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VARIABLE VISCOSITY AND THERMAL CONDUCTIVITY EFFECTS OF TWO DIMENTIONAL STAGNATION POINT FLOW IN PRESENCE OF MAGNETIC FIELD OF A MICROPOLAR FLUID

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ABSTRACT

The effect of variable viscosity and thermal conductivity of the two dimentional stagnation point flow of an incompressible non-Newtonian micropolar fluid impinging on a permeable plate in presence of magnetic field is investigated. By means of similarity transformation the governing equations are reduced to boundary value problem of nonlinear coupled ordinary differential equations and solved numerically. The effects of different dimensionless parameters such as temperature dependent viscosity parameter and thermal conductivity parameter, microrotation parameter, Prandtl number etc. on flow and heat transfer has been studied. The results are presented graphically for velocity distribution, temperature distribution and micropolar distributions for various values of non-dimensional parameters. It is found that the effects of the parameters giving variable property of viscosity and thermal conductivity are significant.

INTRODUCTION

The theory of micro polar fluids was originally formulated by Eringen [1]. For a resume of the theory and more recent literature see the book by Lukaszewiez [2].The two dimensional flow of a fluid near a stagnation point is a classical problem in fluid mechanics. It was first examined by Hiemenz [3], who demonstrated that the Navier-Stokes equations governing the flow can be reduced to an ordinary third order differential equation using similarity transformation. Owing to the nonlinearities in the reduced differential equation, no analytical solution is available and the nonlinear equation is usually solved numerically, subject to two-point boundary conditions, one of which is prescribed at infinity.

The problem of stagnation point flow was extended in numerous ways to include various physical effects. The axisymmetric three-dimensional stagnation point flow was studied by Homman [4]. The results of these studies are of great technical importance; for example, in the prediction of skin-friction, as well as heat/mass transfer near stagnation regions of bodies of high speed flows, and also in the design of thrust bearings and radial diffusers, drag reduction, transpiration cooling, and thermal oil recovery. In either the two or three dimensional case, Navier-Stokes equations governing the flow are reduced to an ordinary differential equation of the third order using a similar transformation. The effect of suction on the Hiemenz flow problem has been considered in the literature. Schlichting and Bussman [5] were the first to give the numerical results. More detailed solutions were later presented by Preston[6]. An approximate solution to the problem of uniform suction is given by Ariel [7]. The effect of uniform suction on the Homman problem, where the flat plate is oscillating in its own plane, is considered by Weidman and Mahalinggam [8]. In hydromantic, the problem of Hiemenz flow was chosen by Na[9] to illustrate the solution of a third order boundary value problem using the technique of finite differences. An approximate solution of the same problem has been provided by Ariel[10]. Attia gave the effect of an externally applied uniform magnetic field on the two or three-dimensional stagnation point flow in the presence of uniform suction or injection.

The study of heat transfer in boundary layer flows is of importance in many engineering application, such as the design of thrust bearings and radial diffusers, transpiration cooling, drag reduction, and thermal recovery of oil. Massoudi and Ramezan [11] used a perturbation technique to solve for the stagnation point flow and heat transfer of a second grade non-Newtonian fluid. Their analysis is valid only for small values of the parameter that determines the behavior of the non-Newtonian fluid. Later, Massoudi and Ramezan [12] extended

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the problem to non-isothermal surfaces. Garg [13] improved the solution obtained by numerically computing the flow characteristics for any value of the non-Newtonian parameter using a pseudo-similarity solution. Non-Newtonian fluids have been considered by many researchers. Thus, among the non-Newtonian fluids, the solution of the stagnation point flow for visco elastic fluids has been given by Rajeshwari and Rathna [14], Beard and Walters [15] Stagnation point flow of a non-Newtonian micro polar fluid was studied by Nath [16] and Nazar *et al*.[17] in the hydrodynamic case.

Through the viscosity and thermal conductivity are assumed as constant properties but these are temperature dependent. There in this paper we considered the effect of variable viscosity and thermal conductivity on the steady laminar flow of an incompressible non-Newtonian micro polar fluid at a two-dimensional stagnation point with heat transfer.

Governing Equations

The equation of motion for incompressible viscous micro polar fluid is given by

$$\rho\left\{\frac{\partial \vec{V}}{\partial t}+(\vec{V}.\nabla)\vec{V}\right\}=-\nabla p+\nabla(\mu\nabla.\vec{V})+\kappa\nabla^2\vec{V}+\kappa(\nabla\times\vec{N})+\vec{F}, \quad (2.1)$$

where ρ is the mass density of the fluid, p is the pressure, μ is the viscosity, $\vec{N}$ is the angular velocity, κ is the material constant and t denotes time. $\vec{F}$ is the body force per unit volume due to flow through porous media given by

$$\vec{F}=\frac{\nu}{\lambda^*}\vec{V}, \quad (2.2)$$

where ν is the kinematic viscosity of the fluid and λ^* is the coefficient of permeability of the porous media.
The equation of angular momentum for incompressible viscous micro polar fluid is given by

$$\rho j\left\{\frac{\partial \vec{N}}{\partial t}+(\vec{V}.\nabla)\vec{N}\right\}=-2\kappa\vec{N}+\kappa(\nabla\times\vec{V})-\gamma\{\nabla\times(\nabla\times\vec{N})\}, \quad (2.3)$$

where j is the micro-inertia per unit mass, γ is the material constants.
The equation of heat transfer is given by

$$\rho C_p\left\{\frac{\partial T}{\partial t}+(\vec{V}.\nabla)T\right\}=\nabla.(\lambda\nabla T)+(\mu+\kappa)\phi, \quad (2.4)$$

where C_p is specific heat at constant pressure, T is the temperature of the fluid, λ is the coefficient of thermal conductivity of the fluid, ϕ is the viscous dissipation function and is given by

$$\phi=2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2 \quad (2.5)$$

Formulation of the problem

Consider the two-dimensional stagnation point flow of an incompressible non-Newtonian micro polar fluid impinging perpendicular on a permeable wall and flowing away along the x-axis. This is a plane potential flow which arrives from the y-axis and impinges on a flat wall placed at $y=0$, divides into two streams on the wall, and leaves in both directions. The flow is through a porous medium, where the Darcy model is assumed. The viscous flow must adhere to the wall, whereas the potential flow slides along it. (u, v) are the components of velocity at any point (x, y) for the viscous flow, whereas (U,V) are the velocity components for the potential flow. The velocity distribution in the frictionless flow in the neighborhood of the stagnation point is given by

$U(x)=ax,\ V(y)=-ay$

Where the constant a (>0) is proportional to the free stream velocity far away from the surface.

Mass equation: $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$

Momentum equation:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}\right)=\rho U\frac{dU}{dx}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+h\frac{\partial^2 u}{\partial y^2}+h\frac{\partial N}{\partial y}+\frac{\mu}{K}(U(x)-axf')-\sigma_0\beta_0^2 u \quad (3.2)$$

Angular momentum equation:

$$\rho\left(u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=\frac{\gamma}{j}\frac{\partial^2 N}{\partial y^2}-\frac{h}{j}\left(2N+\frac{\partial u}{\partial y}\right) \quad (3.3)$$

Energy equation:

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+(\mu+k)\left(\frac{\partial u}{\partial y}\right)^2+\sigma_0\beta_0^2(u^2+v^2) \quad (3.4)$$

where N is the micro rotation or angular velocity whose direction of rotation is in the *x-y* plane, μ is the velocity of the fluid, ρ is the density of the fluid, $\bar{K}$ is the Darcy permeability and j, γ and h are the micro-inertia per unit mass, spin gradient viscosity, and vortex viscosity respectively which are assumed to be constant, λ is the thermal conductivity and C_p is the specific heat at constant pressure. Here γ is assumed to be given by

$\gamma=(\mu+h/2)l$

and we take $l=\nu/\alpha$ as a reference length and ν is the kinematic viscosity.
The appropriate physical boundary conditions of equations are

$u(x,0)=0$, $v(x,0)=0$, $N(x,0)=-n\frac{\partial u}{\partial y}$ $\quad T(x,0)=T_w$

$y\rightarrow\infty:\quad u(x,y)\rightarrow U(x)=ax,\quad v(x,y)\rightarrow 0,\ N(x,y)\rightarrow 0,\ T(x,y)\rightarrow T_\infty$

Where n is a constant and $0 \le n \le 1$. The case n = ½ indicates the vanishing of the anti-symmetric part of the stress tensor and denotes week concentration of microelements. The governing equations subject to the boundary conditions can be expressed in a simpler form by introducing the following transformation

$\eta = \sqrt{\frac{a}{\nu}} y$, $u = axf'(\eta)$, $v = -\sqrt{a}\,\nu f(\eta)$, $N = ax\sqrt{\frac{a}{\nu}}\, g(\eta)$,

$g(\eta) = -\frac{1}{2} f''(\eta)$ (3.7) and

$$\theta(\eta) = \frac{T - T_\infty}{T_W - T_\infty} \quad (3.8)$$

The fluid viscosity is assumed to be inverse linear function of temperature as

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left[1 + \alpha\left(T - T_\infty\right)\right], \frac{1}{\mu} = a\left(T - T_r\right),$$

$$a = \frac{\alpha}{\mu_\infty} \text{ and } T_r = T_\infty - \frac{1}{\alpha} \quad (3.9)$$

where a and T_r are constants and their values depends on the reference state and the thermal property of the fluid. In general a>0 for liquids and a<0 for gases. T_r is transformed reference temperature related to viscosity parameter. α is constant based on thermal property and μ_∞ is the viscosity at $T=T_\infty$ similarly, consider the variation of thermal conductivity as,

$$\frac{1}{\lambda} = \frac{1}{\lambda_\infty}\left[1 + \xi\left(T - T_\infty\right)\right], \frac{1}{\lambda} = b\left(T - T_k\right), \ b = \frac{\xi}{\lambda_\infty} \text{ and}$$

$$T_k = T_\infty - \frac{1}{\xi} \quad (3.10)$$

where b and T_k are constants and their values depends on the reference state and thermal property of the fluid ξ is constant based on thermal property and λ_∞ is the viscosity at $T=T_\infty$.

Using equation (12), it can be easily verified that the continuity equation is satisfied automatically and using equation (12)-(15) in the equations (7) –(9) become,

$$f'''\left(1 + \frac{K}{2}\frac{\theta_r - \theta}{\theta_r}\right) - \left(f'^2 - ff'' + f'\right)\frac{\theta_r - \theta}{\theta_r} + \frac{\theta_r - \theta}{\theta_r} + f''\theta' + \left(m - Mf''\frac{\theta_r - \theta}{\theta_r}\right)\frac{\theta_r - \theta}{\theta_r} = 0 \quad (3.11)$$

$$\left(1 + \frac{K}{2}\right) g'' = K(2g + f'') + f'g - fg' \quad (3.12)$$

and

$$\left\{P_r f + \frac{\theta_k}{(\theta_k - \theta)^2}\right\}\theta'^2 + \frac{\theta_k}{(\theta_k - \theta)}\theta'' - M\left(f^2 + f'^2\right)\frac{(\theta_k - \theta)}{\theta_k} = 0 \quad (3.13)$$

Where $K = \frac{h}{\mu}(>0)$ is the material parameter, $m = \frac{\nu}{a\bar{K}}$ is the porosity parameter.

Transform boundary conditions are

$$f = 0,\ f' = 0,\ g = -nf'',\ \theta = 1 \quad (3.15)$$
$$f' = 1,\ f \to 0,\ g \to 0,\ \theta = 1$$

RESULTS AND DISCUSSION

In this paper we have attempted to develop a mathematical analysis for investigating the effect of variable viscosity and thermal conductivity of stagnation point flow and heat transfer of a micro polar fluid in a porous medium in presence of magnetic field.

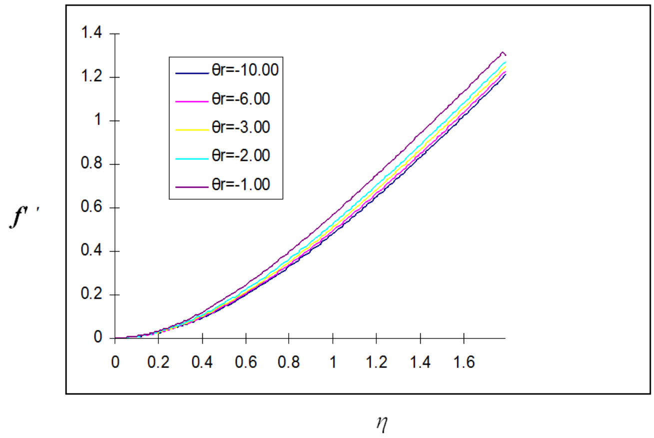

Fig 1 Velocity distribution profiles f' along against η for various values of viscosity parameter θ_r taking Pr=0.70 Ec=0.10 go=1.00 m=0.10 K=0.10 M=0.10 θ_k=-10.00

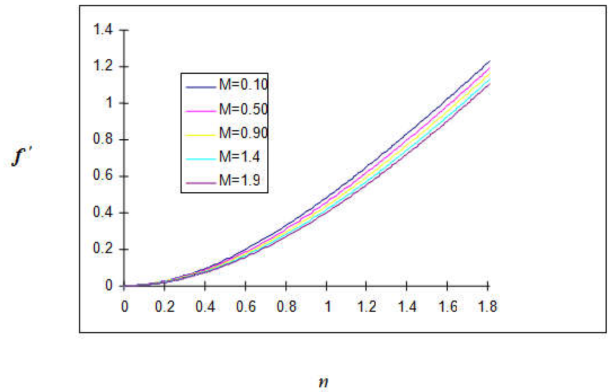

Fig 2 Velocity distribution profiles f' along against η for various values of parameter M taking Pr=0.70 Ec=0.10 go=1.00 K=0.10 θr=-10.00 θ_k=-10.00

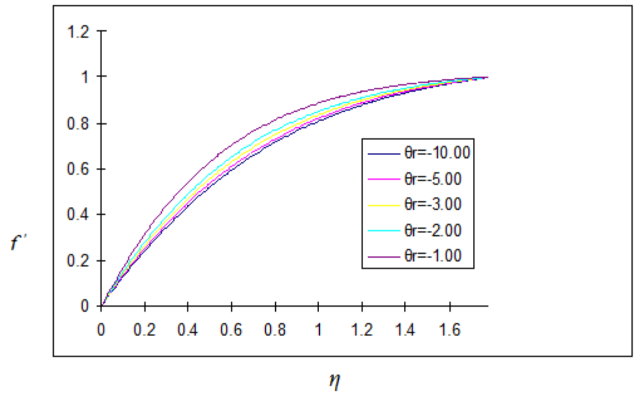

Fig 3 Velocity distribution profiles f' along against η for various values of parameter θ_r taking Pr=0.70 Ec=0.10 go=1.00 M=0.10 K=0.10 θ_k=-10.00

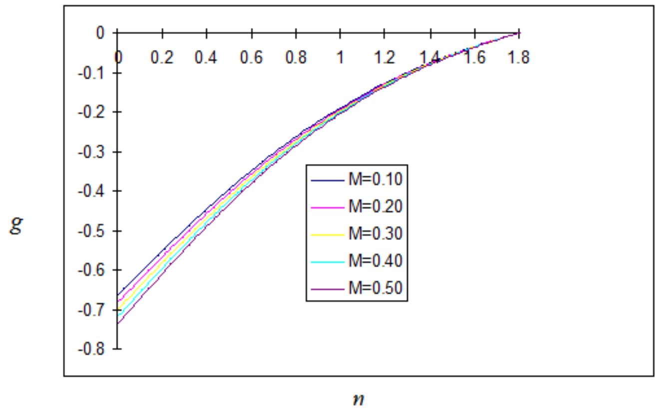

Fig 4 Micro rotation distribution profiles g along against η for various values of parameter M taking Pr=0.70 Ec=0.10 go=1.00 K=0.10 θr=-10.00 θ_k=-10.00

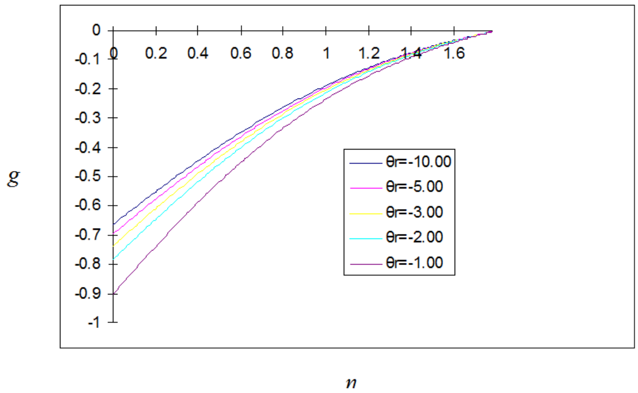

Fig 5 Micro rotation distribution profiles g along against η for various values of parameter θ_r taking Pr=0.70 Ec=0.10 go=1.00 m=0.10 K=0.10 M=0.10 θ_k=-10.00

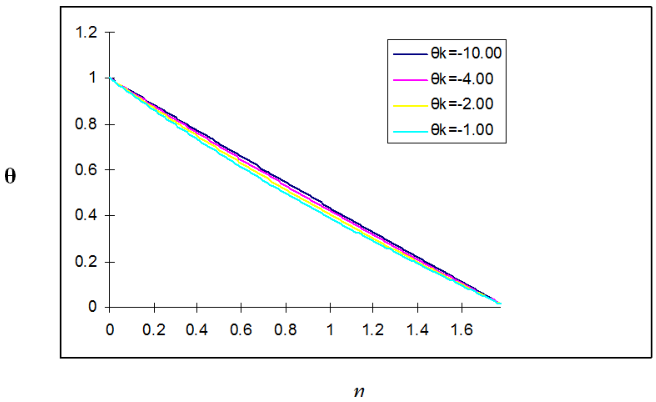

Fig 6 Temperature distribution profiles along against η for various values of parameter θ_k taking Pr=0.70 Ec=0.10 go=1.00 m=0.10 K=0.10 M=0.10 θr=-10.00

The system of differential equations (3.2), (3.3) and (3.4) governed by the boundary conditions (3.6) is solved numerically by applying numerical techniques based on the common Runge-Kutta shooting method [1980]. Numerical computation for velocity and temperature field was carried out for various values of the viscosity parameter and thermal conductivity parameter and M.

The graphical representation of velocity distribution (f), axial velocity distribution (f'), micro rotation distribution (g), temperature distribution against η are presented in figures (Fig. 1–Fig. 6) for different values of the parameters as mentioned in the respective figures.

Initially solution was taken for constant values of Pr=0.70, Ec=0.10, go=1.00, m=0.10, M=0.10, θr=-10.00, θ_k=-10.00 with the viscosity parameter θr ranging from -12 to -1 at the certain values of θ_k=-10. Similarly the solutions have been found with varying the thermal conductivity parameter θ_k ranging from -12 to -1 at the certain values of θr=-10 keeping other values remaining same. Solutions have also been found for different values of parameters M and m. The Fig. (1-3) represents the variation in velocity distribution with the variation of viscosity parameter θr and parameter M at prescribed surface temperature.

From Fig. (1) it is seen that the velocity distribution increases as viscosity parameter

θr increases. From Fig. (2 and 3) show the variation of velocity distribution with the variation of magnetic parameter M and viscosity parameter θr at prescribed surface temperature. From Fig. (2) and (3) it is seen that the velocity (u) decreases as parameter M increases and increases as parameter θr increases. From Fig. (4), it is seen that the micro rotation distribution g decreases for smaller values of η for the increasing values of magnetic parameter (M).

From Fig. (5), it is seen that the microrotion distribution decreases as θr increases. From Fig. (6), it is seen that the temperature distribution increases as temperature corresponding to the thermal conductivity parameter increases.
The presented analysis has shown that the flow field is appreciably influenced by the viscosity temperature-variation, thermal conductivity-temperature variation. Therefore, we can conclude that to predict more accurate results the variable viscosity, thermal conductivity effects have to be taken in to consideration in the electrically conducting fluid.

CONCLUSIONS

In this study, the effect of variable viscosity and thermal conductivity of of the two-dimensional stagnation point flow of an non-Newtonian micropolar fluid with heat transfer through porous medium in presence of magnetic field is examined. The resulting partial differential equations, which describe the problem, are transformed into ordinary differential equations by using similarity transformations. Numerical evaluations are performed and graphical results are obtained. The results presented demonstrate clearly that the viscosity and thermal conductivity parameters have a substantial effect on velocity distribution, micropolar distribution and temperature distribution.

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