RESEARCH ARTICLE
ASYMPTOTIC PROPERTY FOR BAYESIAN MODIFIED CHAIN SAMPLING PLAN
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ABSTRACT
Asymptotic expansions clearly shows how the sampling plan depends on the parameters specified, such as quality levels, cost parameters and the parameters of the prior distribution. The minimum average regret function has been derived for Bayesian Modified Chain Sampling Plan(BMChSP-1).

INTRODUCTION
The dissimilarity between the conventional and Bayesian approach is associated with the utilization of prior process history or knowledge in selection of distribution to describe the random fluctuations involved in acceptance sampling. The Modified Chain Sampling Plan was developed by Govindaraju and Lai(1998) which always utilizes the recently available lot quality history. The operating procedure of the MChSP-1 plan is given below.

1. From each of the submitted lots, draw a random sample of size n. Reject the lot if one or more nonconforming units are found in the sample.
2. Accept the lot if no nonconforming units are found in the sample, provided that the preceding i samples also contained no nonconforming units except in one sample, which may contain at most one nonconforming unit. Otherwise, reject the lot.

The OC function \( P_a(p) \) of the MChSp-1 plan was derived by Govindaraju and Lai(1998) as

\[
P_a(p) = P_{0,n}(P_{0,n}^i + iP_{0,n}^{i-1}P_{1,n})
\]

With gamma as prior distribution, \( w(p) = e^{-np}p^{s-1}t^s/s \), \( s,t,p > 0 \) with parameters \( s \) and \( t \). The average probability of acceptance is given as

\[
\bar{P} = \int_0^1 P_a(p)w(p)dp = \frac{s^t}{(s + n\mu(1+i))^s} + \frac{s^{t+1}n\mu}{(s + n\mu(1+i))^{s+1}}
\]

where \( \mu = s/t \) the mean value of the product quality \( p \). Latha and Rajeswari(2012) have given the regret values for BChSP-1 for different sample sizes and represented graphically. Latha and Rajeswari(2012) have given the average regret function for Bayesian Special Type Double Sampling Plan. Hald (1981) has stated that the quantity most difficult to determine is normally the cost of accepting a defective item.

Average costs per item for acceptance \( k_a = A_1 + A_2 \bar{P} \)

Average costs per item for sampling inspection \( k_s = S_1 + S_2 \bar{P} \)

Average costs per item for rejection \( k_r = R_1 + R_2 \bar{P} \)

Let \( \mu \) be the mean value of the lot quality distribution. Each cost function contains two parameters. The first one describes the cost due to accepting, inspecting and rejecting an item without considering quality, whereas the second gives the additional cost per defective item. Assumptions considered towards the average cost function are that all these functions are non-negative and none of them is equal to zero. For a 100% good lot the cost of acceptance per item is the least compared with the cost of rejection whereas in a 100% defective lot the cost of acceptance per item is the highest compared with the cost of rejection. It is observed that

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The posterior distribution of $\theta$ becomes
\[
\int_0^\infty (A_1 + A_2 P)dw(P) + \int_0^\infty (R_1 + R_2 P)dw(P)
\]
\[= k_0 (A_2 - R_2) \int_0^\infty (P_t - P) \, dw(P).
\]

Where $w(p)$ is the distribution function of the product quality $P$. The regret is defined as the cost minus unavoidable cost and can be used for comparing the economic consequence for selection of different sampling plans.

The average function for Single Sampling Plan has been studied by Hald (1981). The average costs accepted and rejected product of quality $p$, for a Modified Chain Sampling Plan 1 is
\[
k(p) = n(S_1 + S_2 P) + (N - n)(A_1 + A_2 P)\Phi(p) + (R_1 + R_2 P)Q(p)
\]
\[= n(S_1 + S_2 P) + (N - n)\{(A_1 + A_2 P)\Phi(p) + (R_1 + R_2 P)\}
\]
Where $(p) = \{S_1 - R_1 + (S_2 - R_2 P)/(A_2 - R_2)\}$ . Averaging over the gamma prior distribution and simplified as
\[
K(n,N) = \int_0^\infty k(p)dw(p)
\]
\[= n k_0 + (N - n)k_0 + (N - n)(A_2 - R_2) \int_0^\infty (P_t - P)dw(p) + n(A_2 - R_2) \int_0^\infty \delta(p)P_t \, dw(p)
\]
where $k_m = N K_{n0} P^{k_1}$. Let the lot size,
\[
N = n + (N - n)(P_0^{k_1} + 1 - P_0 + \Phi(p)) + n P_0 + (N - n - n)(P_t P_0 + (P_0^{k_1} - 1))
\]
where $P_t$ is the breakeven quality level. Hald(1981) has given the asymptotic properties for Bayesian Double Sampling Plan. Pandey(1986) has given the asymptotic solution to Bayesian Three Decision Plan by attributes. Latha and Rageswari (2013) has given the asymptotic property for Bayesian Special Type Double Sampling Plan. Considering $\theta = n$ and rewriting equation (1),
\[
R(n,N) = \int_0^{\infty} \delta(\theta)P_t^{k_1}(\theta)w(\theta) \, d\theta
\]
\[+ \delta t \int_0^{\infty} \delta(\theta)P_t^{k_1}(\theta)w(\theta) \, d\theta
\]
where $P_t^{k_1}(\theta) = \Phi(\theta - h, n^{1/2}/\sigma(\theta))$. $P_t^{k_1}(\theta) = \Phi(\theta - h, n^{1/2}/\sigma(\theta))$ and changing the variable of integration from $\theta$ to $u = (\theta - h, n^{1/2}/\sigma(\theta)$ and expanding $\sigma(\theta)$ and $w(\theta)$ in Taylor’s series around $\theta$, we get,
\[
R = \int_0^{\infty} \delta(\theta)P_t^{k_1}(\theta)w(\theta) \, d\theta
\]
\[+ \delta t \int_0^{\infty} \delta(\theta)P_t^{k_1}(\theta)w(\theta) \, d\theta
\]
\[N - n = R \text{ is replaced by } N \text{ since } n = \text{ is at most of order } N^{1/2}. \text{ The problem is to determine the values of } n, \theta \text{ and } z \text{ minimizing } R. \text{ When } z \to \infty, \text{ the minR is obtained for } z \propto (ln N)^{1/2}. \text{ The posterior distribution of } \theta \text{ becomes } w_1(\theta) \sim \rho. \text{ Considering the regret value for first sample and multiplying it with the continuation probability we get,}
\[
R = \int_0^{\infty} \delta(\theta)P_t^{k_1}(\theta)w(\theta) \, d\theta
\]
\[+ \delta t \int_0^{\infty} \delta(\theta)P_t^{k_1}(\theta)w(\theta) \, d\theta
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\[N - n = R \text{ is replaced by } N \text{ since } n = \text{ is at most of order } N^{1/2}. \text{ The problem is to determine the values of } n, \theta \text{ and } z \text{ minimizing } R. \text{ When } z \to \infty, \text{ the minR is obtained for } z \propto (ln N)^{1/2}. \text{ The posterior distribution of } \theta \text{ becomes } w_1(\theta) \sim \rho. \text{ Considering the regret value for first sample and multiplying it with the continuation probability we get,}
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R = \int_0^{\infty} \delta(\theta)P_t^{k_1}(\theta)w(\theta) \, d\theta
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\[+ \delta t \int_0^{\infty} \delta(\theta)P_t^{k_1}(\theta)w(\theta) \, d\theta
\]

Setting the derivatives with respect to $n, \delta$ and $z$ equal to zero, we get,
\[
d_\delta - 2w_2\theta^{n^2/3}M_n(z) + 2w_2\theta^{n^2/3}M_n(z) + 2w_2\theta^{n^2/3}(\theta - n)\theta(z) = 0
\]
\[+ \frac{1}{2}w_2\theta^{n^2/3}(\delta, a_n)^{(2/2)} - 2w_2\theta^{n^2/3}M_n(z)\]
\[w_2\theta^{n^2/3}(N - \theta)\theta(z) = 0
\]
\[+ \frac{1}{2}w_2\theta^{n^2/3}(\delta, a_n)^{(2/2)} - 2w_2\theta^{n^2/3}M_n(z)\]
\[+ \frac{1}{2}w_2\theta^{n^2/3}(\delta, a_n)^{(2/2)} - 2w_2\theta^{n^2/3}M_n(z)\]

Multiplying equation (4) by $\frac{1}{2}w_2\theta^{n^2/3}N$, inserting into equation (5) and solving that
\[
\frac{m_2(z) + 2w_2\theta^{n^2/3}M_n(z)}{N/d_n}
\]

Solving equation (4) for $n$, we find,
\[
n^{-1/2}\delta_n^{1/2} - \frac{1}{2}w_2\theta^{n^2/3}(\delta, a_n)^{(2/2)} - 2w_2\theta^{n^2/3}M_n(z)
\]

Using, $m_2(z) = \frac{1}{2}(z)z^{-1}(1 + O(z^{-2}))$, $m_2(z) = \frac{1}{2}(z)z^{-1}(1 + O(z^{-2}))$, $m_2(z) = \frac{1}{2}(z)z^{-1}(1 + O(z^{-2}))$ and $m_2(z) = \frac{1}{2}(z)z^{-1}(1 + O(z^{-2}))$, we get,
\[
\left(\frac{1}{2}(z)z^{-1}(1 + O(z^{-2}))\right)^{1/2} - \frac{1}{2}w_2\theta^{n^2/3}(\delta, a_n)^{(2/2)} - 2w_2\theta^{n^2/3}M_n(z)
\]

Taking logarithm, we get,
\[
z \sim (2 \ln N)^{1/2}
\]

To find min $R$, we note that
\[
m_2(z) \sim 2(z)z^{-3} - 2m_0(z)z^{-2}
\]

so that the second term of equation(2) becomes,
\[
w_2\theta^{n^2/3}(\delta, a_n)^{(2/2)} - 2w_2\theta^{n^2/3}M_n(z)
\]

(5)
Since, $m_3(z) - 2m_2(z)z^{-2}m_1(z)z^{-1}$ the third term of equation(2) becomes,

$$s\sigma^2 n^{-1} Wm_0(z)z^{-2} - 8d_n z^{-3}$$

Since, $m_4(z) - 2m_2(z)z^{-2}2m_1(z)z^{-1}$ the fourth term of equation(2) becomes,

$$2\sigma^2 n^{-1} (N - \theta)m_0(z)z^{-1} - 2t_d z$$

To evaluate fifth term of equation(2), we use equation(5) to eliminate $(\delta, \sigma^2 N)^{1/2}$ and noting that, $zm_4(z) - m_0(z)$ we get,

$$4i\sigma^2 n^{-1} (N - \theta)m_1(z)z^{-1} - 4t_d z$$

From the sum of the five terms,

$$\min R - 11nd_s + 6\theta d_s$$

Illustration – 1

For the given value of N=300, $d_s = d_x = 0.05145$, $n=17$ and $i=1$ the minimum regret for BMChSP is 14.86905 where, the minimum regret for BChSP is 11.37045.

CONCLUSION

There is a need to control the quality of the product. This need arises from the desire of the manufacturer, the contractor, to produce a product for the purchaser, the state, in the most economical manner possible while meeting the specifications for the product. In that means, it is necessary to reduce the unavoidable cost. The formula obtained here gives the minimum regret which will be useful to the producers to obtain the benefit by cost reduction.

Reference