THE ECONOMIC TRAINEE BATCH – SIZE MANPOWER MODEL WITH VARIABLE COST FUNCTION

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ABSTRACT

A single grade deterministic-trainee inventory problem with a constant demand is considered in this paper. A Manpower inventory model where the trainees are trained at a constant rate and are inspected at regular intervals of time. Inefficient persons are identified and retrained to bring them to the expected level of efficiency. The single trainee cost is assumed to be a function of trainee rate. We formulate a generalized single trainee cost function, which brings flexibility in deciding the number of persons to be trained over a training period. The objective of this paper is to obtain Economic Trainee batch-size and optimal trainee rate that minimizes the total cost function. The model solution is justified through a numerical example and a study of impact on optimal batch-size, optimal trainee rate and total cost function for variations in \( \alpha \) is presented.

Key words: Trainee inventory problem, trainee rate, trainee cost, Manpower inventory model.

1. INTRODUCTION

The classical Economic Production Quantity (EPQ) Model has been in use for a long time in research field. It can be considered as an extension of Economic Order Quantity (EOQ) Model. In classical Economic Production Lot Size (EPL) Model the production rate of a machine is assumed to be pre determined and inflexible (Hax, 1984). However the variation in the machine production rate was proposed by (Schweitzer et al., 1991). Alder and Nanda (1974, 1981) extended the EPL Model to both single production and multi production cases to situations where learning effects cause the production rate to increase. Cheng (1991) proposed an extension to the EPL Model in which demand exceed supply and the production process is imperfect. The EPL Model under volume flexibility was presented by (Moutaz, 1995). Lot sizing capacity utilization in a production process with defective item, process correction and rework was given by (Lee, 1992). Bhandari et al., (1999) have developed an extension of EPQ in which production flexibility of manufacturing system is proposed. Lee et al., (1997) developed a model of batch quantity in a multi-state production system considering various proportions of defective items produced in every state while they ignored the rework situation. Gupta and Chakrabarty, (1984) considered an imperfect multi-stage manufacturing system wherein the defectives are collected and reworked (2004) Jamal et al., (1991) considered a single production system with rework option incorporating two cases of rework process.

In the first case they considered the rework is done within the same cycle where the item is produced. In the second case the defective items are accumulated up to \( N \) cycles and accumulated items are reworked after the \( N \)th cycle. They developed optimal batch quantity models to minimize the system cost. This research deals with manpower inventory model wherein we consider human resource as inventory for an organization. Here we consider a training consultancy (organization) that selects trainees and absorbs persons after they complete the training process successfully. Inspection is done after every regular interval of time and inefficient persons are identified and are retrained to make them efficient. The retraining is done within the training cycle. The objective is to determine optimal batch size and optimal trainee lot that minimizes the total cost.

2. MODEL DESCRIPTION

In any organization the required staff strength is maintained through new recruitments. In current scenario first selection of persons are made, trained according to the job requirement of the organization and finally are made permanent employees of the organization. During the training cycle inspection is made and screening test is conducted to check the competence and efficiency level of the person. During the training period if the person’s performance is not found satisfactory, they are retrained in order to improve their performance. Frequent inspection are carried over, this leads to inspection cost added to the total cost. In addition we have retraining cost too. With a view to minimize the total cost incurred by the organization, the optimal batch size i.e. optimal number.
of person getting trained per unit time is attempted here. Also we always maintain optimal number of person as reserve to meet any probable demand during the period of training. This is made in order to maintain the goodwill of the company. These concepts are demonstrated with the help of Fig.1 and Fig.2. The single trainee cost depends on trainee rate. As the number of person getting trained per unit time increases leads to reduction in the single trainee cost or the cost of training one person, the salient feature of this model is to determine optimal batch size and Economic trainee size that is to find the optimal number of persons to be trained under some specific assumptions using the concept of production models in inventory management. Here a Training Retraining and Absorption (TRA) policy is developed to maintain total quality of persons employed in an organization.

3. MODEL FORMULATION AND ASSUMPTIONS

In this research, the inventory model is developed for the operational policy in which we retrain the persons whenever they fail in their efficiency level and it is assumed that all the persons are brought up to the expected level of efficiency once retraining is completed. As retraining cost is comparatively less than training a fresh candidate, the organization prefers and believes in retraining.

3.1 Assumptions

- A single grade deterministic inventory system is considered
- Trainee rate is a decision variable
- Trainee rate is greater than demand rate
- In each cycle the proportion of inefficient person is constant
- Shortages are not allowed
- Single trainee cost is a function of trainee rate
- Inefficient persons are trained to reach their expected efficiency level
- Inspection cost is included as inspection takes place frequently to identify the inefficient persons and retrained them

3.2 Notations

1. k – Training rate, the number of persons getting trained per cycle
2. Q – Batch size per cycle, number of persons trained per cycle
3. R – Demand rate, number of trained persons required per cycle
4. E T B – Economic Trainee Batch
5. S – Set up cost per cycle
6. H – Holding cost per person per unit time
7. T – Cycle time
8. T C – Total cost
9. α – Proportion of inefficient persons identified in each cycle with respect to trainee rate
10. L – Labour cost per cycle
11. F – Positive constant
12. E(k) = \frac{k}{k} + F k, E(k) – Single trainee cost function as a function of trainee rate
13. T_{T} – Training time in each cycle
14. T_{R} – Retraining time in each cycle
15. T_{A} – absorption time in each cycle
16. N_{x} – Number of trained persons remaining after absorption at the end of \( T_{T} \) after removal of inefficient persons
17. \( \hat{N}_{x} \) – Number of trained persons remaining after absorption without identification of inefficient persons
18. \( \hat{N}_{R} \) – Number of persons remaining after absorption at the end of \( T_{R} \), when retraining is completed.
19. I – Inspection cost per unit batch per cycle.
20. B – Bench cost paid for the reserve employees per cycle.
21. N_{IE} – Number of inefficient persons identified at the end of \( T_{T} \).
22. N_{RA} – Number of trained persons absorbed during retraining time \( T_{R} \).

4. MODEL ANALYSIS

We assume that the inefficient persons are identified at the rate of \( \alpha(0<\alpha<1) \) during training time. The total replenishment rate during training time is \([k (1 - \alpha) – R]\) and total number of inefficient persons are \( \alpha Q \). These inefficient persons are retained at the rate of \( k \) persons per unit time. Hence, inventory gets built up during retraining time. During absorption time all the trained persons are absorbed by the organization.
Fig 2 represents the behavior of the inventory level during the first cycle. RT = \( Q \) 

\[
t_f = \frac{Q}{k}
\]

\[
N'_T = (k - R) t_f = (k - R) \frac{Q}{k}
\]

\[
N'_A = (k - k + \alpha - R) t_f = (1 - \alpha - \frac{R}{k}) Q
\]

Number of inefficient persons (untrained persons) identified = \( N_{IE} = N'_T - N'_A = \alpha Q \)

\[
I_R = \frac{N_{IE}}{k} = \frac{\alpha Q}{k}
\]

\[
N_{RA} = R t_R = R \frac{\alpha Q}{k}
\]

\[
N'_A = N'_T - N_{RA} = (k - R) \frac{Q}{k} - R \frac{\alpha Q}{k} = (k - R - \alpha) Q
\]

\[
I_A = \frac{N'_A}{k} = (1 - \frac{R}{k} - \alpha) \frac{Q}{k}
\]

\[
T = t_f + t_R + I_A = \frac{Q}{k} + \frac{\alpha Q}{k} + (1 - \frac{R}{k} - \alpha) \frac{Q}{k} = \frac{Q}{k}
\]

**Fig 2** A schematic diagram of our model

### 4.1 Set up cost

Every training cycle needs the prerequisite of arrangement of place for training etc. Hence the setup cost per unit time is equal to:

\[
\text{Set up cost} = \frac{SR}{Q}
\]

(1)

### 4.2 Inventory holding cost

During the training period (inventory) persons are trained at the rate of \( k \) persons per unit time which is much greater than the demand rate \( R \). Thus the organization has to bear the cost of retaining them until the occurrence of next demand. Hence the retaining cost is given by,

Inventory carrying cost = \( H \bar{I} \)

Where \( \bar{I} = \frac{I}{T} \) gives the average inventory.

#### Total inventory

\[
i = \frac{1}{2} N_A + \frac{t_f N'_A}{k} + \frac{1}{2} (N'_B - N'_A) t_A = \frac{1}{2} \frac{Q}{k} (1 - \frac{R}{k} - \alpha) \frac{Q}{k}
\]

(2)

This gives the average inventory as

\[
\bar{I} = \frac{1}{T} = \frac{I}{T}
\]

#### Inventory carrying cost

\[
\text{Inventory carrying cost} = H \bar{I} = H \frac{I}{T} = \frac{H}{2} \left(\frac{1}{k} + \alpha^2\right) \frac{Q}{R} \quad \text{(2)}
\]

#### Training cost

\[
\text{Training cost} = \frac{E(k) Q}{T} = \frac{E(k) QR}{Q} = E(k) R
\]

(3)

Where, \( E(k) = \frac{L}{k} + F k \)

\( E(k) \) – Single trainee cost function as a function of trainee rate, is a variable cost function \( \frac{L}{k} \). This cost decreases with increasing trainee rate. Some cost like labour cost decreases with increase in trainee rate that is the number of persons are getting trained per batch increases the training cost per person reduces. \( F k \) – This cost increases proportionally with trainee rate. Cost like
machinery cost, stationery cost, etc. increases with increase in number of persons

4.4 Inspection cost

During the training period Inspection takes place frequently to identify the inefficient persons. Hence the inspection cost per unit time is given by:

\[
\text{Inspection cost} = I \frac{Q}{T} = I \frac{QR}{Q} = I \text{R}
\]

\[\text{(4)}\]

4.5 Retraining cost

Whenever inefficient persons are identified they are given extra training in order to bring them to their expected level of efficiency. It is assumed in each cycle the \((0<\alpha<1)\) percentages of inefficient persons are identified. This leads to the additional cost called the to the organization. The retraining cost for \(\alpha Q\) person per unit time is given by,

\[
\text{Retraining cost} = \frac{E(k)\alpha Q}{T} = \frac{E(k)\alpha QR}{Q} = E(k)\alpha \text{R}
\]

\[\text{(5)}\]

4.6 Reserve employees holding cost

Since we assume that inefficient persons are identified at the rate of \(\alpha\) of trained persons, in order to maintain the good will of the company \(Q\) employees are maintained as reserve employees. Where these persons are paid bench staff cost which is more than the holding cost since they are completely trained.

\[
\text{Reserve employees holding cost} = \frac{\alpha Q}{T} \text{B} = \alpha Q \text{B}
\]

\[\text{(6)}\]

4.7 Total cost

The total cost per cycle can be expressed as,

\[
\text{TC} = \frac{SR}{Q} + \frac{H}{2} \left\{ \frac{1}{Q} - \frac{R}{k} \right\} (1 + a + a^2) + E(k) R + E(k) \left[ R + I R + \alpha \text{QB} \right]
\]

\[\text{TC} = \frac{SR}{Q} + \frac{H}{2} \left\{ \frac{1}{Q} - \frac{R}{k} \right\} (1 + a + a^2) + E(k) R + \left[ R + I \alpha + I R + \alpha \text{QB} \right]
\]

\[\text{TC}(Q,k) = \frac{SR}{Q} + \frac{H}{2} \left\{ \frac{1}{Q} - \frac{R}{k} \right\} (1 + a + a^2) + \frac{L}{k} + E(k) R + \left[ I \alpha + I R + \alpha \text{QB} \right]
\]

\[\text{(7)}\]

4.8 Optimal Solution

The optimal values of \(Q\) and \(k\) can be obtained by setting the first order partial derivative of TC to zero,

\[
\frac{\partial \text{TC}}{\partial Q} = 0, \quad \frac{\partial \text{TC}}{\partial k} = 0.
\]

This gives after simplification,

\[
Q^* = \frac{2RSk}{H \left[k-R(1+\alpha + \alpha^2)\right] + 2k\alpha B}
\]

\[
k^* = \frac{2L(1+\alpha) - HQ(1+\alpha + \alpha^2)}{2F(1+\alpha)}
\]

The pair \((Q^*, k^*)\) will be an optimal solution, as the following conditions are satisfied,

\[
\frac{\partial^2 \text{TC}}{\partial Q^2} > 0, \quad \frac{\partial^2 \text{TC}}{\partial k^2} > 0 \quad \text{and} \quad \left(\frac{\partial^2 \text{TC}}{\partial Q \partial k}\right) (Q^* k^*) = 0.
\]

Hence, \(Q^* = \sqrt{\frac{2RSk}{H \left[k-R(1+\alpha + \alpha^2)\right] + 2k\alpha B}}\) and \(k^* = \sqrt{\frac{2L(1+\alpha) - HQ(1+\alpha + \alpha^2)}{2F(1+\alpha)}}\).

Substituting these values in equation (7) we get minimum TC\((Q^*, k^*)\).

The algorithm to solve the optimal batch size and optimal trainee rate are as follows:

Step 0. Initialize and store \(S, H, R, \alpha, L, F, B\) and I

Step 1. Compute the optimal batch size and the optimal trainee rate by solving the simultaneous equations

\[
Q^* = \sqrt{\frac{2RSk}{H \left[k-R(1+\alpha + \alpha^2)\right] + 2k\alpha B}}\quad \text{and} \quad k^* = \sqrt{\frac{2L(1+\alpha) - HQ(1+\alpha + \alpha^2)}{2F(1+\alpha)}}
\]

Step 2. If \(Q^*\) and \(k^*\) are integers then go to Step 4. else continue with step 3.

Step 3. If \(Q^*\) and \(k^*\) are not integers then find the combination of integer approximation of \(Q^*\) and \(k^*\) that minimizes TC.

Step 4. Compute TC\((Q^*, k^*)\) Stop.

5. Numerical example

To illustrate the usefulness of the model developed, consider a training consultancy where in the parameters are:

\(S = \text{Rs.10000/cycle}, H = \text{Rs.2000/per person/year}, R = 100\) persons /year, \(\alpha = 2\%\),

\(L = \text{Rs 50000/year}, F = 0.1, I = \text{Rs.3000/year}, B = \text{Rs.3000/person/year}\) the optimal batch size \(Q\) is found to be 46.9668 and the Optimal trainee rate \(k\) is 168.792.

It is necessary that \(Q\) and \(k\) must be integers, as it specifies the number of persons. So, we try to find the...
best combination of integer values in the neighborhood that will minimize the total cost.

<table>
<thead>
<tr>
<th>Calculated values</th>
<th>$Q$</th>
<th>$k$</th>
<th>$TC(Q,k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values with upper integer</td>
<td>46.9668</td>
<td>168.792</td>
<td>374520</td>
</tr>
<tr>
<td>Values with lower integer</td>
<td>47</td>
<td>169</td>
<td>374520</td>
</tr>
<tr>
<td>Values with combination</td>
<td>46</td>
<td>168</td>
<td>374532</td>
</tr>
<tr>
<td>Values with combination</td>
<td>47</td>
<td>168</td>
<td>374520</td>
</tr>
<tr>
<td>Optimal solution</td>
<td>47</td>
<td>168</td>
<td>374520</td>
</tr>
</tbody>
</table>

From the above table we infer that $Q^* = 47$ persons, $k^* = 168$ persons, are the optimal values that gives minimum cost $TC(Q^*, k^*) = Rs.374520$year.

6. Impact on optimal batch size, optimal trainee rate and total cost for variations in $\alpha$

From the above table we infer that $Q^* = 47$ persons, $k^* = 168$ persons, are the optimal values that gives minimum cost $TC(Q^*, k^*) = Rs.374520$year.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$Q^*$</th>
<th>$k^*$</th>
<th>$TC(Q^<em>, k^</em>)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>47.691</td>
<td>168.792</td>
<td>374520</td>
</tr>
<tr>
<td>0.1</td>
<td>47.9159</td>
<td>168.792</td>
<td>374520</td>
</tr>
<tr>
<td>0.15</td>
<td>47.721</td>
<td>168.792</td>
<td>374520</td>
</tr>
<tr>
<td>0.2</td>
<td>47.266</td>
<td>168.792</td>
<td>374520</td>
</tr>
<tr>
<td>0.25</td>
<td>46.6334</td>
<td>168.792</td>
<td>374520</td>
</tr>
</tbody>
</table>

7. CONCLUSION

In this paper, an inventory Manpower model is being proposed in which the trainee rate is taken as the variable. The single trainee cost function is taken as the function of trainee rate. The study not only brings flexibility in deciding the number of persons to be trained over a training period but also demonstrates the importance of taking in to account the quality of the trained persons.

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