



## CONSTRUCTION OF GROUP ACCEPTANCE SAMPLING PLAN USING WEIGHTED BINOMIAL DISTRIBUTION

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### ABSTRACT

Acceptance sampling plans developed relating to groups of items on testers is called the group acceptance sampling plan (GASP). In this scheme, a sample of items is distributed into different groups and a lot of product is rejected if more than a specified number of failures are recorded in any group. A group acceptance sampling schedule can be used to save the time and cost in inspection as compared with classical acceptance sampling. In this paper, a procedure for constructing a group acceptance sampling plan (GASP) using weighted binomial distribution in the construction of sampling plan is proposed. Suitable tables and examples are also provided for easy selection of the plans.

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**Key words:** Acceptance sampling plan, Group sampling, Pareto distribution of the second kind, consumer's risk, producer's risk, weighted binomial distribution..

### INTRODUCTION

In life testing more than one characteristic is tested in the experiment simultaneously and in these situations experimenters cannot use the ordinary acceptance sampling plan to test the items as it consumes more time and cost. Experimenters can treat items as a group and group acceptance sampling plans are applied. In this scheme, a sample of items is distributed into different groups for inspection and a lot is rejected if more than a specified number of failures are recorded in any group. Aslam *et.al.*, (2009) constructed group acceptance sampling plan using binomial distribution with Pareto distribution of the second kind in life testing. Radhakrishna Rao, C (1977) suggested weighted Binomial Distribution can be used in the construction of sampling plans. Sudeswari (2002) constructed single sampling plan using weighted binomial distribution as the base line distribution, Radhakrishnan and Mohanapriya (2008) suggested the weighted Poisson distribution can be used in the construction of sampling plans Radhakrishnan (2009) constructed six sigma based sampling plan with intervened random effect Poisson distribution and weighted Poisson distribution as the base line distributions. In this paper an attempt is made to construct group acceptance sampling plans using weighted binomial distribution as the base line distribution with Pareto distribution of the second kind in life testing.

Tables are also constructed for the easy selection of the plan, for the Engineers working in the assembly.

### 2. Glossary of symbols

p	-	Quality of submitted lots
$\alpha$	-	producer's risk
$\beta$	-	consumer's risk
g	-	Group size
r	-	Sample size in each group
c	-	Acceptance number
a	-	Termination ratio
$\lambda$	-	Shape parameter
$\mu_0$	-	Specified mean life
Pa (p)	-	Probability of acceptance of the lot quality p
n	-	Sample size

### 3. Design of Group Acceptance Sampling Plans Weighted binomial Distribution

The probability mass function of the weighted binomial distribution is given by

$$P(X: n, p, \alpha) = x^k p(X, n, p, \alpha) / \sum x^k p(X, n, p, \alpha), X = 0, 1, 2, \dots, n, k \geq 0$$

The probability mass function of the weighted binomial distribution for  $k = 1$  is given by

$$L(P) = \binom{r-1}{i-1} p^{i-1} (1-p)^{r-i} \quad i = 1, 2, 3, \dots, r$$

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**The Pareto Distribution of the Second Kind**

The probability density function and the cumulative distribution function (cdf) of the Pareto distribution of the second kind are given respectively in (1) and (2)

$$f(t, \sigma, \lambda) = \frac{\lambda}{\sigma} \left( t + \frac{t}{\sigma} \right)^{-(\lambda+1)}, t > 0, \lambda, \sigma, > 0 \dots (1)$$

$$F(t, \sigma, \lambda) = 1 - \left( t + \frac{t}{\sigma} \right)^{-\lambda}, t > 0, \lambda, \sigma, > 0 \dots (2)$$

Where  $\lambda$  and  $\sigma$  are shape and the scale parameters respectively. The mean life of the model (2) is given by

$$\mu = \sigma / (\lambda-1), \lambda > 1 \dots (3)$$

It is important to note that the mean function is only valid when  $\lambda > 1$ .

**4. Conditions for the application of GASP with WBD in the product control**

- Production is continuous, so that results of past, present and future lots are broadly indicative of a continuing process.
- Lots submitted may be sequentially.
- Inspection is by attributes, with the lot quality defined as the proportion defective.
- Items are to be submitted for inspection in groups.

**5. The operating procedure of Group Acceptance sampling plan**

The producer's risk and the consumer's risk are represented by  $\alpha$  and  $\beta$  respectively in the development of acceptance sampling plans. The producer's risk is defined as the probability of rejecting a good lot. The consumer's risk is the probability of accepting a bad lot. We are interested in constructing a GASP to ensure that the mean life  $\mu$  is greater than a specified mean life  $\mu_0$ .

The following scheme is used to develop the GASP:

- Determine the group size  $g$ . Sample  $n=gr$  items from a lot randomly and allocate  $r$  items to each group for the life test.
- Determine the acceptance number  $c$  for every group and specify the termination time of the life test  $t_0$  and
- Implement the life test based on the  $g$  groups of items simultaneously. Accept the lots if at most  $c$  failed items are found in every group by the termination time. Truncate the life test and reject the lot if more than  $c$  failures are found in any group.

The parameters in the GASP to be determined are the group size  $g$ , the sample size in each group  $r$ , the

acceptance number  $c$  and the termination ratio  $t/\mu_0$ . If  $r = 1$ , the GASP reduces to the ordinary acceptance sampling plan. As the decision is either to accept or to reject the lot on the basis of a sample selected from a big lot, the weighed binomial distribution can also be used to develop the GASPs. The lot acceptance probability for the proposed plan is given by

$$L(p) = \left[ \sum_{i=1}^c \binom{r-1}{i-1} p^{i-1} (1-p)^{r-i} \right]^g \dots (4)$$

Where  $p$  = function of cumulative distribution function given in (2).

It would be convenient to take the termination time as a multiple of the specified number  $a$ , i.e.,  $t_0 = a\mu_0$ . Therefore  $p$  can be derived from (2) as

$$p = \left[ 1 + \frac{a}{(\lambda - 1)(\mu / \mu_0)} \right]^{-\lambda} \dots (5)$$

The parameters  $g$ ,  $r$ ,  $c$  and  $\mu/\mu_0$  are to be determined. The minimal group size ( $g$ ) and minimal ratio ( $\mu/\mu_0$ ) of the true mean life to the specified mean life can be determined from the following equations (6) and (7)

$$L(p) \leq \beta \dots (6)$$

$$L(p) \geq 1 - \alpha \dots (7)$$

When consumer risk ( $\beta$ ) and producers risk ( $\alpha$ ) are specified.

**6. Construction of Tables**

The minimum group size ( $g$ ) is obtained using equations (4), (5), (6) & (7) for various values of  $\beta$ ,  $\alpha$ , termination ratio ( $a$ ) with  $r = 4$ ,  $a = 0.7$  and  $c = 2$  for a specified value of  $r = 4$ ,  $c = 2$ ,  $\alpha = 0.05$ ,  $\lambda = 2$  &  $3$  and with the help of Excel package the results are calculated and presented in table1, table2 and table3.

Table1: Minimal Group Sizes for GASP for  $r = 4$ ,  $c = 2$ , where  $\lambda=2$

a \ β	0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	2	2	2	2	2
0.10	4	3	4	4	4	4
0.05	8	6	7	8	8	7
0.01	11	10	9	7	7	9

Table2: Minimal Group Sizes for GASP for  $r = 4$ ,  $c = 2$ , where  $\lambda = 3$

a \ β	0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	2	3	3	3	3
0.10	4	4	5	5	5	5
0.05	5	5	6	7	7	6
0.01	7	8	9	10	10	9

Table 3: Minimal Values of GASP for Weighted Binomial Distribution for  $r = 4, c = 2$ , when  $\lambda = 2, \alpha = 0.05$

$\beta \backslash a$	0.7	0.8	1.0	1.2	1.5	2.0
0.25	13.2	14.1	17.1	20.5	25.7	380
0.10	20	20.5	28	33	42	860
0.05	22	23	32	38	47	100
0.01	28	30	37	48	60	1000

**7. Construction of GASP**

**Example1**

For a specified  $\beta = 0.05, a = 0.8, r = 4, c = 2$  and  $\lambda = 2$ , the value of  $g$  can be obtained from table 1 as 6. The relevant GASP is  $n = 24, g = 6, r = 4, c = 2$  for a specified  $\beta = 0.05$  and  $a = 0.8$ .

**Explanation**

If the consumer fixes  $\beta = 0.05$  (5 non-confirmative at of 100) the manufacturer has to select 24 items from the lot and allocate them into 6 groups of 4 items each and allow for life test till the time  $t_0$  and the life test is stopped when more than 2 failed items are recorded during the time and the lot is rejected, otherwise the lot is accepted. If the lot is rejected inform the management for corrective action.

**CONCLUSION**

In this paper a new procedure for the construction of

GASP using weighted binomial distribution as the base line distribution with Pareto distribution of the second kind in life testing is presented. Tables are also presented for the easy selection of groups by the engineers and additional tables can also be developed depending on the choice of  $a, \alpha, \beta, \lambda$  and  $c$ .

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