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Research Article

ON THE HEPTIC DIOPHANTINE EQUATION WITH THREE UNKNOWNS $5(x^2+y^2)-9xy=35z^7$

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ABSTRACT

We obtain three different patterns of non-zero integral solutions of the Heptic Diophantine equation with three unknowns $5(x^2+y^2)-9xy=35z^7$ by employing suitable transformations.

INTRODUCTION

Diophantine equations, homogeneous and non- homogeneous have aroused the interest of numerous mathematicians since antiquity as can be seen from [1-2]. The problem of finding all integral solutions of on Interminate equation with three or more variables in general presents a good deal of difficulties. Cubic equations in two variables fall into the theory of elliptic curves which is a very developed theory but still an important topic of current research [3-15]. For equations with degree at least three very little is known. In this communication a Heptic Polynomial equation with three variables represented by $5(x^2+y^2)-9xy=35z^7$ is considered and three different patterns of non-zero integral solutions have been presented.

Method of Analysis

The equation under consideration is

$$5(x^2+y^2)-9xy=35z^7 \qquad \text{--- (1)}$$

Assigning the transformations

$$x=u+v,\ y=u-v \qquad \text{--- (2)}$$

in (1) leads to

$$u^2+19v^2=35z^7 \qquad \text{--- (3)}$$

The equation (3) is solved through different approaches and they, one obtains distinct sets of solutions is (1)

Case 1

Assume that $z=a^2+19b^2$ --- (4)

Write $35=\frac{(4n+n\sqrt{19}i)(4n-n\sqrt{19}i)}{n^2}$

Where n = 1, 2, 3 --- (5)

use (5) & (4) in (3) and applying the method of factorization, define

$$u+i\sqrt{19}v=\frac{1}{n}\left(4n+n\sqrt{19}i\right)(a+i\sqrt{19}b)^7 \qquad \text{--- (6)}$$

Equating the real and imaginary parts, we have

$$u=u(a,b)=4a^7-1596a^5b^2+50540a^3b^4-192052ab^6-133a^6b$$
$$+12635a^4b^3-144039a^2b^5+130321b^7$$
$$v=v(a,b)=a^7-399a^5b^2+12635a^3b^4-48013ab^6+28a^6b$$
$$-2660a^4b^3+30324a^2b^5-27436b^7$$

Substituting the above values of u and v in equation (2), and hence the non-zero integral solutions of (1) are

$$x=5a^7-1995a^5b^2+63175a^3b^4-240065ab^6-105a^6b$$
$$+9975a^4b^3-113715a^2b^5+102885b^7$$

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$$y = 3a^7 - 1197a^5b^2 + 37905a^3b^4 - 144039ab^6 - 161a^6b \quad (7)$$

$$+15295a^4b^3 - 174363a^2b^5 + 157757b^7$$
$$z = a^2 + 19b^2$$

Case 2

Equ (3) can be written as

$$u^2 + 19v^2 = 35z^7 * 1 \quad \text{--- (8)}$$

Instead of (5), we write as

$$35 = \frac{(11+i\sqrt{19})(11-i\sqrt{19})}{4} \quad \text{--- (9)}$$

and also 1 as

$$1 = \frac{(9+i\sqrt{19})(9-i\sqrt{19})}{100} \quad \text{--- (10)}$$

use (4), (10), (9) in (8) and applying the method of factorization, define

$$u + i\sqrt{19}v = \frac{1}{20}\left\{(11 + i\sqrt{19})(9 + i\sqrt{19})(a + i\sqrt{19}b)^7\right\} \quad \text{--- (11)}$$

Equating the real and imaginary part, we have

$$u = u(a,b) = 4a^7 - 159a^5b^2 + 50540a^3b^4 - 192052ab^6$$
$$-133a^6b - 12635a^4b^3 - 144039a^2b^5 + 130321b^7\}$$
$$v = v(a,b) = 28a^6b - 2660a^4b^3 + 30324a^2b^5 - 27436b^7$$
$$+a^7 - 399a^5b^2 + 12635a^3b^4 - 48013ab^6\}$$

Substituting the values of u and v in equ (2), then the values of x and y are given by

$$\left.\begin{aligned} x &= 5a^7 - 1995a^5b^2 + 63175a^3b^4 - 240065ab^6 \\ &\quad -105a^6b + 9975a^4b^3 - 113715a^2b^5 + 102885b^7 \\ y &= 3a^7 - 1197a^5b^2 + 37905a^3b^4 - 144039ab^6 \\ &\quad -161a^6b - 15295a^4b^3 - 174363a^2b^5 + 157757b^7 \\ z &= a^2 + 19b^2 \end{aligned}\right\} \quad (12)$$

Case 3:

Let $35 = \frac{(8+2\sqrt{19}i)(8-2\sqrt{19}i)}{4}$ --- (13)

And also 1 as

$$1 = \frac{(5+3\sqrt{19}i)(5-3\sqrt{19}i)}{196} \quad \text{--- (14)}$$

Following the same procedure as in Case 2, the non-zero integral solutions of (1) are

$$\left.\begin{aligned} x &= 28^6\{-40A^7 + 15960A^5B^2 - 505400A^3B^4 + 1920520AB^6 \\ &\quad -5040A^6B + 478800A^4B^3 - 5458320A^2B^5 + 4938480B^7\} \\ y &= 28^6\{-108A^7 + 43092A^5B^2 - 1364580A^3B^4 + 5185404AB^6 \\ &\quad -4004A^6B + 380380A^4B^3 - 4336332A^2B^5 + 3923348B^7\} \\ z &= 784A^2 + 14896B^2 \end{aligned}\right\} \quad (15)$$

Note

1 can be also written as

$$1 = \frac{(3+5\sqrt{19}i)(3-5\sqrt{19}i)}{484}$$

CONCLUSION

In this paper we have presented three different patterns of non-zero integral solutions of the Heptic Diophantine equation with three unknown (1). One may search for other patterns of solutions and their corresponding properties.

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