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ARITHMETIC LABELING OF $P_m \otimes P_n$ and $P_3 \otimes S_k$

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ABSTRACT

Acharya.B.D. and Hedge.S.M. was introduced the concept of arithmetic labeling and many research articles have published in this topic. In this paper, we have proved that the Kronecker product of $P_m \otimes P_n$ and $P_3 \otimes S_k$ are arithmetic graphs. Also we established a general formula to label the vertices and edges of the graph G.

INTRODUCTION

In 1989 Acharya.B.D. and Hedge.S.M. Introduced a new version of sequential graph known as arithmetic graph and is defined as follows: Let G be a graph with q edges, a and d are the positive integers, the labeling f of G is said to be (a, d) – arithmetic if the vertices are labeled by distinct nonnegative integers and the edge labels are induced by $f(x) + f(y)$ for each edges xy are in the form of $a, a + d, a + 2d, \ldots,$ $a + (q - 1)d$. A graph is called arithmetic if it is an (a, d) - arithmetic for some a and d.

Definition

A graph G is an ordered pair $(V(G), E(G))$ consisting of a non empty set $V(G)$ of vertices and a set $E(G)$, disjoint from $V(G)$ of edges, together with an incidence function ψ_G that associates with each edge of G *is* an unordered pair of vertices of G.

Definition

Walk is an alternating sequence of vertices and edges starting and ending with vertices. A walk in which all the vertices are distinct is called a path.

Definition

A Star S_k is the complete bipartite graph $K_{1,n}$. A tree with one internal node and k leaves (but, no internal nodes and k+1 leaves when $k \leq 1$). Alternatively some authors define S_k to be the tree of order k with maximum diameter 2; in which case a star of $k > 2$ has k-1 leaves.

Definition

If G_1 and G_2 are two graphs with vertex sets V_1 and V_2 respectively then their product graph is a graph denoted by $G_1 \otimes G_2$ with it's vertex set as $V_1 \times V_2$ where (u_1,v_1) is adjacent with (u_2,v_2) if and only if $u_1u_2 \in E_1$ and $v_1v_2 \in E_2$ is called the ***kronecker product*** of graphs.

Definition

A labeling or valuation of a graph G is an assignment f of labels to the vertices of G that induces for each edge xy a label depending on the vertex labels $f(x)$ and $f(y)$.

Definition

A graph is said to be *arithmetic* if it's vertices can be assigned distinct non negative integers in such a way that the value of the edges are obtained as sum of the values assigned in an arithmetic progression.

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Theorem

Let the graph G be the Kronecker product of P_m and P_n that is $P_m \otimes P_n$ Where $m, n > 0$ is an (a,d) - arithmetic graph.

Proof

Let G = $P_m \otimes P_n$ Where $m, n > 0$. Then the graph is given in (Figure:1) as below.

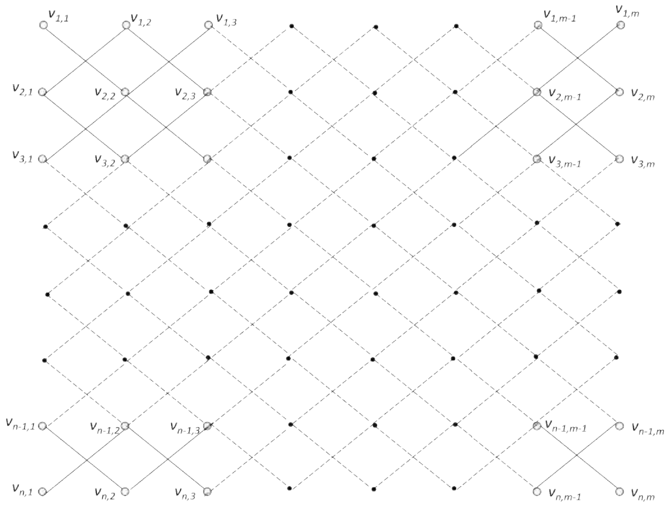

Figure 1

Let V be the vertex set of G and is denoted by
V(G) = $\{V_{i,j} / i \le i \le n;\ 1 \le j \le m\}$
Define f:V (G)→N. Now we are giving the label to the vertices of G as below.

If both 'i' and 'j'are odd

$f(v_{i,j}) = 4\left[3\left(i - \left(\frac{i-1}{2}+1\right)\right) + (m-1)(j-1)\right] + 1$

Where$1 \le i \le m$ ∀odd m(or) $1 \le i \le m-1$ ∀even m;
$1 \le j \le n$ ∀odd n(or) $1 \le j \le n-1$ ∀even n ……….(1)

If 'i' is even and 'j' is odd

$f(v_{i,j})= 4\left[3\left(i - \left(\frac{i}{2}+1\right)\right) + (m-1)(j-1)\right] + 3$
Where$2 \le i \le m-1$ ∀odd m (or) $2 \le i \le m$ ∀even m;
$1 \le j \le n$ ∀odd n (or) $1 \le j \le n-1$ ∀even n. ……….(2)
If 'i' is odd and 'j' is even

$f(v_{i,j}) = 2(i+1) + 4(j-2)(m-1)$where$1 \le i \le m$ ∀odd m (or) $1 \le i \le m-1$ ∀even m;

$2 \le j \le n-1$ ∀odd n (or)$2 \le j \le n$ ∀even n ……….(3)

If both 'i' and 'j' are even

$f(v_{i,j}) = 2(i-1) + 4(j-2)(m-1)$where$2 \le i \le m-1$ ∀odd m (or) 2 ≤ i ≤m ∀ even m; 2≤ j ≤ n-1 ∀ odd n (or) 2 ≤j ≤ n ∀even n. ……….(4)

The edge labels induced by $f(uv) = f(u) + f(v)$ are as follows:

If *r,t* odd and *k,l* even

$$f(v_{r,t}v_{k,l}) = 4m(t+l-3) - 2(2t+2l-k) + 12\left(r - \left(\frac{r-1}{2}+1\right)\right) + 11$$

Where $1 \le r \le m$ ∀odd m(or)$1 \le r \le m-1$ ∀even m;
$1 \le t \le n$ ∀odd n (or) $1 \le j \le n-1$ ∀ even n;
2 ≤ k ≤ m- 1 ∀ odd m (or)$2 \le k \le m$ ∀even m;
$2 \le l \le n-1$ ∀odd n ;
$2 \le l \le n$ ∀even n. ……….(5)

If *r, l* are odd and *t, k* are even

$$f(v_{r,t}v_{k,l}) = 4m(t+l-3) - 2(2t+2l-r) + 12\left(k - \left(\frac{k}{2}+1\right)\right) + 17$$

Where$1 \le r \le m$ ∀ odd m (or) $1 \le r \le m-1$ ∀ even m;
$2 \le t \le (n-1)$ ∀odd n (or) $2 \le t \le n$ ∀even n;
$2 \le k \le m-1$ ∀odd m (or) $2 \le k \le m$ ∀even m;
$1 \le l \le n$ ∀odd n (or)$1 \le l \le n-1$ ∀even n………. (6)

Clearly the edges are labeled as $f(E(G)) = \{a, a+d, a+2d, \dots, a+(q-1)d\}$

Therefore, f is an arithmetic labeling. Hence the graph $G = P_m \otimes P_n$ is an
(a, d) – arithmetic graph.

Example

Consider the graph *G* = $P_8 \otimes P_7$.
Here, m = 8; n=7 and q = 84

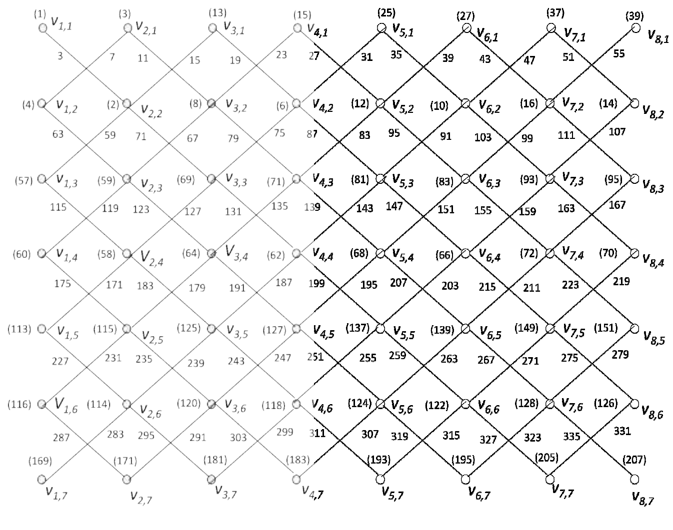

Figure 2

The vertex labels are given below.
Equation (1) ⇒$f(v_{i,j}) = 4\left[3\left(i - \left(\frac{i-1}{2}+1\right)\right) + (m-1)(j-i)\right] + 1$

for allodd i and j where $1 \le i \le m-1$ *for all* even m;
$1 \le j \le n$ *for all* odd n .
when *i = 1,3, 5, 7 ; j= 1, 3, 5, 7* ⇒$(v_{1,1})$= 1;$f(v_{3,1})$ =13; $f(v_{5,1})$ =25; $f(v_{7,1})$ = 37;
$f(v_{1,3})$ = 57;$f(v_{3,3})$ = 69; $f(v_{5,3})$ = 81; $f(v_{7,3})$ = 93 ; $f(v_{1,5})$ = 113;$f(v_{3,5})$ = 125;
$f(v_{5,5})$ = 137; $f(v_{7,5})$ = 149; $f(v_{1,7})$ = 169; $f(v_{3,7})$ = 181 ; $f(v_{5,7})$= 193 ;
$f(v_{7,7})$ = 205.

Equation (2) ⇒$f(v_{ij}) = 4\left[3\left(i - \left(\frac{i}{2}+1\right)\right) + (m-1)(j-i)\right] + 3$

for all even i and odd j where $2 \le i \le m$ for all even m ;
$1 \le j \le n$ *for all* odd n.

When $i = 2, 4, 6, 8; j= 1, 3, 5, 7 \Rightarrow f(v_{2,1}) = 3; f(v_{4,1}) = 15; f(v_{6,1}) = 27$;
$f(v_{8,1}) = 39; f(v_{2,3}) = 59; f(v_{4,3}) = 71; f(v_{6,3}) = 83; f(v_{8,3}) = 95$;
$f(v_{2,5}) = 115; f(v_{4,5}) = 127; f(v_{6,5}) = 139; f(v_{8,5}) = 151; f(v_{2,7}) =171; f(v_{4,7}) = 183; f(v_{6,7}) = 195; f(v_{8,7}) = 207$.

Equation (3)$\Rightarrow f(v_{i,j}) = 2(i+1) + 4(j-2)(m-1)$ for all odd i and even j
Where $1 \le i \le m-1$ *for all* even m ; $2 \le j \le n-1$ *for all* odd n
When$i = 1,3,5,7; j=2,4,6 \Rightarrow f(v_{1,2}) = 4; f(v_{3,2}) = 8; f(v_{5,2}) = 12; f(v_{7,2}) =16$;
$f(v_{1,4})=60; f(v_{3,4})= 64; f(v_{5,4})= 68; f(v_{7,4})= 72; f(v_{1,6})=116; f(v_{3,6})=120$;
$f(v_{5,6}) = 124; f(v_{7,6})=128$.

Equation (4) $\Rightarrow f(v_{i,j}) = 2(i-1) + 4(j-2)(m-1)$ *forall* even i and j
where $2 \le i \le m$ ∀even m; $2 \le j \le n-1$ ∀odd n.
When$i =2,4,6,8 ; j=2,4,6 \Rightarrow f(v_{2,2})= 2; f(v_{4,2})= 6; f(v_{6,2})= 10; f(v_{8,2})=14; f(v_{2,4})=58; f(v_{4,4})= 62; f(v_{6,4})= 66; f(v_{8,4})=70; f(v_{2,6})=114; f(v_{4,6})=118$;
$f(v_{6,6}) =122; f(v_{8,6}) = 126$.

The edge labels are given below.

Equation (5) $\Rightarrow f(v_{r,t}v_{k,l}) = 4m(t+l-3) - 2(2t+2l-k) + 12\left(r - \left(\frac{r-1}{2}+1\right)\right) 11$
for all odd r, t and even k, l where $1 \le r \le m-1$ for all even m;
$1 \le t \le n$ *for all* odd n ; $2 \le k \le m$ for all even m;
$2 \le l \le n-1$ *for all* odd n

When $r= 1,3,5,7$; $t= 1,3,5,7$; $k= 2,4,6,8$; $l= 2,4,6$ $\Rightarrow f(v_{1,1} v_{2,2}) =3; f(v_{3,1} v_{2,2})=15$;
$f(v_{3,1} v_{4,2})= 19; f(v_{5,1} v_{4,2})= 31; f(v_{5,1} v_{6,2})=35; f(v_{7,1} v_{6,2})= 47; f(v_{7,1} v_{8,2})=51$;
$f(v_{1,3} v_{2,2}) =59; f(v_{3,3} v_{2,2})=71; f(v_{3,3} v_{4,2})=75; f(v_{5,3} v_{4,2})=87; f(v_{5,3} v_{6,2})=91$;
$f(v_{7,3} v_{6,2})= 103; f(v_{7,3} v_{8,2})=107; f(v_{1,3} v_{2,4}) =115; f(v_{3,3} v_{2,4}) = 127; f(v_{3,3} v_{4,4})=131$;
$f(v_{5,3} v_{4,4})= 143; f(v_{5,3} v_{6,4})=147; f(v_{7,3} v_{6,4})=159; f(v_{7,3} v_{8,4})= 163; f(v_{1,5} v_{2,4})=171$;
$f(v_{3,5} v_{2,4})=183; f(v_{3,5} v_{4,4})= 187; f(v_{5,5} v_{4,4})= 199; f(v_{5,5} v_{6,4})=203; f(v_{7,5} v_{6,4})=215; f(v_{7,5} v_{8,4})= 219; f(v_{1,5} v_{2,6})=227; f(v_{3,5} v_{2,6})=239; f(v_{3,5} v_{4,6})= 243; f(v_{5,5} v_{4,6})= 255$;
$f(v_{5,5} v_{6,6})= 259; f(v_{7,5} v_{6,6})= 271; f(v_{7,5} v_{8,6})=275; f(v_{1,7} v_{2,6}) = 283; f(v_{3,7} v_{2,6})=295; f(v_{3,7} v_{4,6})=299; f(v_{5,7} v_{4,6})= 311; f(v_{5,7} v_{6,6})= 315; f(v_{7,7} v_{6,6})=327; f(v_{7,7} v_{8,6}) =331$

Equation(6)$\Rightarrow f(v_{r,t}v_{k,l}) = 4m(t+l-3) - 2(2t+2l-r) + 12\left(k - \left(\frac{k}{2}+1\right)\right) + 17$
where $1 \le r \le m-1$ *for all* even m; $2 \le t \le (n-1)$ *forall* odd n;
$2 \le k \le m$ *for all* even m; $1 \le l \le n$ *for all* odd n.
When$r = 1,3,5,7; t=2,4,6 ; k=2,4,6,8 ; l=1,3,5,7 \Rightarrow f(v_{1,2} v_{2,1})= 7; f(v_{3,2} v_{2,1})=11; f(v_{3,2} v_{4,1})=23; f(v_{5,2} v_{4,1})= 27; f(v_{5,2} v_{6,1})=39; f(v_{7,2} v_{6,1})=43; f(v_{7,2} v_{8,1})=55$;
$f(v_{1,2} v_{2,3})=63; f(v_{3,2} v_{2,3})=67; f(v_{3,2} v_{4,3})= 79; f(v_{5,2} v_{4,3})= 83; f(v_{5,2} v_{6,3})=95$;
$f(v_{7,2} v_{6,3})=99; f(v_{7,2} v_{8,3})= 111; f(v_{1,4} v_{2,3})= 119; f(v_{3,4} v_{2,3})= 123; f(v_{3,4} v_{4,3})=135; f(v_{5,4} v_{4,3})=139; f(v_{5,4} v_{6,3})=151; f(v_{7,4} v_{6,3})= 155; f(v_{7,4} v_{8,3})=167; f(v_{1,4} v_{2,5})=175; f(v_{3,4} v_{2,5})=179; f(v_{3,4} v_{4,5})=191; f(v_{5,4} v_{4,5})=195; f(v_{5,4} v_{6,5})=207; f(v_{7,4} v_{6,5})= 211$;
$f(v_{7,4} v_{8,5})=223; f(v_{1,6} v_{2,5})=231; f(v_{3,6} v_{2,5})=235; f(v_{3,6} v_{4,5})= 247; f(v_{5,6} v_{4,5})=251; f(v_{5,6} v_{6,5})=263; f(v_{7,6} v_{6,5})=267; f(v_{7,6} v_{8,5})= 279; f(v_{1,6} v_{2,7})=287; f(v_{3,6} v_{2,7})=291; f(v_{3,6} v_{4,7})= 303; f(v_{5,6} v_{4,7})=307; f(v_{5,6} v_{6,7})= 319; f(v_{7,6} v_{6,7})= 323; f(v_{7,6} v_{8,7})=335$;
In this graph a=3 and d=7-3=4

The edge labels are in the arithmetic progression

a=3,a+d=7,a+2d =11,a+3d=15,a+4d=19,a+5d=23,a+6d=27, a+7d=31,a+8d=35,a+9d=39,a+10d=43,a+11d=47, a+12d=51,a+13d=55,a+14d=59,a+15d=63, a+16d=67,a+17d=71,a+18d=75,a+19d=79,a+20d=83,a+21d=87,a+22d=91, a+23d=95, a+24d=99, a+25d=103,a+26d=107,a+27d=111, a+28d=115,a+29d=119,a+30d=123,a+31d=127, a+32d=131,a+33d=135,a+34d=139,a+35d=143, a+36d=147,a+37d=151,a+38d=155,a+39d=159, a+40d=163, a+41d=167,a+42d=171,a+43d=175,a+44d=179,a+45d=183, a+46d=187, a+47d=191,a+48d=195,a+49d=199,a+50d=203, a+51d=207,a+52d=211, a+53d=215,a+54d=219,a+55d=223,a+56d=227,a+57d=231, a+58d=235, a+59d=239,a+60d=243,a+61d=247,a+62d=251, a+63d=255,a+64d=259,a+65d=263, a+66d=267,a+67d=271,a+68d=275,a+69d=279,a+70d=283, a+71d=287, a+72d=291, a+73d=295, a+74d=299, a+75d=303, a+76d=307, a+77d=311, a+78d=315,a+79d=319, a+80d=323, a+81d=327, a+82d=331,a+(q-1)d=a+83d=335.

Therefore, the graph $G=P_8 \otimes P_7$ is an (3, 4)- arithmetic graph.

Theorem

Let the graph G be the Kronecker product of $P_3 \otimes S_k$. That is $G = P_3 \otimes S_k$
where $k > 0$ is an (a, d) - arithmetic graph.

Proof

Let G= $P_3 \otimes S_k$ where $k > 0$. Then the graph is given in (Figure:3) as below.
Consider k = n

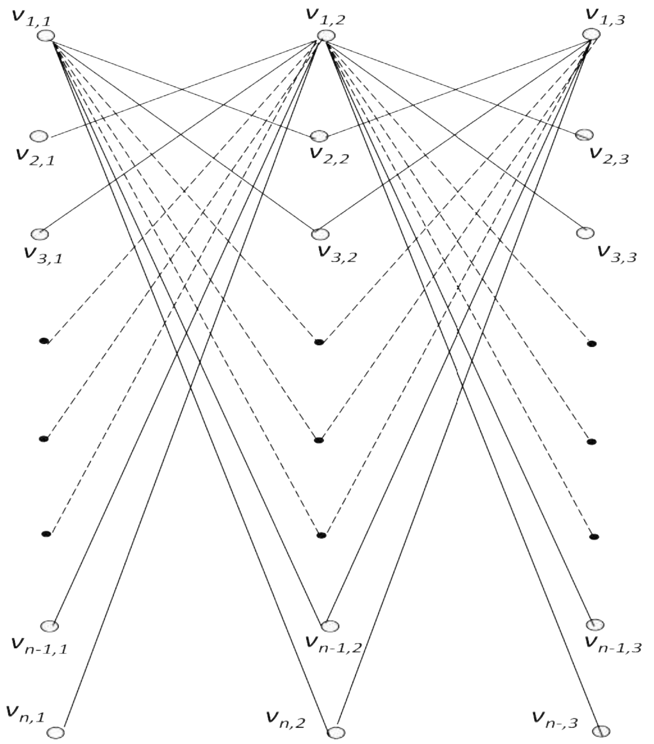

Figure 3

Let V be the vertex set of G and is denoted by
$V(G) = \{v_{i,j}/1 \leq i \leq n\,;\ 1 \leq j \leq 3\}$
Define $f: V(G) \rightarrow N$. Now we are giving the label to the vertices of G as below.

If 'j' is odd
$f(v_{i,j}) = (j+3) + 8(i-2)$ where $2 \leq i \leq n$; $1 \leq j \leq 3$ ……….(A)
If 'j' is even:
$f(v_{i,j}) = (j+1) + 8\,(i-1)$ where $1 \leq i \leq n\,; j = 2$ ……….(B)
If 'i = 1'and 'j' is odd:
$f(f(v_{i,j})) = j - i$ where $i = 1$; $1 \leq j \leq 3$ ……….(C)
The edge labels induced by $f(uv) = f(u) + f(v)$ are as follows.
If 'l' is odd and 't' is even:

$$f(v_{r,t}v_{e,l}) = 8(r+e) + t + l - 20$$

where $r = 1\,;\ t = 2;\ 2 \leq e \leq n\,;\ 1 \leq l \leq 3$ ……….(D)
If 't' is odd and 'l'is even:

$$f(v_{r,t}v_{e,l}) = t + l + 8e - r - 7$$

where $r = 1;\ 1 \leq t \leq 3;\ 2 \leq e \leq n; l=2$ ……….(E)
Clearly the edges are labeled as

$f(G(E)) = \{a, a+d, a+2d, \ldots, a+(q-1)d\}$
Therefore, f is an arithmetic labeling
Hence, the graph G= $P_3 \otimes S_k$ is an (a,d) - arithmetic graph,

Example

Consider the graph G= $P_3 \otimes S_6$

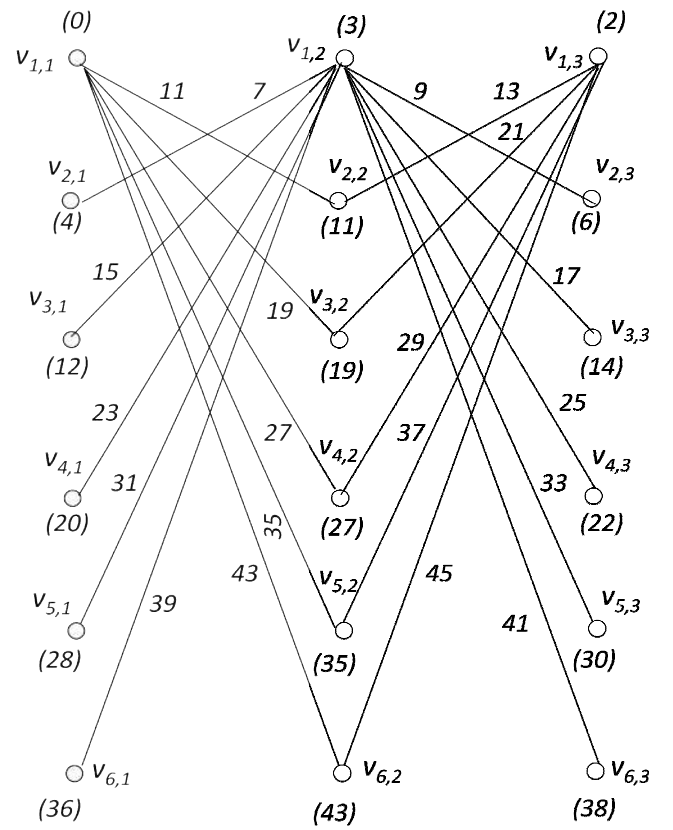

Figure 4

Here, k=6 and q= 20
The vertex labels are given below
Equation $(A) \Rightarrow f(v_{i,j}) = (j+3) + 8(i-2)$ ∀odd j

Where $2 \leq i \leq n\,;\ 1 \leq j \leq 3$
When i = 2,3,4,5,6 ; j=1,3 ⇒$f(v_{2,1})$ =4; $f(v_{3,1})$=12; $f(v_{4,1})$= 20; $f(v_{5,1})$=28;
$f(v_{6,1})$=36;$f(v_{2,3})$= 6; $f(v_{3,3})$=14; $f(v_{4,3})$=22; $f(v_{5,3})$=30;$f(v_{6,3})$=38.
Equation (B) $\Rightarrow f(v_{i,j}) = (j+1) + 8\,(i-1)$ where $1 \leq i \leq n;\ j = 2$
When$i$ =1,2,3,4,5,6; j=2 ⇒ $f(v_{1,2})$=3;$f(v_{2,2})$=11; $f(v_{3,2})$=19 ; $f(v_{4,2})$=27;
$f(v_{5,2})$=35;$f(v_{6,2})$=43.
Equation (C) $\Rightarrow f(v_{i,j}) = j - i$ ∀ odd j where $i = 1;\ 1 \leq j \leq 3$
when i=1, j=1,3 ⇒$f(v_{1,1})$=0; $f(v_{1,3})$=2
The edge labels are given below,
Equation(D)$\Rightarrow f(v_{r,t}v_{e,l}) = 8(r+e) + t + l - 20$ ∀odd l
where $r = 1;\ t = 2;\ 2 \leq e \leq n;\ 1 \leq l \leq 3$
When$r$=1; t=2; e=2,3,4,5,6 ;l=1,3 ⇒$f(v_{1,2}\,v_{2,1})$=7; $f(v_{1,2}\,v_{3,1})$= 15;
$f(v_{1,2}\,v_{4,1})$=23;$f(v_{1,2}\,v_{5,1})$=31; $f(v_{1,2}\,v_{6,1})$=39; $f(v_{1,2}\,v_{2,3})$= 9 ; $f(v_{1,2}\,v_{3,3})$= 17;
$f(v_{1,2}\,v_{4,3})$=25; $f(v_{1,2}\,v_{5,3})$=33;$f(v_{1,2}\,v_{6,3})$=41.
Equation(E) $\Rightarrow f(v_{r,t}v_{e,l}) = t + l + 8e - r - 7$ ∀odd t

Where $r = 1;\ 1 \leq t \leq 3;\ 2 \leq e \leq n\,;\ l = 2$
when r= 1; t=1,3; e=2,3,4,5,6; l=2 ⇒$f(v_{1,1}\,v_{2,2})$= 11; $f(v_{1,1}\,v_{3,2})$=19;
$f(v_{1,1}\,v_{4,2})$=27; $f(v_{1,1}\,v_{5,2})$= 35; $f(v_{1,1}\,v_{6,2})$=43;$f(v_{1,3}\,v_{2,2})$= 13; $f(v_{1,3}\,v_{3,2})$= 21;

$f(v_{1,3}\, v_{4,2})=29; f(v_{1,3}\, v_{5,2})= 37; f(v_{1,3}\, v_{6,2})=45$.
Here, a=7 and d=9 – 7=2
The edge labels are in the arithmetic progression
a=7, a+d= 9, a+2d=11, a+3d=13, a+4d=15, a+5d=17, a+6d=19, a+7d=21, a+8d=23, a+9d=25, a+10d=27, a+11d =29, a+12d=31, a+13d=33, a+14d=35,
a+15d=37, a+16d=39, a+17d=41, a+18d=43, a+(q-1) d=a+19d=45.
Therefore, the graph $G=P_3 \otimes S_6$ is an (7, 2)- arithmetic graph.

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