$
$+ 3n_1^2n_2M_1(G_2) - 12n_1m_1M_1(G_2)$
$+3M_1(G_1)M_1(G_2) + 3n_1n_2m_1m_2 + 6m_1n_2^2 + 6m_2n_1^2$
$- 12m_1m_2 + n_1^3n_2^3 + n_1n_2$
$= (8k+1)(n_2^2-6m_2)n_2^2F(G_1)$
$+ (8k+1)(n_1^2-6m_1)n_1^2F(G_2)$
$- (8k+1)F(G_1)F(G_2)$
$+(2n_1n_2^2m_2 + n_1n_2^2 - 4n_2m_2)(8k+1)3M_1(G_1)$
$+ (2n_2n_1^2m_1 + n_2n_1^2$
$- 4n_1m_1)(8k+1)3M_1(G_2)$
$+(8k+1)3n_2F(G_1)M_1(G_2) + (8k+1)3n_1F(G_2)M_1(G_1)$
$- (8k+1)6n_1n_2M_1(G_1)M_1(G_2)$
$+3n_1n_2m_1m_2 + 6(m_1n_2^2 + m_2n_1^2 - 2m_1m_2)$
$+ n_1n_2(n_1^2n_2^2+1).$

Theorem 4.21 Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple connected graphs. Then

$F(\mu_k(G_1+G_2)) = (8k+1)F(G_1) + (8k+1)F(G_2)$
$+ (3n_2+3)M_1(G_1) + (3n_1+3)M_1(G_2)$
$+ 6\{(8k+1)n_2^2m_1 + (8k+1)n_1^2m_2$
$+ n_1^2n_2 + n_1n_2^2 + 2m_1n_2 + 2m_2n_1 + m_1$
$+ m_2$
$+n_1n_2\} + (8k+1)n_1n_2^3 + (8k+1)n_1^3n_2 + n_1^3 + n_2^3$
$+ n_1 + n_2$

Proof Clearly $n = |V(G_1+G_2)| = n_1 + n_2$ and $= |E(G_1+G_2)| = m_1 + m_2 + n_1n_2$. By lemma 2.3, 4.13 and 4.14

$F(\mu_k(G_1+G_2)) = (8k+1)[F(G_1) + F(G_2) + 3n_2M_1(G_1)$
$+ 3n_1M_1(G_2) + 6n_2^2m_1 + 6n_1^2m_2 + n_1n_2^3$
$+n_2n_1^3] + 3\{M_1(G_1) + M_1(G_2) + n_1n_2[n_1+n_2] + 4n_2m_1$
$+ 4n_1m_2\}$
$+6\{m_1+m_2+n_1n_2\} + (n_1+n_2)[(n_1+n_2)^2+1]$
$= (8k+1)F(G_1) + (8k+1)F(G_2) + (3n_2+3)M_1(G_1)$
$+ (3n_1+3)M_1(G_2) + S_2$

where $S_2 = (8k+1)6n_2^2m_1 + (8k+1)6n_1^2m_2 +$
$(8k+1)n_1n_2^3 + (8k+1)n_1^3n_2 + 3n_1n_2(n_1+n_2)$
$+12m_1n_2 + 12m_2n_1 + 6m_1 + 6m_2 + 6n_1n_2$
$+ (n_1+n_2)[n_1^2+n_2^2+2n_1n_2+1]$
$= (8k+1)6n_2^2m_1 + (8k+1)6n_1^2m_2 + (8k+1)n_1n_2^3$
$+ (8k+1)n_1^3n_2 + 3n_1^2n_2 + 3n_1n_2^2$
$+12m_1n_2 + 12m_2n_1 + 6m_1 + 6m_2 + 6n_1n_2 + n_1^3 + n_1^2n_2$
$+ n_1n_2^2 + n_2^3 + 2n_1^2n_2$
$+2n_1n_2^2 + n_1 + n_2$
$= (8k+1)6n_2^2m_1 + (8k+1)6n_1^2m_2 + (8k+1)n_1n_2^3$
$+ (8k+1)n_1^3n_2 + 6n_1^2n_2 + 6n_1n_2^2$
$+12m_1n_2 + 12m_2n_1 + 6m_1 + 6m_2 + 6n_1n_2 + n_1^3 + n_2^3 + n_1$
$+ n_2$
$= 6\{(8k+1)n_2^2m_1 + (8k+1)n_1^2m_2 + n_1^2n_2 + n_1n_2^2$
$+ 2m_1n_2 + 2m_2n_1 + m_1 + m_2 +$
$n_1n_2\} + (8k+1)n_1n_2^3 + (8k+1)n_1^3n_2 + n_1^3 + n_2^3 + n_1 + n_2$

$$F\big(\mu_k(G_1+G_2)\big) = (8k+1)F(G_1) + (8k+1)F(G_2) + (3n_2+3)M_1(G_1) + (3n_1+3)M_1(G_2) + 6\{(8k+1)n_2^2m_1 + (8k+1)n_1^2m_2 + n_1^2n_2 + n_1n_2^2 + 2m_1n_2 + 2m_2n_1 + m_1 + m_2 + n_1n_2\} + (8k+1)n_1n_2^3 + (8k+1)n_1^3n_2 + n_1^3 + n_2^3 + n_1 + n_2$$

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