ON THE NONIC DIOPHANTINE EQUATION WITH THREE UnknownS
8x^2+8y^2-15xy=32z^9

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**Abstract**

We obtain two different patterns of non-zero integral solutions of the Nonic Diophantine equation with three unknowns 8x^2+8y^2-15xy=32z^9 by employing suitable transformations.

**Method of Analysis**

The equation under consideration is

\[ 8x^2 + 8y^2 - 15xy = 32z^9 \]  

Assigning the transformations

\[ x = u + v, \quad y = u - v \]

in (1) leads to

\[ u^2 + 31v^2 = 32z^9 \]  

**Case 1**

Assumethatz = a^2 + 31b^2 \quad (4)

Write \[
2 = \frac{(n+ni\sqrt{3})+(n-ni\sqrt{3})}{n} \]

\[ a = a^9 - 279a^7b + 3246756a^5b^3 + 80724a^6b^4 + 121086a^6b^4 - 3753666a^6b^3 - 2502444a^6b^4 + 2604a^6b^5 + 121086a^6b^5 - 2502444a^6b^6 - 1072476a^6b^7 + 8311689a^7b^5 + 923521b^9 \]

Substituting the above values of u and v in equation (2), and hence the non-zero integral solutions of (1) are

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x = 2a^6 - 270a^b - 232a^b + 78120a^b + 242172a^b - 3632580a^b - 5004888a^b + 32174280a^b^2 + 16623378ab^2 - 27705630b^9 (6)

y = -288a^b + 83328a^b^2 - 3874752a^b^3 + 34319232a^b^4 - 29552672b^9 (7)

z = a^2 + 31b^2

Case 2

Equation (3) can be written as

\[ u^2 + 3 \left( \sqrt{2} \right)^2 = 2^9 + 1 \] (8)

Instead of (5), we write as

\[ \left( \frac{2 + \sqrt{2}}{2} \right) \left( \frac{2 - \sqrt{2}}{2} \right) = 2^4 \] (9)

and also 1 as

\[ \left( \frac{1 + \sqrt{2}}{2} \right) \left( \frac{1 - \sqrt{2}}{2} \right) = 2^{\frac{5}{6}} \] (10)

use (4), (10), (9) in (8) and applying the method of factorization, define

\[ u + i\sqrt{3}v = \frac{1}{2} \left[ (a + i\sqrt{3}b)^2 (2 + 2\sqrt{2}) (15 + i\sqrt{3}) \right] \] (11)

Equating the real and imaginary part, we have

\[ u = u(a,b) = -a^2 - 279a^b + 1116a^b^2 + 80724a^b^3 - 121086a^b^4 - 3753666a^b^5 + 2502444a^b^6 + 33246756a^b^7 - 8311689a^b^8 - 82692151b^9 \]

\[ v = v(a,b) = a^2 - 9a^b + 1116a^b^2 + 2604a^b^3 + 121086a^b^4 - 2502444a^b^5 + 33246756a^b^6 - 8311689a^b^7 + 82692151b^9 \]

Substituting the values of u and v in equ (2), then the values of x and y are given by

\[ x = -288a^b + 83328a^b^2 - 3874752a^b^3 + 34319232a^b^4 - 29552672b^9 \]

\[ y = -28a^6 - 270a^b + 232a^b^2 + 78120a^b^3 - 242172a^b^4 - 3632580a^b^5 + 5004888a^b^6 + 32174280a^b^7 - 16623378ab^2 + 27705630b^9 \]

\[ z = a^2 + 31b^2 \]

CONCLUSION

In this paper we have presented two different patterns of nonzero integral solutions of the Nonic Diophantine equation with three unknown (1). One may search for other patterns of solutions and their corresponding properties.

**How to cite this article:**


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