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RESEARCH ARTICLE

ON SOME PROPERTIES OF UNITARY AND NORMAL BIMATRICES

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ABSTRACT

The characterization of unitary and normal bimatrices are given as a generalization of the results for unitary and normal matrices. Also the unitarily equivalence of unitary and normal bimatrices are found.

INTRODUCTION

Let $_{n\times n}$ be the set of all matrices with complex entries of order $n\times n$. Let $_n$ be the collection of vectors with complex entries. A matrix $A_B = A_1 \cup A_2$ is called a bimatrix if A_1 and A_2 are matrices of same or different orders. we consider here only matrices of same order.

In this paper we analyse the characteristics of unitary and normal bimatrices analyses to that of the results in [1,2,5,6] **.**

Definition [7]

Let $A_B = A_1 \cup A_2$ be a bimatrix. If both A_1 and A_2 are square matrices then A_B is called the square bimatrix.

Definition [7]

An eigen bivalue of a bimatrix A_B is defined to be a zero of the polynomial $\det(\alpha_B I_B - A_B) = 0$.

Definition [7]

A nonzero bivector $(x_B \neq 0)$ in $_n$ is said to be a eigen bivector of a complex bimatrix A_B associated with a eigen bivalue α_B if it satisfies $A_B x_B = \alpha_B x_B$.

Unitary Bimatrices

In this section some important results of unitary matrices are generalized to unitary bimatrices.

Definition

Let $A_B = A_1 \cup A_2$ be a square bimatrix over $_{n\times n}$. A_B is called an unitary bimatrix if $A_B A_B^* = A_B^* A_B = I_B$ (or) $\overline{A}_B^{\,T} = A_B^{-1}$. That is $A_1 A_1^* \cup A_2 A_2^* = A_1^* A_1 \cup A_2^* A_2 = I_1 \cup I_2$.

Proposition

Let A_B be a square bimatrix. Then the following conditions are equivalent.

1. A_B is unitary bimatrix.
2. The rows of A_B form an orthonormal set of vectors.
3. The columns of A_B form an orthonormal set of vectors.
4. A_B preserves the inner product ,that is, for all vectors $x, y \in$ $_n$,we have

$$\langle A_B x, A_B y\rangle = \langle x, y\rangle_1 \cup \langle x, y\rangle_2 .$$

Proof

Write the matrix A_B column-wise.

$A_B = A_1 \cup A_2 = [a_1\ a_2\ \dots\ a_n] \cup [b_1\ b_2\ \dots\ b_n]$ so that

$$A_B^* = (A_1 \cup A_2)^* = A_1^* \cup A_2^* = \begin{bmatrix} a_1^{1^*} \\ a_2^{1^*} \\ . \\ . \\ . \\ a_n^{1^*} \end{bmatrix} \cup \begin{bmatrix} a_1^{2^*} \\ a_2^{2^*} \\ . \\ . \\ . \\ a_n^{2^*} \end{bmatrix}$$

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Now

$$A_B^* A_B = \left(\begin{bmatrix} a_1^{1^*} \\ a_2^{1^*} \\ . \\ . \\ . \\ a_n^{1^*} \end{bmatrix} \cup \begin{bmatrix} a_1^{2^*} \\ a_2^{2^*} \\ . \\ . \\ . \\ a_n^{2^*} \end{bmatrix} \right) \times \left(\begin{bmatrix} a_1^1 & a_2^1 & \dots & a_n^1 \end{bmatrix} \cup \begin{bmatrix} a_1^2 & a_2^2 & \dots & a_n^2 \end{bmatrix} \right)$$

$$= \left(\begin{bmatrix} a_1^{1^*} \\ a_2^{1^*} \\ . \\ . \\ . \\ a_n^{1^*} \end{bmatrix} \times \begin{bmatrix} a_1^1 & a_2^1 & \dots & a_n^1 \end{bmatrix} \right) \cup \left(\begin{bmatrix} a_1^{2^*} \\ a_2^{2^*} \\ . \\ . \\ . \\ a_n^{2^*} \end{bmatrix} \times \begin{bmatrix} a_1^2 & a_2^2 & \dots & a_n^2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} a_1^{1^*}a_1^1 & a_1^{1^*}a_2^1 & . & . & . & a_1^{1^*}a_n^1 \\ a_2^{1^*}a_1^1 & a_2^{1^*}a_2^1 & . & . & . & a_2^{1^*}a_n^1 \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ a_n^{1^*}a_1^1 & a_n^{1^*}a_2^1 & . & . & . & a_n^{1^*}a_n^1 \end{bmatrix} \cup \begin{bmatrix} a_1^{2^*}a_1^2 & a_1^{2^*}a_2^2 & . & . & . & a_1^{2^*}a_n^2 \\ a_2^{2^*}a_1^2 & a_2^{2^*}a_2^2 & . & . & . & a_2^{2^*}a_n^2 \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ a_n^{2^*}a_1^2 & a_n^{2^*}a_2^2 & . & . & . & a_n^{2^*}a_n^2 \end{bmatrix}$$

Hence $A_B^* A_B = I_B$ iff $(a_i^* a_j) \cup (b_i^* b_j) = 0$ for $i \neq j$ and $(a_i^* a_i) \cup (b_i^* b_i) = 1$ for $i = 1, 2, \dots, n,$ iff the columns vectors of A_B form an orthonormal set.

This proves $(i) \Leftrightarrow (ii)$. Since $A_B^* A_B = I_B$ implies $A_B A_B^* = I_B$, the proof of $(i) \Leftrightarrow (iii)$ follows.

To prove $(i) \Leftrightarrow (iv)$.

Let A_B be unitary bimatrix. Then $A_B^* A_B = A_B A_B^* = I_B$ and

$$\langle A_B x, A_B y \rangle = \langle (A_1 \cup A_2)x, (A_1 \cup A_2)y \rangle$$

$$= \langle (A_1 x \cup A_2 x), (A_1 y \cup A_2 y) \rangle$$

$$= \langle A_1 x, A_1 y \rangle_1 \cup \langle A_2 x, A_2 y \rangle_2$$

$$= \langle x, A_1^* A_1 y \rangle_1 \cup \langle x, A_2^* A_2 y \rangle_2$$

$$= \langle x, I_1 y \rangle_1 \cup \langle x, I_2 y \rangle_2$$

$$= \langle x, y \rangle_1 \cup \langle x, y \rangle_2$$

Thus $\langle A_B x, A_B y \rangle = \langle x, y \rangle_1 \cup \langle x, y \rangle_2$.

Conversely, iff A_B preserves inner product take $x = e_i$ and $y = e_j$ to get

$$e_i^* (A_B^* A_B) e_j = e_i^* e_j = ¢_{ij}$$

where $¢_{ij}$ are kronecker symbols ($¢_{ij} = 1$ if $i = j$; =0 otherwise). This means the $(i, j)^{th}$ entry of $A_B^* A_B$ is $¢_{ij}$.

Thus $A_B^* A_B = I_B$. (1)

Similarly, we can prove that $A_B A_B^* = I_B$. (2)

From (1) and (2), we get $A_B^* A_B = A_B A_B^* = I_B$.

Hence, A_B is a unitary bimatrix.

Corollary

Let A_B be a unitary bimatrix. Then

1. For all $x, y \in {}_n$, $\langle A_B x, A_B y \rangle = \langle x, y \rangle_1 \cup \langle x, y \rangle_2$. Hence $\|A_B x\| = \|x\|_1 \cup \|x\|_2$.
2. If a_B is an eigen bivalue of A_B then $|a_B| = 1$.
3. Eigen bivectors corresponding to different eigen bivalues are orthogonal.

Proof

(i) we have $\|A_B x\|^2 = \langle A_B x, A_B x \rangle$

$$= \langle (A_1 \cup A_2)x, (A_1 \cup A_2)x \rangle$$

$$= \langle (A_1 x \cup A_2 x), (A_1 x \cup A_2 x) \rangle$$

$$= \langle A_1 x, A_1 x \rangle_1 \cup \langle A_2 x, A_2 x \rangle_2$$

$$= \langle x, A_1^* A_1 x \rangle_1 \cup \langle x, A_2^* A_2 x \rangle_2$$

$$= \langle x, I_1 x \rangle_1 \cup \langle x, I_2 x \rangle_2$$

$$= \langle x, x \rangle_1 \cup \langle x, x \rangle_2$$

$$\|A_B x\|^2 = \|x\|_1^2 \cup \|x\|_2^2$$

Hence $\|A_B x\| = \|x\|_1 \cup \|x\|_2$.

(ii) If $a_B = a_1 \cup a_2$ is an eigen bivalue of A_B with eigen bivectors x_B then $A_B x_B = a_B x_B$.

$$\|x_B\| = \|A_B x_B\|$$

$$= \|(A_1 \cup A_2)(x_1 \cup x_2)\|$$

$$= \|A_1 x_1 \cup A_2 x_2\|$$

$$= \|a_1 x_1 \cup a_2 x_2\|$$

$$= \|(a_1 \cup a_2)(x_1 \cup x_2)\|$$

$$= |a_1 \cup a_2| \|x_1 \cup x_2\|$$

$$\|x_B\| = |a_B| \|x_B\|$$

Hence, $|\lambda_B| = 1$.

(iii) Let $A_B x_B = \lambda_B x_B$ and $A_B y_B = \mu_B x_B$ where x_B, y_B are eigenvectors with distinct eigen bivalues λ_B and μ_B respectively. Then

$$\langle x_B, y_B \rangle = \langle A_B x_B, A_B y_B \rangle$$
$$= \langle (A_1 \cup A_2)(x_1 \cup x_2), (A_1 \cup A_2)(y_1 \cup y_2) \rangle$$
$$= \langle A_1 x_1 \cup A_2 x_2, A_1 y_1 \cup A_2 y_2 \rangle$$
$$= \langle \lambda_1 x_1 \cup \lambda_2 x_2, \mu_1 y_1 \cup \mu_2 y_2 \rangle$$
$$= \langle (\lambda_1 \cup \lambda_2)(x_1 \cup x_2), (\mu_1 \cup \mu_2)(y_1 \cup y_2) \rangle$$
$$= \langle \lambda_B x_B, \mu_B y_B \rangle$$
$$= \bar{\lambda}_B \mu_B \langle x_B, y_B \rangle$$

Hence $\bar{\lambda}_B \mu_B = 1$ (or) $\langle x_B, y_B \rangle = 0$.

Since $\bar{\lambda}_B \lambda_B = 1$ we cannot have $\bar{\lambda}_B \mu_B = 1$.

Hence, $\langle x_B, y_B \rangle = 0$. That is, x_B and y_B are orthogonal.

Normal Bimatrices

In this section some of the properties of normal matrices are extended to normal bimatrices [3, 4]. Some important results of normal matrices are generalized to normal bimatrices.

Definition

A square bimatrix A_B is called normal bimatrix if $A_B A_B^* = A_B^* A_B$.

That is, $A_1 A_1^* \cup A_2 A_2^* = A_1^* A_1 \cup A_2^* A_2$.

Example

Let $A_B = A_1 \cup A_2 = \begin{bmatrix} 1 & 1 \\ i & 3+2i \end{bmatrix} \cup \begin{bmatrix} 2 & i \\ i & 2 \end{bmatrix}$

$A_B^* = \begin{bmatrix} 1 & -i \\ 1 & 3-2i \end{bmatrix} \cup \begin{bmatrix} 2 & -i \\ -i & 2 \end{bmatrix}$

$$A_B A_B^* = \left(\begin{bmatrix} 1 & 1 \\ i & 3+2i \end{bmatrix} \cup \begin{bmatrix} 2 & i \\ i & 2 \end{bmatrix} \right) \times \left(\begin{bmatrix} 1 & -i \\ 1 & 3-2i \end{bmatrix} \cup \begin{bmatrix} 2 & -i \\ -i & 2 \end{bmatrix} \right)$$
$$= \left(\begin{bmatrix} 1 & 1 \\ i & 3+2i \end{bmatrix} \times \begin{bmatrix} 1 & -i \\ 1 & 3-2i \end{bmatrix} \right) \cup \left(\begin{bmatrix} 2 & i \\ i & 2 \end{bmatrix} \times \begin{bmatrix} 2 & -i \\ -i & 2 \end{bmatrix} \right)$$
$$= \begin{bmatrix} 1+1 & -i+3-2i \\ i+3+2i & 1+9+4 \end{bmatrix} \cup \begin{bmatrix} 4+1 & -2i+2i \\ 2i-2i & 1+4 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 3-3i \\ 3+3i & 14 \end{bmatrix} \cup \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \qquad (3)$$

$$A_B^* A_B = \left(\begin{bmatrix} 1 & -i \\ 1 & 3-2i \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ i & 3+2i \end{bmatrix} \right) \cup \left(\begin{bmatrix} 2 & -i \\ -i & 2 \end{bmatrix} \times \begin{bmatrix} 2 & i \\ i & 2 \end{bmatrix} \right)$$
$$= \begin{bmatrix} 1+1 & 1-3i+2 \\ 1+3i+2 & 1+9+4 \end{bmatrix} \cup \begin{bmatrix} 4+1 & 2i-2i \\ -2i+2i & 1+4 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 3-3i \\ 3+3i & 14 \end{bmatrix} \cup \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \qquad (4)$$

From (3) and (4), we get $A_B A_B^* = A_B^* A_B$.

Hence, A_B is a normal bimatrix.

Lemma

For a normal bimatrix A_B we have $\|A_B x\|^2 = \|A_B^* x\|^2$ for all $x \in {}_n$.

Proof

Given that A_B is a normal bimatrix , That is, $A_B A_B^* = A_B^* A_B$.

Consider $\|A_B x\|^2 = \langle A_B x, A_B x \rangle$

$$= \langle (A_1 \cup A_2) x, (A_1 \cup A_2) x \rangle$$
$$= \langle A_1 x \cup A_2 x, A_1 x \cup A_2 x \rangle$$
$$= \langle A_1 x, A_1 x \rangle_1 \cup \langle A_2 x, A_2 x \rangle_2$$
$$= \langle x, A_1^* A_1 x \rangle_1 \cup \langle x, A_2^* A_2 x \rangle_2$$
$$= \langle x, A_1 A_1^* x \rangle_1 \cup \langle x, A_2 A_2^* x \rangle_2$$
$$= \langle A_1^* x, A_1^* x \rangle_1 \cup \langle A_2^* x, A_2^* x \rangle_2$$
$$= \langle A_1^* x \cup A_2^* x, A_1^* x \cup A_2^* x \rangle$$
$$= \langle (A_1^* \cup A_2^*) x, (A_1^* \cup A_2^*) x \rangle$$
$$= \langle (A_1 \cup A_2)^* x, (A_1 \cup A_2)^* x \rangle$$
$$= \langle A_B^* x, A_B^* x \rangle$$
$$= \|A_B^* x\|^2$$

Hence, $\|A_B x\|^2 = \|A_B^* x\|^2$ for all $x \in {}_n$.

Lemma

For a normal bimatrix A_B, $A_B x = 0$ if and only if $A_B^* x = 0$.

Proof

For a normal matrix A_B, $A_B A_B^* = A_B^* A_B^*$. That is , $A_1 A_1^* \cup A_2 A_2^* = A_1^* A_1 \cup A_2^* A_2$.

From the above lemma, we have $\|A_B x\|^2 = \|A_B^* x\|^2$

$\|A_B x\|^2 = \|A_B^* x\|^2 = 0$

$\|A_B x\| = \|A_B^* x\| = 0$

$A_B x = A_B^* x = 0$.

Hence, $A_B x = 0$ if and only if $A_B^* x = 0$.

Lemma

If A_B is normal bimatrix, then V_B is an eigen bivector of A_B with eigen bivalue $«_B$ iff V_B is an eigenbivector of A_B^* with eigenbivalue $\overline{«}_B$.

Proof

If A_B is normal bimatrix , then $A_B - «_B I_B$ is also nomal bimatrix.

Now $(A_B - «_B I_B)V_B = 0$ iff $\|(A_B - «_B I_B)V_B\| = 0$

iff $\left\|\left[(A_1 \cup A_2) - («_1 \cup «_2)(I_1 \cup I_2)\right](V_1 \cup V_2)\right\| = 0$

iff $\left\|\left[(A_1 \cup A_2) - («_1 I_1 \cup «_2 I_2)\right](V_1 \cup V_2)\right\| = 0$

iff $\left\|\left[(A_1 - «_1 I_1) \cup (A_2 - «_2 I_2)\right](V_1 \cup V_2)\right\| = 0$

iff $\left\|\left[(A_1 - «_1 I_1) \cup (A_2 - «_2 I_2)\right]^* (V_1 \cup V_2)\right\| = 0$

iff $\left\|\left[(A_1 - «_1 I_1)^* V_1 \cup (A_2 - «_2 I_2)^*\right] V_2\right\| = 0$

iff $\left\|\left[\left(A_1^* V_1 - («_1 I_1)^* V_1\right) \cup \left(A_2^* V_2 - («_2 I_2)^* V_2\right)\right]\right\| = 0$

iff $\left\|\left[\left(A_1^* - \overline{«}_1 I_1^*\right) V_1 \cup \left(A_2^* - \left(\overline{«}_2 I_2^*\right) V_2\right)\right]\right\| = 0$

iff $\left\|\left[\left(A_1^* - \overline{«}_1 I_1\right) V_1 \cup \left(A_2^* - \left(\overline{«}_2 I_2\right) V_2\right)\right]\right\| = 0$

iff $\left\|\left[\left(A_1^* \cup A_2^*\right) - \left(\overline{«}_1 I_1 \cup \overline{«}_2 I_2\right)\right](V_1 \cup V_2)\right\| = 0$

iff $\left\|\left[(A_1 \cup A_2)^* - \left(\overline{«}_1 \cup \overline{«}_2\right)(I_1 \cup I_2)\right](V_1 \cup V_2)\right\| = 0$

iff $\left\|\left(A_B^* - \overline{«}_B I_B\right) V_B\right\| = 0$

iff $\left(A_B^* - \overline{«}_B I_B\right) V_B = 0$.

Proposition

An upper triangular normal bimatrix is diagonal bimatrix.

Proof

Let A_B be an upper triangular normal bimatrix .

we shall show that $a_{ij}^1 \cup a_{ij}^2 = 0$ for $j > i$.

we have $A_B e_1 = \left(a_{11}^1 \cup a_{11}^2\right) e_1$

$\|A_B e_1\|^2 = \left\|\left(a_{11}^1 \cup a_{11}^2\right) e_1\right\|^2$

$= \left\|\left(a_{11}^1 \cup a_{11}^2\right)\right\|^2 \|e_1\|^2$.

Hence $\|A_B e_1\|^2 = \left\|\left(a_{11}^1 \cup a_{11}^2\right)\right\|^2$.

On the other hand this equal to

$\|A_B^* e_1\| = \left|a_{11}^1 \cup a_{11}^2\right|^2 + \left|a_{12}^1 \cup a_{12}^2\right|^2 + \ldots + \left|a_{1n}^1 \cup a_{1n}^2\right|^2$.

Hence $a_{12}^1 \cup a_{12}^2 = a_{13}^1 \cup a_{13}^2 = \ldots = a_{1n}^1 \cup a_{1n}^2 = 0$.

Inductively suppose we have shown $a_{ij}^1 \cup a_{ij}^2 = 0$ for $j > i$ for all $1 \le i \le k-1$. Then it follows that $A_B e_k = \left(a_{kk}^1 \cup a_{kk}^2\right) e_k$. Exactly as in the first case, this implies that

$\|A_B^* e_1\| = \left|a_{k,k}^1 \cup a_{k,k}^2\right|^2 + \left|a_{k,k+1}^1 \cup a_{k,k+1}^2\right|^2 + \ldots + \left|a_{k,n}^1 \cup a_{k,n}^2\right|^2$

$= \left|a_{k,k}^1 \cup a_{k,k}^2\right|^2$.

Hence, $a_{k,k+1}^1 \cup a_{k,k+1}^2 = \ldots = a_{k,n}^1 \cup a_{k,n}^2 = 0$.

Definition

Two bimatrices $A_B, B_B \in {}_{n \times n}$ are called unitarily equivalent bimatrix if there exists a unitary bimatrix $C_B \in {}_{n \times n}$ such that $B_B = C_B^* A_B C_B$. That is , $B_1 \cup B_2 = C_1^* A_1 C_1 \cup C_2^* A_2 C_2$

Proposition

$A_B \in {}_{n \times n}$ is normal bimatrix if and only if every bimatrix unitarily equivalent to A_B is normal bimatrix.

Proof

Suppose A_B is normal bimatrix and $B_B = C_B^* A_B C_B$ where C_B is unitary bimatrix.

Then $B_B B_B^* = \left(C_B^* A_B C_B\right)\left(C_B^* A_B C_B\right)^*$

$= \left(C_1^* A_1 C_1 \cup C_2^* A_2 C_2\right)\left(C_1^* A_1 C_1 \cup C_2^* A_2 C_2\right)^*$

$= \left(C_1^* A_1 C_1 \cup C_2^* A_2 C_2\right)\left(\left(C_1^* A_1 C_1\right)^* \cup \left(C_2^* A_2 C_2\right)^*\right)$

$= \left(C_1^* A_1 C_1 \cup C_2^* A_2 C_2\right)\left(\left(C_1^* A_1^* C_1\right) \cup \left(C_2^* A_2^* C_2\right)\right)$

$= C_1^* A_1 C_1 C_1^* A_1^* C_1 \cup C_2^* A_2 C_2 C_2^* A_2^* C_2$

$= C_1^* A_1 I A_1^* C_1 \cup C_2^* A_2 I A_2^* C_2$

$= C_1^* A_1 A_1^* C_1 \cup C_2^* A_2 A_2^* C_2$

$= C_1^* A_1^* A_1 C_1 \cup C_2^* A_2^* A_2 C_2$

$= C_B^* A_B^* A_B C_B$

$$= B_B^* B_B$$

Therefore, every bimatrix unitarily equivalent to A_B is normal bimatrix.

Conversely, if $C_B^* A_B C_B$ is normal bimatrix, then

$$C_B^* A_B A_B^* C_B = C_B^* A_B^* A_B C_B.$$

That is,

$$C_1^* A_1 A_1^* C_1 \cup C_2^* A_2 A_2^* C_2 = C_1^* A_1^* A_1 C_1 \cup C_2^* A_2^* A_2 C_2$$

Multiply this equation on the right by $(C_1 \cup C_2)^*$ and on the left by $C_1 \cup C_2$ to get

$$(C_1 \cup C_2)(C_1^* A_1 A_1^* C_1 \cup C_2^* A_2 A_2^* C_2)(C_1 \cup C_2)^*$$

$$= (C_1 \cup C_2)(C_1^* A_1^* A_1 C_1 \cup C_2^* A_2^* A_2 C_2)(C_1 \cup C_2)^*$$

$$(C_1 \cup C_2)(C_1^* A_1 A_1^* C_1 \cup C_2^* A_2 A_2^* C_2)(C_1^* \cup C_2^*)$$

$$= (C_1 \cup C_2)(C_1^* A_1^* A_1 C_1 \cup C_2^* A_2^* A_2 C_2)(C_1^* \cup C_2^*)$$

$$C_1 C_1^* A_1 A_1^* C_1 C_1^* \cup C_2 C_2^* A_2 A_2^* C_2 C_2^* = C_1 C_1^* A_1^* A_1 C_1 C_1^* \cup C_2 C_2^* A_2^* A_2 C_2 C_2^*$$

$$(I A_1 A_1^* I) \cup (I A_2 A_2^* I) = (I A_1^* A_1 I) \cup (I A_2^* A_2 I)$$

$$(A_1 A_1^* \cup A_2 A_2^*) = A_1^* A_1 \cup A_2^* A_2$$

$$(A_1 \cup A_2)(A_1^* \cup A_2^*) = (A_1^* \cup A_2^*)(A_1 \cup A_2)$$

$$(A_1 \cup A_2)(A_1 \cup A_2)^* = (A_1 \cup A_2)^* (A_1 \cup A_2)$$

$$A_B A_B^* = A_B^* A_B$$

Hence, A_B is normal bimatrix.

CONCLUSION

Some of the characterization of unitary and normal matrices can be verified for unitary and normal bimatrices. Also the unitarily equivalence of unitary and normal bimatrices are found.

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