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Research Article

APPLICATION OF NON-LINEAR CONTROL UNDER A DYNAMIC ROTOR SIGNAL

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ABSTRACT

In this work the aim is to control the chaotic unstable behavior of a dynamic rotor using the optimal linear control theory OLC, minimizing the chaotic unstable behavior arising from the vibration system, to become a stable signal. To develop this work, there were several steps ranging from the characterization of the experiment consisting of the acquisition of the rotor signal compared with the theoretical model and justified by the stability criteria of Routh-Hurwitz. In a second step, we were introduced an unbalanced mass and analyzed the vibration signal, characterizing the stability of the system by Lyapunov method. In this phase were also developed, mathematically, the equations of characteristics solutions derived from the equations of the space system. Finally, in the third step, for effective control was used linear state feedback control, linear control design minimizing the chaotic signal in a stable point. The results found, the application of control theory, was the effective control of the chaotic unstable signal of the dynamic rotor, thus justifying the result of this work.

INTRODUCTION

The dynamics, the concepts and theory that involves physics, can be related to the characteristics of the movement and deformation of a particular body or system whose behavior is analyzed on the principles of analytical mechanics, kinematics and kinetics. Mathematical concepts through the governing equations are also applied to understanding the dynamics of a system, and the results of the solution of these balances between, describe the different states for the specific conditions.

The studies of dynamical systems begin, when Aristotle (384-322 BC) becomes the precursor of the concept of teleology, which can be defined as the explanation for something according to their purpose or intended [1]. Aristotle (384-322 BC) developed different theories involving the field of physics, one of them describing the four elements earth, air, fire and water, and its dynamic relationships exemplified in nature, which described the characteristics that constituted the movement object, namely, the function and its purpose, [1].

After, different researchers contributed information to classical mechanics as Galileo (1564-1642), Isaac Newton (1642-1727) and Leibniz (1646-1716), Leonhard Paul Euler (1707-1783), Jean le Rond d ' Alembert (1717-1783), Joseph-Louis Lagrange (1736-1813), Pierre Simon Marquis de Laplace (1749-1827), Alexander Mikhailovich Lyapunov (1857-1918), Poincaré (1854-1912), [2; 3; 4; 5; 6].

Currently modern dynamics, derived from classical dynamics can be called deterministic dynamics, which aims to describe the state of a system, adopting the idea that the next state in time is determined solely by the current state [7]. The theory of dynamical systems applied to dynamic rotor is an excellent example of use. For a dynamic rotor system, the rotating shaft of the flotation given by the coupling of the lateral displacement and gyroscopic effect is a phenomenon difficult to control and often occurs in applications related to aircraft [8]. The rotor misalignment is one of the most common problems encountered when studying rotating machines, which also remains incomplete in his understanding, which, so far, no satisfactory analysis explains the range of different phenomena observed in this process [9].

Different researchers have applied the criterion of Routh-Hurwitz to identify the stability of a rotor, and one work are studied three methods of algorithms that perform the Routh-

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Hurwitz test, namely, a stable and simple, one unstable and fast, and an intermediary between the two commented, and all applied in an ecosystem [10]. The criterion of Routh-Hurwitz stability proving the method of elementary geometric considerations in the complex plane, being applied in control systems theory [11].

In the other study the application of a new robust control method in a nonlinear system, whose purpose is to control the variation of the horizontal speed of the wind turbine through a signal without the knowledge of the general parameters of this turbine [12]. Other case, it seeks to ensure traced speed control and regulation of reactive stator power, for it was built a controller based on the concept of Lyapunov, which ensured uniformity in the boundary region, all the signs the closed loop system [13].

The dynamics of a system whose qualitative model is represented by a system of several degrees of freedom and uses different theories vibration to finally obtain the system control using a proportional–integral–derivative(PID) controller [14]. Was developed a linear quadratic regulator LQR control research to analyze the dynamic behavior of vehicles, proposing solutions considering the torque and angles of the wheels [15]. Already it is presents a new methodology using LQR theory considering the parametric optimization with piezoelectric actuators installed in a laminar composite structure [16].

Dynamic Rotors

Studies of dynamic impellers come from 1895, and in Föppl research, generally consisted of a single disk positioned symmetrically between the rotary axis and undamped, demonstrating the stability of the system at supercritical speed [17]. The result of this study showed results as: a) a shaft may have several critical speeds, and depending on certain conditions, those speeds are the natural frequencies of a non-rotatable shaft; b) the introduction of the general theory of Reynolds to meet the critical speed; c) gyroscopic effect regarding the dependence of speed, and even the calculations of vibrations unbalancing. But even considering all this information, the shaft behavior analysis for different speeds, they were not enough to fully understand this dynamic system [18].

Jeffcott in 1919 used the same model introducing the concept of damping, and published this work in an English newspaper, and since then, this model is considered Jeffcott's rotor[17].

Dynamic rotors are designed for specific areas, while many of the results presented in various research works are difficult to analyze and thus to obtain a possible result, the general understanding of a setting is required [17]. The areas of application of the dynamic rotor may be, turbines, generators, compressors, among others, whose forces acting on the shaft are bending and torsion forces both vibrations are considered. The dynamic vibrations of the rotor depend on the geometry, structure, bearing type, and excitation forces, among others, which can result to system instability[18; 19].

“The instability of a dynamic rotor is a special case of the general theory of dynamic instability” [19].

The dynamic phenomena of a rotor, provided that modeled, can be studied considering a system as a single degree of freedom, SDOF [18], and multi dregree of freedom, MDOF [20].

Nowadays vibration theory for dynamic rotor systems, is called Jeffcott Rotors theory, and contains a placed disk symmetrically in the middle of the axis and can be modeled mathematically considering the physical conditions of relative movement of the axis of flexion, and the gyroscopic effect [18,21]. The details of Jeffcott rotor system on a cartesian plane and the phase angle displacement from the point G (disc center of mass), can be describe as ω is the angular velocity, $t$is time, $\zeta$is the damping factor, and C and G points, referenced to cartesian planex, y, are respectively represented by u_{xC}, u_{yC} and u_{xG}, u_{yG}, and segment is $\overline{AC} = u_c$, and $\overline{CG} = e_u$. $\theta = \omega t$ is the phase angle, and φ is the angle of displacement of the mass center disk coupled to the shaft [18; 21]. It considers that the system has no loss, and assuming that the disk mass does not affect shaft stiffness, and bending is uniform, and the shaft simply supported shaft stiffness equation $K_e$it is written as, $K_e = \frac{48.E.I}{L^3}$, where$E$is the modulus of elasticity, $I$is the moment of inertia. C_e is the axis of the damping coefficient. Therefore, considering that the axis is uniform in diameter d, the moment of inertia of the equation $I$is written as, $I = \frac{\pi d^4}{64}$.

The equation of motion for a homogeneous solution [18; 21], they are written as,

$$m\ddot{u}_{xG} + C_e\dot{u}_{xC} + K_e u_{xC} = 0 \quad (1)$$

$$m\ddot{u}_{yG} + C_e\dot{u}_{yC} + K_e u_{yC} = 0 \quad (2)$$

Considering that $u_{xC} = A_x e^{st}$, and $u_{yC} = A_y e^{st}$, and$A$is amplitude and $e^{st} = e^{i\omega t} = \cos(\omega t) + isen\,(\omega t)$ which is the notation of a complex exponential, which can simultaneously treat two forms of harmonic excitation [22].The homogeneous solution by Laplace method is [22],

$$(ms^2 + C_s + K_s)A_x e^{st} = 0 \quad (3)$$

$$(ms^2 + C_s + K_s)A_y e^{st} = 0 \quad (4)$$

where, C_s is damping, and K_s is the stiffness of the shaft in complex form, and the characteristic damped equation rotor system written in general is

$$ms^2 + C_s + K_s = 0 \quad (5)$$

and its roots, the eigenvalues are,

$$s_{1,2} = -\frac{C_s}{2.m} \pm i\sqrt{\frac{K_s}{m} - \left(\frac{C_s}{2.m}\right)} \quad (6)$$

It is possible to obtain information from the behavior of a linear dynamic rotor through the governing equations, such as natural vibrations, unbalancing and vibrational transients, using the modal analysis technique [23]. The application of this technique results as a response of an inelastic rotation with symmetric matrices and proportional damping.

The gyroscopic effect occurs misalignment of the shaft relative to its bearing members. The movements which occur in the gyroscopic effect, result in the axis of rotation around the X and Y axis, the disc center of mass G [21], A mathematical model for a dynamic rotor, and also consider the conditions of the physical forces acting on the system, so the coupled equations representing the system can be written vectorially, and thus represented by the following mathematical model [21; 24],

$$m\ddot{X} + \omega G\dot{X} + KX = 0 \quad (7)$$

Generalizing the state vector, we have, $X = \begin{bmatrix} x_G \\ y_G \\ \theta_{xG} \\ \theta_{yG} \end{bmatrix}$, the array of mass m, the gyroscopic mass matrix G and K the stiffness matrix, are respectively written as, $m = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & J_t & 0 \\ 0 & 0 & 0 & J_t \end{bmatrix}$, $G = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_p \\ 0 & 0 & -J_p & 0 \end{bmatrix}$, $K = \begin{bmatrix} \alpha & 0 & 0 & \gamma \\ 0 & \alpha & \gamma & 0 \\ 0 & \gamma & \delta & 0 \\ \gamma & 0 & 0 & \delta \end{bmatrix}$.

and m the mass of the shaft, $\ddot{x}_G, \ddot{y}_G$ represents the second derivative of displacement in the X axis, and displacement Y, in relation to the geometric center, respectively; $\ddot{\theta}_{xG}, \ddot{\theta}_{yG}$ representing the second derivative of the angular displacement relative to the center of mass G of the disk axes X and Y, respectively; $\dot{\theta}_{xG}, \dot{\theta}_{yG}$ representing the first derivative of the angular displacement relative to the center of mass G of the disk axes X and Y, respectively; and finally, ω is the angular frequency. The α, γ, δ are equations that define the stiffness parameter, therefore, $\alpha = K_1 + K_2$, $\gamma = -K_1 a + K_2 b$, $\delta = K_1 a^2 + K_2 b^2$, being K_1, K_2 stiffness constants; and a, b represent the distance between the mass center of the shaft [21; 24]. The x_G, y_G are the rotational movements relative to the disc center of mass G, the equations are $x_G = \frac{1}{L}(bx_1 + ax_2)$, $y_G = \frac{1}{L}(by_1 + ay_2)$. The θ_{xG}, θ_{yG} are the angular displacements relative to the disc center of mass G treated as the result of gyroscopic forces, the equation are $y_G = \frac{1}{L}(by_1 + ay_2)$, $\theta_{yG} = \frac{1}{L}(x_2 - x_1)$, being L is the total distance between the bearings of the shaft, and a and b correspond to the distance from the center axis of mass G until the end of the shaft. The J_t, J_p are respectively the transverse moment of inertia, and moment of rotational inertia, and can be calculated as $J_t = \frac{m}{4}\left(R^2 + \frac{1}{3}L^2\right)$, and $J_p = \frac{mR^2}{2}$.

The precession is calculated by the symmetrical inclination of the gyroscopic matrix G representing the coupling between the motion of the x and y axis. In the case of uncoupling of the translational movement and angular movement, $\gamma = 0$, the matrix condition, should be considered that $K = K_1 = K_2$, and the rotor is axially symmetrical about the center of mass, where $\frac{L}{2} = a = b$ [21; 24]. The stiffness matrix is then written

as, $K = \begin{bmatrix} \alpha & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \delta & 0 \\ 0 & 0 & 0 & \delta \end{bmatrix}$.

Experimental Methods - System Characterization

The experimental method was developed based on a practical experiment designed to provide a signal of a dynamic rotor. The experiment is based on a 0,5 hp motor of the 60 Hz; the frequency inverter is able to operate in the frequency 0-60 Hz; the shaft is 1045 steel and has a diameter of 6,0 mm; shaft length 120,0 mm; two rolling bearings P205-UC205. The equipment used for measuring the vibration, operates in the frequency range of 10 at 1 kHz, with an approximate sensitivity vibration sensor 15 mA DC, with all these devices were manufactured satisfying the standard ISO2954-2012 [25].

This procedure for the characterization of the system, consisting of a simple analysis is to verify that the difference between the amplitudes of the numerical simulation with the experimental values, given the same initial conditions [26]. The initial conditions for this experiment and numerical simulation are: mass $m = 0{,}5\, Kg$; stiffness $k = 20.10^3 \frac{kgf}{mm^2}$; frequency $f = 60\, Hz$; shaft diameter $d = 6{,}0\, mm$; distance to the shaft center $a = b = 60{,}0\, mm$; shaft length $L = 120{,}0\, mm$; and damping $\zeta = 0{,}01$.

The numerical simulation was performed using the coupled equations that make up the system, and considering the initial conditions.

The signals of the accelerometer were obtained considering two different experimental applications, the first being, signal capture using new bearings, and the second was using a good and bad bearing. The method for capturing the acceleration of experimental applications, first and second signal was placing the accelerometer in the bearing 1 and after 2.

The results of this analysis show that the amplitudes of the numerical simulation and the experimental signal are similar in size and that when the amplitudes have different sizes it is possible to state that the bearing-bearing system has interference. Therefore, the characterization of the system is valid when the amplitudes of the numerical model have the same size as the signal of the experimental model [26].

The error in this analysis was done using the statistical technique of error analysis by finite sampling, which can Table 1 show the difference between the results.

Thus, it can be said that the results found is valid and the experimental method can be applied for future experiments [26].

Table 1 Error analysis sampling finite - 60Hz

Variance Table and Standard Deviation

Aceleration	Characteristic	Variance	Standart Deviation
Theoretical	Theoretical Model	0,102741	0,320533
point 1	flawless	0,151893	0,389734
point 2	flawless	0,150246	0,387616
point 1	flawless	0,263224	0,513054
point 2	faulty	0,369774	0,608091

Analysis Using Stability Routh-Hurwitz Criteria

The method of Routh-Hurwitz, applied to the dynamic system under study aims to show the result of qualitative stability.

The equations of the dynamic system equation 7 can also be written as, $m\ddot{x}_G + \alpha x_G - \gamma\theta_{yG} = 0$; $m\ddot{y}_G + \alpha y_G - \gamma\theta_{xG} = 0$;

$J_t\ddot{\theta}_{xG} + J_p\omega\dot{\theta}_{yG} + \gamma x_G + \delta\theta_{xG} = 0$; $J_t\ddot{\theta}_{yG} - J_p\omega\dot{\theta}_{xG} + \gamma y_G + \delta\theta_{yG} = 0$, considered equations of the dynamic rotor, [18; 21]. Using the initial conditions of the system, and applying them in the dynamic equations, the result of stability, analytically solved, can be found using the criteria of Routh-Hurwitz.

Thus, replacing the values of the initial conditions in the equations of the dynamic system, we have these equations rewritten,

$$0{,}5\ddot{x}_G + 0{,}92443x_G = 0 \quad (8)$$

$$0{,}5\ddot{y}_G + 0{,}92443y_G = 0 \quad (9)$$

$$0{,}19744\ddot{\theta}_{xG} + 0{,}32197 \cdot 3{,}0815\dot{\theta}_{yG} + 0{,}053121\theta_{xG} = 0 \quad (10)$$

$$0{,}19744\ddot{\theta}_{yG} - 0{,}32197 \cdot 3{,}0815\dot{\theta}_{xG} + 0{,}053121\theta_{yG} = 0 \quad (11)$$

Then, the equations 8-11, these are respectively rewritten as,

$0{,}5s^2 + 0{,}92443s = 0$;
$0{,}5s^2 + 0{,}92443s = 0$;
$0{,}19744s^2 + 0{,}053121s = 0$;
$0{,}19744s^2 + 0{,}053121s = 0$.

Transferring the values of the coefficients of the polynomial equations for different tables, and applying the condition$\Delta = \begin{vmatrix} a_1 & a_3 \\ 1 & a_2 \end{vmatrix}$, we have the solution to the equations, respectively,

equation solution 8, $\Delta = \begin{vmatrix} 0{,}5 & 0 \\ 1 & 0{,}92443 \end{vmatrix}$;
equation solution 9, $\Delta = \begin{vmatrix} 0{,}5 & 0 \\ 1 & 0{,}92443 \end{vmatrix}$;
equation solution 10, $\Delta = \begin{vmatrix} 0{,}19744 & 0 \\ 1 & 0{,}053121 \end{vmatrix}$;
equation solution 11, $\Delta = \begin{vmatrix} 0{,}19744 & 0 \\ 1 & 0{,}053121 \end{vmatrix}$.

Observe that in the result,Δ , the leftmost column is positive in all terms, and the results of the roots of polynomial equations, these equations can be shown in table 2.

Table 2 Roots of polynomial equations

Equation Roots		
Axis x	Δ_1	0
	Δ_2	-66667
Axis y	Δ_1	0
	Δ_2	-66667
Angular Displacement x	Δ_1	-3327,1
	Δ_2	0
Angular Displacement y	Δ_1	-3327,1
	Δ_2	0

The criterion of Ruth-Hurwtiz ensures that all the roots of this polynomial have negative real part if $a_j$coefficients are positive [27].

Whereas the system is characterized, after, it is done the stability analysis by Routh-Hurwitz method, allowing you to create criteria of the stability behavior. The Rout-Hurwitz method is a qualitative analysis method that determines that the results can be stable or unstable.

The results of the solution of the dynamical system equations shows that all the equations of the first column on the left side are positive values, and table 2, the roots of the equations are negative, and thus, it can be stated that the system study is asymptotically stable.

Another significant analytical approach shows that the result of the characterization is similar amplitudes and associated forces balance condition in the system, and the result of asymptotically stable stability, the overall result is seen as more efficient and higher quality.

Therefore, it is understood that the results so far, not only show the state of stability of the initial conditions, but also serves as an analysis parameter of different results for the signals acquired dynamic rotors.

Mechanics Instability

In a mechanical system design is necessary to consider phenomena linked to stability, and that can be analyzed mathematically through equations of motion representing the spatial model [8].

The dynamic system disturbed by external agents is stable when the response to disturbance, stands close to its equilibrium position, while instability considering the same condition of disturbance of external agents, it can be understood as the removal of the equilibrium point [8]. The system with one degree of freedom and having random movement may result in chaotic oscillations [28].

The notation often used for the analysis of stability is the theory of Lyapunov. Considering the equations of motion of a dynamical system, you can determine if this system is stable or unstable by direct and indirect method of Lyapunov [8].

In a linear system, $\dot{x} = ax + by = f(x,y)$; $\dot{y} = cx + dy = g(x,y)$; the general solution, considering the equilibrium point $(x^*, y^*) = (0,0)$, can be written as, $x(t) = e^{\lambda t}x_0$; $y(t) = e^{\lambda t}y_0$. Therefore, the homogeneous system can be written as$(a-\lambda)x_0 + by_0 = 0$; $cx_0 + (d-\lambda)y_0 = 0$, and that a system is non-homogeneous, the determinant of the matrix, which is the system must be equal to zero,$det\begin{bmatrix} (a-\lambda) & b \\ c & (d-\lambda) \end{bmatrix} = 0$ [29]. Mathematically have a polynomial $\lambda^2 + ad - bc - a\lambda - d\lambda = 0$; the result of which the roots are eigenvalues$\lambda_i$with real and imaginary.

The stability criterion for a linear system depends on the real part of the eigenvalue $Re(\lambda_i)$, so using the Lyapunov parameter, we have the conditions for $Re(\lambda_i) \neq 0$the hyperbolic equilibrium or not degenerated; for $Re(\lambda_i) < 0$the asymptotic stability; for $Re(\lambda_i) > 0$the asymptotic instability; for $Re(\lambda_i) = 0$ equilibrium is not hyperbolic, elliptical or degenerated [29; 30]. This method is known as the indirect method of Lyapunov [8].

The Lyapunov exponent, when applied, evaluates the sensitivity in the initial conditions of a discrete or continuous time system, the result shows the divergence of neighboring trajectories corresponding to an average rate in time [7; 29].

"A strange attractor can be seen as a result of an infinite number of stretching in one direction and contraction in other directions, combined with unfolding" [29].

Measurement of contraction and elongation can be measured and if $y\,(i+1) = f(y(i))$, and the factor of the Lyapunov exponent e^L measures the distance of δstretching toδ', then $\delta' = \delta e^L$. The equation $|f(y+\delta) - f(y)| = |\delta| e^L$, shows the dependence of the variation of elongation with$y$, ande^Lhas direct dependency relationship of the δ distance, measured in the plane of the two-dimensional axis [7; 29; 40]. Whereas N repetitions occur, the equation representing this system is written as, $|f^N(y+\delta) - f^N(y)| = |\delta| e^{NL}$, and evidencing the Lyapunov exponent L and we will then$L = \frac{1}{N} ln \left|\frac{(f^N(y+\delta) - f^N(y))}{\delta}\right|$. It is observed that the elongation still depends on N andδ, considering the elongation of a small distance infinitesimal, $\delta \to \infty$for$N \to \infty$, the equation is written as, $L(y) = \lim_{N\to\infty} \lim_{\delta\to 0} \frac{1}{N} ln \left|\frac{(f^N(y+\delta) - f^N(y))}{\delta}\right|$. This equation can also be written as $L(y) = \lim_{N\to\infty} \frac{1}{N} ln \left|\frac{df^N(y)}{dy}\right|$.

The result of Lyapunov exponent is then defined as, if $L > 0$the exponential divergence is considered an unstable node; if$L < 0$then the exponential is convergent, considered a stable node [7;29].

Instability Analysis System with Imbalance Mass

To from the system to be considered characterized, the sequence is to study the vibration signal of the rotor dynamic system by introducing anunbalancing mass. The result of this study is to see if the signal from the vibration of the dynamic rotor has a chaotic behavior, and classifies it according to the conditions of Lyapunov exponent.

At first, the experiment, a mass unbalanced the mounting disc is inserted, secured to the shaft, and the initial conditions of the system are, $m = 0{,}6\,kg$; $k = 20.10^3 \frac{kgf}{mm^2}$; $f = 60Hz$; $d = 6{,}0\,mm$; $a = b = 60{,}0\,mm$; $L = 120{,}0\,mm$; $\zeta = 0{,}01$. The experiment was the same system used for the characterization process, and with the same components and measurement apparatus. Only the working frequency was changed in procedure, $f = 20\,Hz$ e $f = 25\,Hz$, therefore, it was observed that frequencies exceeding that were no longer necessary because of the result found. The numerical simulation was made considering the same equations and only by changing the mass, note figures 1 and 2

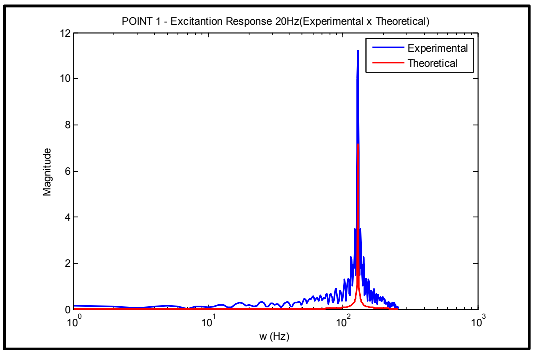

Fig 1 Comparison between the numerical simulation and experimental signal - Position 1 (20Hz)

It is observed that figures 1 and 2, both contain differences in amplitudes, between the results of the theoretical and experimental model, and can be explained by the introduction of the unbalance mass, which was introduced in the disk centered in the middle of the axis, between the Bearings. If the unbalance mass was not introduced into the system, the amplitudes would be the same between the result of the numerical simulation and the experimental signal.

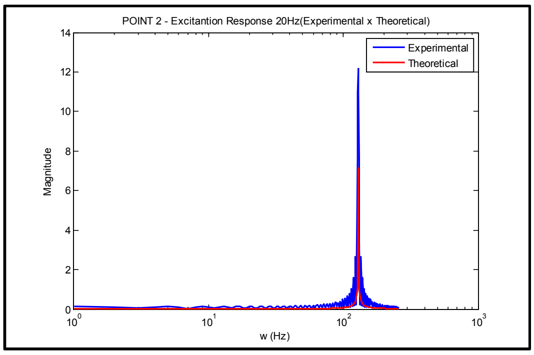

Fig 2 Comparison between the numerical simulation and experimental signal - Position 2 (20Hz)

The same procedure was done for the frequency of 25Hz, and it was noted that the result between the amplitudes increased when compared to the frequency of 20Hz.

Observing the increasing amplitude analysis, it can be seen that when the frequency is 60 Hz, the amplitudes would be even larger, hence for this experiment, the maximum frequency was 25 Hz.

Considering also that the sampling procedure was the same standard, developed in the characterization of the experiment, the error calculated by the error analysis method was 2%. Table 3 shows the standard deviation and variance for each measured point, which shows the difference between the theoretical and experimental variance for each frequency, 20Hz and 25Hz, together with the standard deviation.

Note that with the introduction of unbalancing mass, the amplitudes of the numerical simulation and experimental signal has difference, therefore, is necessary to investigate the behavior of the state system stability.

Table 3. Error Analysis Sampling Finite - 20 Hz and 25 Hz

Variance Table and Standard Deviation

Characteristic	Variance	Standart Deviation
Theoretical	0,015829	0,124549
point 1, 2 - 20Hz	0,039139	0,197836
point 1, 2 - 25Hz	1,356900	1,164860

Thus, using the indirect method of Lyapunov, the result of analysis of the state of stability (see Figure 1-table 4).

Analytical Solution Stability

The differential equations of the dynamic rotor, which represent the spatial model [18; 21], will be developed analytically by applying the Lyapunov technique [8; 28; 29; 30]. Therefore, considering the system of equations, and substituting the terms with the initial conditions of the system, we will,

$$0{,}6\ddot{x}_G + 0{,}92443 x_G = 0 \quad (12)$$

$$0{,}6\ddot{y}_G + 0{,}92443 y_G = 0 \quad (13)$$

$$0{,}19744\ddot{\theta}_{xG} + 0{,}32197 \cdot 3{,}0815\dot{\theta}_{yG} + 0{,}053121\theta_{xG} = 0 \quad (14)$$

$0{,}19744\ddot{\theta}_{yG} - 0{,}32197 \cdot 3{,}0815\dot{\theta}_{xG} + 0{,}0531 21\theta_{yG} = 0$ (15)

The solution characteristic of the equations 12-15 are written respectively as,

$f(x) = 1[(k_1 \cdot \cos(1{,}241\ x) + (k_2 \cdot \text{sen}(1{,}241\ x)]$
$f(y) = 1[(k_1 \cdot \cos(1{,}241\ y) + (k_2 \cdot \text{sen}(1{,}241\ y)]$
$f(x) = 1[(k_1 \cdot \cos(4{,}3525\ y) + (k_2 \cdot \text{sen}(4{,}3525\ y)]$
$f(y) = C_1 e^{18{,}944\ x} + C_2 \cdot x \cdot e^{-18{,}944\ x}$

For analytical solution, the equation is that the lateral displacement, the characteristic equation was found and the result is the equilibrium point in the x and y axes, and the method used was a solution to ordinary differential equation of a single variable. For the equation representing the angular displacement, was also found a characteristic equation for each situation, with solution of the equilibrium point in the x and y axes, however, the theorem was used for ordinary differential equation of two variables, with the direct method of Lyapunov working with its eigenvalues and eigenvectors (see Figure 2-table 5).

It is observed that for the lateral displacement phenomenon in x and y, the eigenvalues are equal to zero and the eigenvalues are positive and negative respectively, it shows that the equilibrium point can be characterized as inflected node (hyperbolic), and stability as asymptotically stable. While for the angular displacement, x, that is, the eigenvalue equal to zero, with positive and negative eigenvalues, respectively, and its state classified as inflected node (hyperbolic), and stability as a stable asymptotically. Already, in y, the eigenvalue is nonzero, and the eigenvector is zero, so the equilibrium point is classified as saddle (hyperbolic), and is unstable.

In general, we can say that the system is unstable stability.
Because the condition of the system under study, be unstable stability, it was numerically simulated the Lyapunov exponent to analyze the existence of chaos in the system. Thus, Figure 3 shows the result of Lyapunov exponent.

It is observed that the Lyapunov exponents from 1 to 5 are positive, and according to the state analysis is unstable node and may be associated with chaos, while the Lyapunov exponents 6 to 8 are negative, with stable node [7;29].

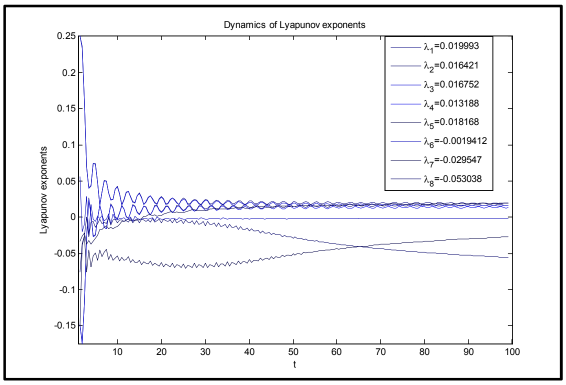

Fig 3 Lyapunov Exponent

Control Project

The system so far has unstable behavior and may be associated with chaos, and so, you can use the optimal control theory in a project that comes from the need for a system to behave in the pre-established form, and also when applied to a dynamic system operates with a minimum cost using the linear quadratic regulator technique LQR [31; 32].

Thus, to control the unstable signal rotor vibration with the same initial conditions, it will be used the technique of linear optimal control, LOC. It is desired that the result is to minimize the chaotic vibrations of the system by reducing the movement until a stable point.

For the chaotic motion reduction solution is used the linear state-feedback control proposed by [33], and is applied to various systems by [34; 35; 36; 37; 38], and a proposed problem by [39]. For $y(0) = y_0$,

$$\dot{y} = A(t)y + h(y) + Bu \quad (16)$$

Were $y \in \Re^{n}$ is a state vector, $A(t) \in \Re^{nxn}$ is a boundary conditions matrix (parameters) which elements are time depending, $B \in \Re^{nxm}$ is a matrix of constants, $u \in \Re^{m}$ is a control vector and $h(y) \in \Re^{n}$ is a vector which the elements are continuous non-linear functions, $h(0) = 0$. It is highlighted that the chosen of $A(t)$ is not the only influence of controller efficiency. For a finite time interval and A, B, Q and R been matrixes of constants elements, the positive defined matrix P is the solution of the algebric non-linear Riccati equation, given by:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \quad (17)$$

Control Theory Application

Through numerical simulation and using the initial conditions of the system, we obtain, $B = \begin{bmatrix}1\\1\\1\\1\\1\\1\\1\\1\end{bmatrix}$; $y = \begin{bmatrix}x_1 - \tilde{x}_1\\x_2 - \tilde{x}_2\\x_3 - \tilde{x}_3\\x_4 - \tilde{x}_4\\x_5 - \tilde{x}_5\\x_6 - \tilde{x}_6\\x_7 - \tilde{x}_7\\x_8 - \tilde{x}_8\end{bmatrix}$; $\tilde{x} = \begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix}$, the matrix $Q = I_8$ is a positive-definite matrix, i.e., the eigenvalues are positive, in this case $\lambda_{1,8} = 1$ and $R = [1]$, (see Figure 3-matrix A), and $M|B|AB|A^2B \ldots |A^{2n}B| \neq 0$, Then, it is considered that the dynamic system is controlled, (see Figure 4-matrix P), and after we obtain simplified optimal control, $u = -1{,}02044x_1 - 0{,}3953x_2 + 0{,}9582x_3 - 0{,}2800x_4 - 0{,}3760x_5 - 0{,}9182x_6 + 5{,}0923x_7 + 4{,}5320x_8$.
System path without control is grayed out, and with control, is black, and can be seen through the figure from 4 to 7.

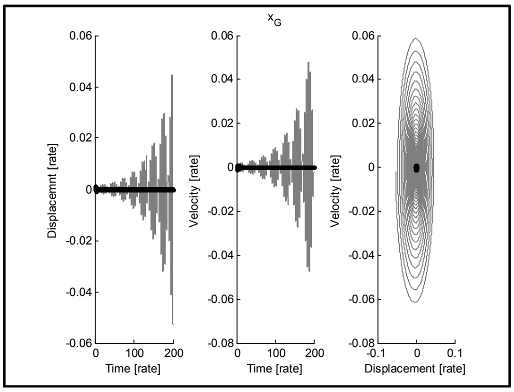

Fig 4 Lateral Shift Control x_G

Figure 4 shows the controlled signal response is black, and the uncontrolled are gray, and the first figure on the left side, displacement in time, and the center, the speed in time, and next the right to the phase space. Figure 5 is the same representation as in Figure 4.

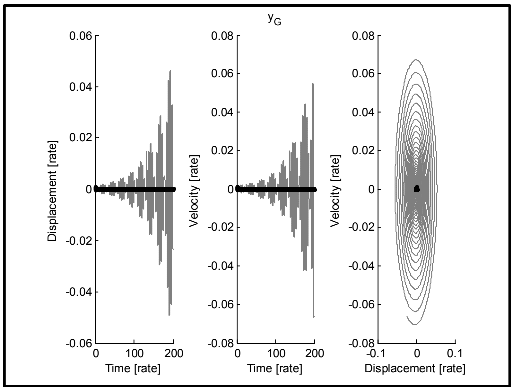

Fig 5 Lateral Shift Controly_G

Figure 6 and 7 are shown for angular displacement in x and y, respectively, and has the same descriptive representation of the positioning of Figures 4 and 5.

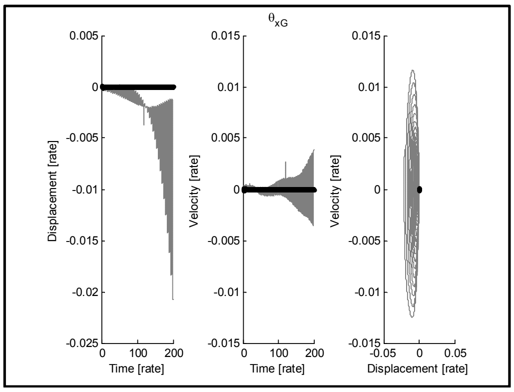

Fig 6 Angular Displacement Controlθ_{x_G}

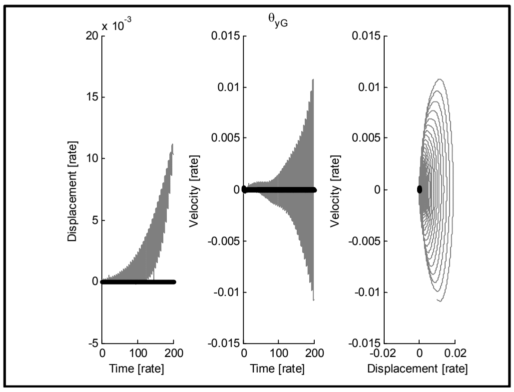

Fig 7 Angular Displacement Control θ_{y_G}

CONCLUSION

It is observed that the results of unstable and chaotic stability behavior were controlled using a control strategy, whose technique used is the optimal linear control, which reduces the chaotic movement of this system to the equilibrium point. Therefore, it can be said that the controller developed in this research meets the needs of stabilizing the signal of an unstable rotor up to the point of equilibrium.

The application of this method can be done in rotary machines of predictive maintenance, improving the effective control and consequently the reduction of costs.

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