A SQUARE MULTIPLICATIVE LABELING FOR SOME FAMILIES OF GRAPHS

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INTRODUCTION

In Mathematics, Graph Theory is the study of graphs, which are mathematical structures that are being used to model pair wise relationship between objects. A graph is made up of vertices, nodes or points which are connected by edges, arcs or lines. The graphs that are taken in this paper are finite and undirected. A linear graph (or simply a graph) \( G = (V,E) \) consists of set of object \( V = \{v_1, v_2, \ldots, v_p\} \) called vertices and another set \( E = \{e_1, e_2, \ldots\} \), whose elements are called edges, such that each edge \( e_k \) is identified with an unordered pairs \( (v_i, v_j) \) of vertices. The symbols \( V(G) \) and \( E(G) \) are denoted as vertex set and edge set of a graph \( G \). The cardinality of the vertex set is called the order of \( G \), denoted by \( p \) and the cardinality of the edge set is called the size \( G \), denoted by \( q \). Hence the graph is referred to as \( (p,q) \) graph. Graph Theory is applied in various fields such as Physics, Chemistry, Biology, Computer Science, Operations Research, Social Networks, etc., Graph labeling is the assignment of integers from its vertices or edges subject to some certain conditions. A dynamic survey on graph labeling is regularly updated by Gallian [4] and it is published by electronic journal of combinatories. Some basic definitions and notations are taken from Bondy and Murthy [3], Beineke and Hedge [1] referred a graph with \( p \) vertices as strongly multiplicative if the vertices of \( G \) can be labeled with distinct integers \( 1, 2, \ldots, p \) such that the labels induced on the edges by the product of the end vertices are distinct. Shalini and Paul Dhayabaran [14] introduced a minimization of multiplicative graphs. Shalini, Visalatchi and Paul Dhayabaran [15] discussed square multiplicative labeling for disconnected graphs. In this paper, we studied some families of graphs which satisfy square multiplicative labeling.

Definition 1.1

A graph \( G \) is said to be square multiplicative labeling if there exists a bijection \( f : V(G) \rightarrow \{1, 2, \ldots, p\} \) such that the induced function \( f : E(G) \rightarrow N \) given by

\[ f(uv) = f(u)^2 \cdot f(v)^2 \]

for every \( uv \in E(G) \) are all distinct. A graph which admits square multiplicative labeling is called square multiplicative graph. In this paper, we have investigated some families of graphs which admit square multiplicative labeling.
A graph \( G \) is said to be \textbf{square multiplicative graph} if it admit a square multiplicative labeling.

**Definition 1.2**

A path in a simple graph is a finite (or) infinite sequence of edges which connect a sequences of vertices are all distinct from one another. A path cannot both start and terminate at the same vertex.

**Definition 1.3**

The double star graph \( (K_{1,n,n}) \) is a tree obtained from the star \( K_{1,n} \) by adding a new pendant edge of the exiting \( n \) pendant vertices. It consists of \( 2n+1 \) vertices and \( 2n \) edges.

**Definition 1.4**

The \textbf{comb} graph is defined as \( \{v_1,v_2,\ldots,v_n\} \) be a path graph with \( n \) vertices. It consists of \( 2n \) vertices and \( 2n-1 \) edges.

**Definition 1.5**

The graph \( (P_m,S_n) \) is obtained from \( m \) copies of the star graph \( S_n \) and the path \( P_m \) \( \{u_1,u_2,\ldots,u_m\} \) by joining \( u_j \) with the vertex \( v_j \) of the \( j^{th} \) copy of \( S_n \) by means of an edge, for \( 1 \leq j \leq m \).

**Square Multiplicative Labeling**

**Theorem 2.1**

A path is a square multiplicative graph for \( n \geq 2 \).

**Proof**

Let \( G \) be a graph of path \( P_n \).

Let \( \{v_1,v_2,\ldots,v_n\} \) be the vertices of \( P_n \) and \( \{e_1,e_2,\ldots,e_{n-1}\} \) be the edges of \( P_n \) which are denoted as in the fig. 2.1.

The path consists of \( n \) vertices and \( n-1 \) edges.

If \( G = P_n \) then \( |V(G)| = n \)

and \( |E(G)| = n-1 \).

Let us define a vertex labeling \( f: V(G) \rightarrow \{1,2,\ldots,p\} \) as follows

\[
f(v_j) = i; \quad 1 \leq i \leq n.
\]

And the edge function \( f: E(G) \rightarrow N \) defined by

\[
f(uv) = f(u)^2 \times f(v)^2 \quad \text{for every} \quad uv \in E(G) \quad \text{are all distinct.}
\]

Then the edges labels are distinct.

Then \( G \) is a square multiplicative graph.

(i.e) Hence, every path is a square multiplicative graph.

**Example 2.1**

![Path \( P_6 \)](image)

**Theorem 2.2**

Every double star \( (K_{1,n,n}) \) is a square multiplicative graph.

**Proof**

Let \( G \) be a graph of double star \( (K_{1,n,n}) \) with vertex set \( \{u,u_i,v_i; \quad 1 \leq i \leq n\} \) obtained from the .

Let \( \{u,u_1,u_2,\ldots,u_n\} \) and \( \{v_1,v_2,\ldots,v_n\} \) be the vertices and \( \{e_1,e_2,\ldots,e_{n-1}\} \) be the edges which are denoted as in the fig. 2.2.

The path consists of \( n \) vertices and \( n-1 \) edges.

If \( G = (K_{1,n,n}) \) then \( |V(G)| = 2n + 1 \)

and \( |E(G)| = 2n \).

Let us define a vertex labeling \( f: V(G) \rightarrow \{1,2,\ldots,p\} \) as follows

\[
f(u) = 1
\]

\[
f(u_i) = i + 1; \quad 1 \leq i \leq n
\]

\[
f(v_i) = n + i + 1; \quad 1 \leq i \leq n
\]

And the edge function \( f: E(G) \rightarrow N \) defined by

\[
f(uv) = f(u)^2 \times f(v)^2 \quad \text{for every} \quad uv \in E(G) \quad \text{are all distinct.}
\]

Then the edges labels are distinct.

Then \( G \) is a square multiplicative graph.

(i.e) Hence, every double star is a square multiplicative graph.
Example 2.2

**Theorem 2.3**

A comb is a square multiplicative graph for \( n \geq 3 \).

**Proof**

Let \( G \) be a graph of comb with vertex set \( \{u_i, v_i; \ 1 \leq i \leq n\} \) obtained from the path.

Let \( \{u_1, u_2, \ldots, u_n\} \) and \( \{v_1, v_2, \ldots, v_n\} \) be the vertices and \( \{e_1, e_2, \ldots, e_{n-1}\} \) be the edges which are denoted as in the fig. 2.3.

The path consists of \( n \) vertices and \( n - 1 \) edges.

If \( G = (P_n, S) \) then \( |V(G)| = 2n \) and \( |E(G)| = 2n - 1 \).

Let us define a vertex labeling \( f: V(G) \rightarrow \{1, 2, \ldots, p\} \) as follows

\[
\begin{align*}
  f(u_i) &= i; \quad 1 \leq i \leq n \\
  f(v_i) &= n + i; \quad 1 \leq i \leq n
\end{align*}
\]

And the edge function \( f: E(G) \rightarrow N \) defined by

\[
f(uv) = f(u)^2 \cdot f(v)^2\]

for every \( uv \in E(G) \) are all distinct.

Then the edges labels are distinct.

Then \( G \) is a square multiplicative graph.

*Hence, every comb is a square multiplicative graph.*

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Example 2.3

**Theorem 2.4**

Every \( P_n^2 \) is a square multiplicative graph for \( n \geq 3 \).

**Proof**

Let \( G \) be a graph of \( P_n^2 \). Let \( \{v_1, v_2, \ldots, v_n\} \) be the vertices and \( \{e_1, e_2, \ldots, e_{n+3}\} \) be the edges which are denoted as in the fig. 2.4.

The graph consists of \( n \) vertices and \( n + 3 \) edges.

If \( G = P_n^2 \) then \( |V(G)| = n \) and \( |E(G)| = 2n - 2 \).

Let us define a vertex labeling \( f: V(G) \rightarrow \{1, 2, \ldots, p\} \) as follows

\[
\begin{align*}
  f(u_i) &= i; \quad 1 \leq i \leq n
\end{align*}
\]

And the edge function \( f: E(G) \rightarrow N \) defined by

\[
f(uv) = f(u)^2 \cdot f(v)^2\]

for every \( uv \in E(G) \) are all distinct.

Then the edges labels are distinct.

Then \( G \) is a square multiplicative graph.

*Hence, every \( P_n^2 \) is a square multiplicative graph.*

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Example 2.4
CONCLUSION

In this paper, we investigated some families of graphs such as path, comb, double star, $P_n^2$ graphs which satisfy the formula $f(\langle uv \rangle) = f(u)^2 \ast f(v)^2$. Finally, we conclude that the above mentioned graphs are square multiplicative graphs.

References

3. Bondy and Murthy. Introduction to Graph theory.

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