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Research Article

ON CHANGING BEHAVIOR OF EDGES OF SOME SPECIAL CLASSES OF GRAPHS II

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ABSTRACT

Let G be a (p,q) graph and $f: V(G) \to \{1,2,\dots,p+q-1,p+q+2\}$ be an injection. For each edge $e = uv$, the induced edge labeling $f^*$is defined as follows:

$$f^*(\text{e}) = \begin{cases} \dfrac{|f(u)-f(v)|}{2} & if\ |f(u)-f(v)|\ is\ even \\ \dfrac{|f(u)-f(v)|+1}{2} & if\ |f(u)-f(v)|\ is\ odd \end{cases}$$

Then f is called Near Skolem difference mean labeling if $f^*(e)$ are all distinct and are from $\{1,2,3,\dots.q\}$. A graph that admits a Near Skolem difference mean labeling is called a Near Skolem difference mean graph. In this paper, a new parameter E^- is introduced and verified for some graphs.

INTRODUCTION

We consider only finite, undirected and simple graphs in this paper. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. For standard terminology and notations, we follow Harary (1) and for graph labeling, we refer to Gallian (2).

In this paper, a Near skolem difference mean graph G is investigated and a new parameter E^- is introduced to find the minimum number of edges that should be deleted from G to convert the non- near skolem difference mean graph G into a near skolem difference mean graph G^*.

Definition: The fan graph $F_n(n \geq 2)$ is obtained by joining all vertices of P_n (Path of n vertices) to a further vertex called the center and contains $(n+1)$ vertices and $(2n-1)$ edges. That is, $F_n = P_n + K_1$.

Definition: The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and $G"$. Join each vertex u' in G' to the neighbours of the corresponding vertex $u"$ in $G"$.

Definition: For a graph G, the splitting graph which is denoted by $spl(G)$ is obtained by adding to each vertex v, a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G.

Definition: The Jewel J_n is the graph with vertex set $V(J_n) = \{u,v,x,y,u_i: 1 \leq i \leq n\}$ and edge set $E(J_n) = \{ux,uy,xy,xv,yv,uu_i,vv_i, 1 \leq i \leq n\}$.

MAIN RESULT

Definition: A graph $G = (V,E)$ with $p$vertices and $q$edges is said to have Nearly skolem difference mean labeling if it is possible to label the vertices x∈V with distinct elements $f(x)$ from $\{1,2,\dots\dots,p+q-1,p+q+2\}$ in such a way that each edge e = uv , is labeled as $f^*(\text{e}) = \frac{|f(u)-f(v)|}{2}$ if $|f(u)-f(v)|$ is even and $f^*(\text{e}) = \frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd. The resulting labels of the edges are distinct and are from $\{1,2,\dots\dots,q\}$. A graph that admits a Near skolem difference mean labeling is called a Near skolem difference mean graph.

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Definition: Let G be a non-near skolem difference mean graph. Then the parameter $E^{-}$of a graph G is defined as the minimum number of edges to be deleted from a G, so that the resulting graph is Near skolem difference mean.

Proposition: Let G be a non-Near skolem difference mean graph.

Then $k = E^{-}(G) \geq q - p - 2$, (where k is the number of edges to be removed from G to make it Near skolem difference mean graph).

Proof: Let G^* be the graph obtained from G, by removing k edges of G.

Then, $|V(G^*)| = |V(G)| = p$ and $|E(G^*)| = |E(G)| - k = q - k$.

Let f be a Near skolem difference mean labeling of G^*, such that

$f: V(G^*) \longrightarrow \{1,2,..,p+q-k-1, p+q-k+2\}$.
Let $uv \in E(G^*)$ such that $f^*(uv) = q - k$.

Then, $\frac{|f(u)-f(v)|+1}{2} = q - k$.
This implies $|f(u) - f(v)| = 2q - 2k - 1$.
This implies $f(u) = 2q - 2k - 1 + f(v)$.
$\geq 2q - 2k$
But, $f(u) \leq p + q - k + 2$.
This implies $2q - 2k \leq f(u) \leq p + q - k + 2$.
This implies $q - k \leq p + 2$
And hence $k \geq q - p - 2$.

This is true even if, $\frac{|f(u)-f(v)|}{2} = q - k$.

Hence in both cases, $E^{-}(G) \geq q - p - 2$.

Theorem:$E^{-}(D_2(P_n)) = q - p - 2 = 2n - 6$ for $n \geq 4$.

Proof: By Preposition, $E^{-}(D_2(P_n)) \geq q - p - 2 = 2n - 6$.

Let G^*be the graph defined by

$G^* = D_2(P_n) - \{u_i v_{i+1}, v_i u_{i+1} / 3 \leq i \leq n-1\}$.
where $V(G^*) = \{u_i, v_i / 1 \leq i \leq n\}$ and
$E(G^*) = \{u_i u_{i+1}, v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{u_1 v_2, u_2 v_3, v_1 u_2, v_2 u_3\}$.
Then $|V(G^*)| = 2n$ and $|E(G^*)| = 2n + 2$.
Let $f: V(G^*) \rightarrow \{1,2, \dots, 4n+1, 4n+4\}$ be defined as follows:
$f(u_1) = 1$
$f(u_2) = 4n + 4$

$$f(u_i) = \begin{cases} i - 1. & i \equiv 1(mod2), 3 \leq i \leq n \\ 4n + 5 - i, & i \equiv 0(mod2), 3 \leq i \leq n \end{cases}.$$

$$f(v_i) = \begin{cases} 2n + 5 - i, & i \equiv 1(mod2), 1 \leq i \leq n \\ 2n + 4 + i, & i \equiv 0(mod2), 1 \leq i \leq n \end{cases}$$

Let f^* be the induced edge labeling of f. Then,
$f^*(u_i u_{i+1}) = 2n + 3 - i, \quad 1 \leq i \leq n - 1$.
$f^*(v_i v_{i+1}) = i, \quad 1 \leq i \leq n - 1$.
$f^*(u_1 v_2) = n + 3$
$f^*(u_2 v_3) = n + 1$
$f^*(v_1 u_2) = n$
$f^*(v_2 u_3) = n + 2$

The induced edge labeling are all distinct and are $\{1,2, \dots, 2n + 2\}$.

Then G^* is a Near Skolem Difference Mean graph.

Hence, $E^{-}(D_2(P_n)) = 2n - 6$ for $n \geq 4$.

Example: Near skolem difference mean labeling of edge deleted graphs obtained from $D_2(P_8)$and $D_2(P_9)$ are given in fig 1 and fig 2 respectively.

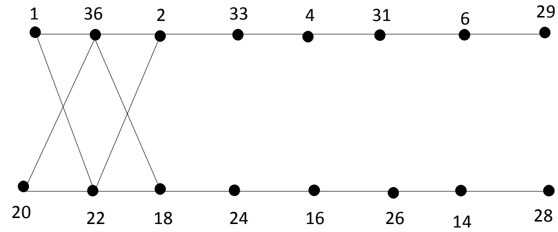


Fig 1

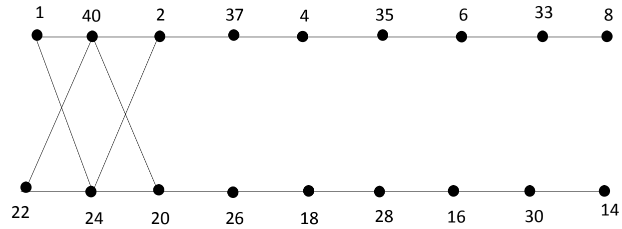


Fig 2

Theorem:$E^{-}(G) \geq q - p - 2 = n - 2$ for $n \geq 3$, where G is the Jewel graph.

Proof: By Preposition, $E^{-}(G) \geq q - p - 2 = n - 2$.
Let G^* be the graph defined by
$G^* = G - \{uw_i, 1 \leq i \leq n - 2\}$, where
$V(G^*) = \{u, v, x, y, w_i / 1 \leq i \leq n\}$ and
$E(G^*) = \{ux, uy, vx, vy, uw_{n-1}, uw_n, vw_i / 1 \leq i \leq n\}$.
Then $|V(G^*)| = n + 4$ and $|E(G^*)| = n + 6$.
Let $f: V(G^*) \rightarrow \{1,2, \dots, 2n + 9, 2n + 12\}$ be defined as follows:

$f(x) = 2n + 12$
$f(y) = 2n + 9$
$f(u)$=1
$f(v) = 3$
$f(w_i) = 2n + 9 - 2i, \ 1 \leq i \leq n - 2$.
$f(w_{n-1}) = 9$
$f(w_n) = 5$
Let f^* be the induced edge labeling of f. Then,
$f^*(ux) = n + 6$
$f^*(uy) = n + 4$
$f^*(vx) = n + 5$
$f^*(vy) = n + 3$
$f^*(uw_{n-1}) = 4$
$f^*(uw_n) = 2$
$f^*(vw_i) = n + 3 - i, \quad 1 \leq i \leq n - 2$
$f^*(vw_{n-1}) = 3$
$f^*(vw_n) = 1$
The induced edge labeling of G^* are all distinct and are $\{1,2, \dots, n + 6\}$.
Hence, $E^{-}(G) = n - 2$, for $n \geq 3$.
Therefore G^* a is Near Skolem Difference Mean graph.

Example: Near skolem difference mean labeling of the edge deleted graphs G^* obtained from the Jewel graph for $n = 5$ and $n = 6$ are given in fig 3 and fig 4 respectively.

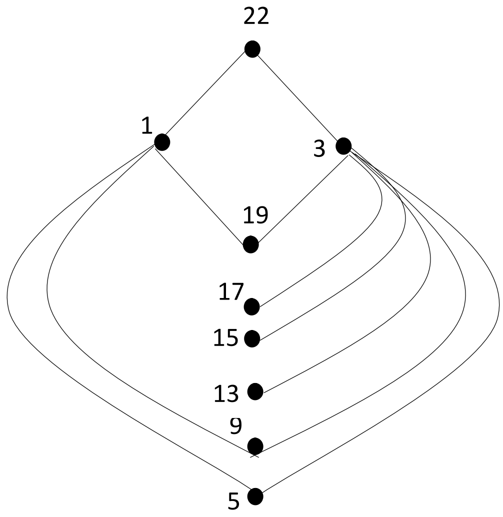


Fig 3

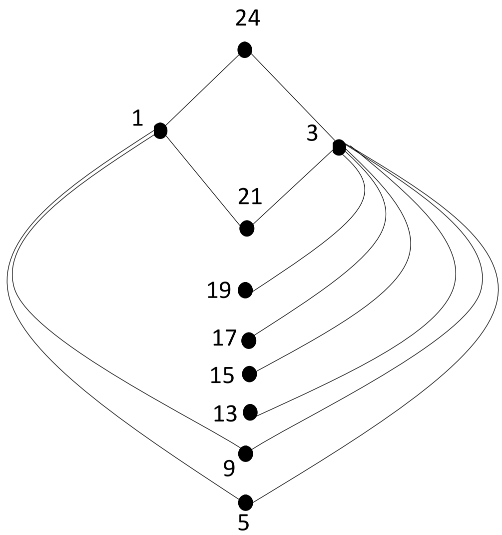


Fig 4

Theorem: $E^{-}(G) = q - p - 2 = n - 4$ for $n \geq 5$ where G is the graph $spl(K_{1,n})$.

Proof: By Preposition, $E^{-}(G) \geq n - 4$.

Let G^*be the graph defined by
Let $G^* = G - \{v'v_i / \quad 1 \leq i \leq n - 4\}$ where
Let $V(G^*) = \{v, v', v_i, v_i' \,/\, 1 \leq i \leq n\}$ and
$E(G^*) = \{vv_i, vv_i', v'v_j / \ 1 \leq i \leq n, \ n-3 \leq j \leq n\}$.
Then $|V(G^*)| = 2n + 2$ and $|E(G^*)| = 2n + 4$.

Let $f: V(G^*) \rightarrow \{1, 2, \ldots, 4n + 5, 4n + 8\}$ be defined as follows:

$f(v) = 4n + 8$.
$f(v_i) = 2i - 1, \ 1 \leq i \leq n$.
$f(v') = 2n$.
$f(v_i') = 2n - 1 + 2i, \ 1 \leq i \leq n$.
Let f^* be the induced edge labeling of f. Then,
$f^*(vv_i) = 2n + 5 - i, \quad 1 \leq i \leq n$.
$f^*(vv_i') = n + 5 - i, \quad 1 \leq i \leq n$.
$f^*(v'v_i) = n + 1 - i, \quad n - 3 \leq i \leq n$.

The induced edge labels are all distinct and are $\{1, 2, \ldots, 2n + 4\}$.

Then the edge deleted graph G^* is Near skolem difference mean for $n \geq 5$.
Hence, $E^{-}(G) = n - 4$.

Example: Near Skolem Difference Mean labeling of the graph $G^*$is given below in fig 5

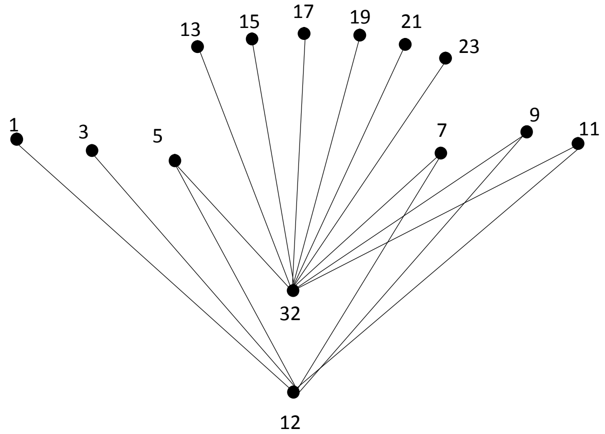


Fig 5

Theorem: The graph

$$E^{-}(P_n + K_1) = \begin{cases} q - p - 2 = n - 4, n \geq 7 \\ 1 \text{ } if \ n = 2, 3 \\ 2 \text{ } if \ n = 4, 5 \\ 3 \text{ } if \ n = 6 \end{cases} \quad \text{for } n \geq 5.$$

Proof: By Preposition, $E^{-}(P_n + K_1) \geq n - 4$.
Let G^* be the graph obtained from $P_n + K_1 \ withV(G^*) = \{v, u_i / 1 \leq i \leq n\}$.
and $E(G^*) = \{u_i u_{i+1}, u_1 v, u_3 v, u_5 v, u_7 v / \ \ 1 \leq i \leq n - 1\}$.
Then $|V(G^*)| = n + 1$ and $|E(G^*)| = n + 3$.

For $n = 2,4,6$, the Near Skolem Difference Mean labeling of G^* for $n = 2,4,6$ are as shown in fig 6, fig 7 and fig 8 respectively.

Similarly, the Near Skolem Difference Mean labeling of G^* for $n = 3, 5$ are as shown in fig 9, and fig 10 respectively.
Let $n \geq 7$.

Let $f: V(G^*) \rightarrow \{1, 2, \ldots, 2n + 3, 2n + 6\}$ be defined as follows:

$$f(v) = 1$$

$$f(u_i) = \begin{cases} 2n + 8 - 2i \ for \ i \equiv 1(mod2), 1 \leq i \leq n \\ 2i - 3 \ for \ \ i \equiv 0(mod2), 1 \leq i \leq n \end{cases}$$

Let f^* be the induced edge labeling of f.Then,
$f^*(vu_i) = n + 3 - i, \quad for \ i = 1, 3, 5, 7$.

Case(i): When n is odd $n \geq 7$

$$f^*(u_i u_{i+1}) = \begin{cases} n + 5 - 2i, \quad 1 \leq i \leq \frac{n+3}{2} \\ 2i - n - 4, \ \frac{n+3}{2} + 1 \leq i \leq n - 1 \end{cases}.$$

Case (ii)When n is even $n \geq 8$

$$f^*(u_i u_{i+1}) = \begin{cases} n + 6 - 2i, \quad 1 \leq i \leq \dfrac{n+4}{2} \\ 2i - n - 4, \quad \dfrac{n+4}{2} + 1 \leq i \leq n - 1 \end{cases}$$

The induced edge labels are all distinct and are $\{1, 2, \ldots, n + 3\}$.
Hence, $E^{-}(P_n + K_1) = n - 4$ for $n \geq 7$.

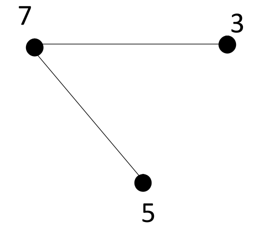


Fig 6

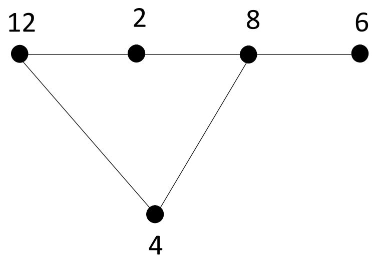


Fig 7

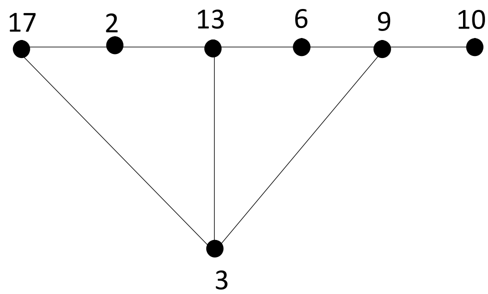


Fig 8

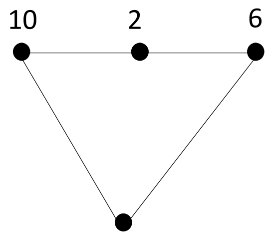


Fig 9

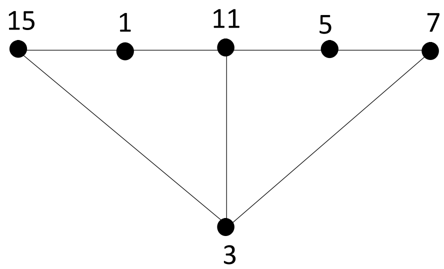
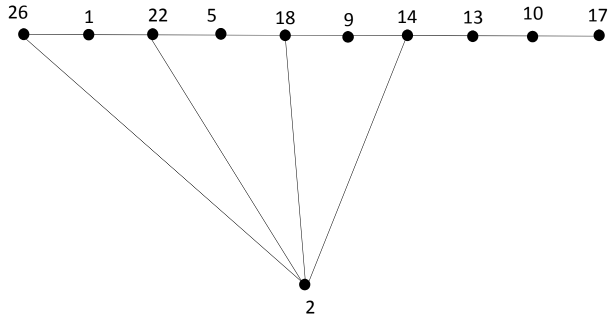


Fig 10

Example: The Near skolem difference mean labeling of the edge deleted graphs G^* obtained from $(P_{10}+K_1)$ and (P_9+K_1) are given in fig 11 and fig 12 respectively.



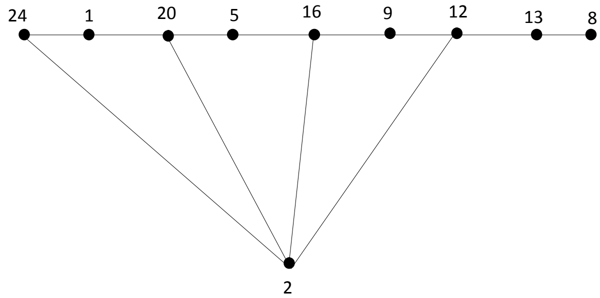


Fig 12

Theorem: $E^-(F_m@2P_n) \geq q-p-2 = m-4$ for $n \geq 5$.

Proof: By Preposition, $E^-(F_m@2P_n) \geq m-4$.

Let $G^* = F_m @ 2P_n - \{uv_i, 5 \leq i \leq n\}$, where
$V(G^*) = \{u, v_i, u_j,\ w_j /\ 1 \leq i \leq m,\ 1 \leq j \leq n-1\}$.
and $E(G^*) = \{uv_i/1 \leq i \leq 4\} \cup \{v_iv_{i+1}/1 \leq i \leq m-1\} \cup \{v_mw_1, w_jw_{j+1}/1 \leq j \leq n-2\} \cup \{uu_1, u_ju_{j+1}/1 \leq j \leq n-2\}$.
Then $|V(G^*)| = m+2n-1$ and $|E(G^*)| = m+2n+1$
Define $f: V(G^*) \to \{1,2,\ldots,2m+4n-1, 2m+4n+2\}$.

***Case (i)m* is odd**

$$f(v_{2i+1}) = \begin{cases} i+1, & 0 \leq i \leq 1 \\ 2m+2n+4-2i, & 2 \leq i \leq \frac{m-1}{2}. \end{cases}$$

$$f(v_{2i}) = \begin{cases} 8-2i, & 1 \leq i \leq 2 \\ 1+2i, & 3 \leq i \leq \frac{m-1}{2}. \end{cases}$$

$$f(w_{2i+1}) = m+2+2i, \quad \begin{matrix} 0 \leq i \leq \frac{n-3}{2}, when\ n\ is\ odd. \\ 0 \leq i \leq \frac{n-2}{2}, when\ n\ is\ even \end{matrix}$$

$$f(w_{2i}) = m+2n+5-2i, \quad \begin{matrix} 1 \leq i \leq \frac{n-1}{2}, when\ n\ is\ odd. \\ 1 \leq i \leq \frac{n-2}{2}, when\ n\ is\ even \end{matrix}$$

$$f(u) = 2m+4n+2.$$

$$f(u_{2i+1}) = 8+2i, \quad \begin{matrix} 0 \leq i \leq \frac{n-3}{2}, when\ n\ is\ odd. \\ 0 \leq i \leq \frac{n-2}{2}, when\ n\ is\ even \end{matrix}$$

$$f(u_{2i}) = 2m+4n+1 - 2i, \quad \begin{matrix} 1 \leq i \leq \frac{n-1}{2}, when\ n\ is\ odd. \\ 1 \leq i \leq \frac{n-2}{2}, when\ n\ is\ even \end{matrix}$$

Case (ii)**m is even**

$$f(v_{2i+1}) = \begin{cases} i+1, & 0 \le i \le 1 \\ 2m+2n+4-2i, & 2 \le i \le \frac{m-2}{2}. \end{cases}$$

$$f(v_{2i}) = \begin{cases} 8-2i, & 1 \le i \le 2 \\ 1+2i, & 3 \le i \le \frac{m}{2}. \end{cases}$$

$$f(w_{2i+1}) = m+2n+4-2i, \quad \begin{cases} 0 \le i \le \dfrac{n-3}{2}, when\ n\ is\ odd. \\ 0 \le i \le \dfrac{n-2}{2}, when\ n\ is\ even \end{cases}$$

$$f(w_{2i}) = m+1+2i, \quad \begin{cases} 1 \le i \le \dfrac{n-1}{2}, when\ n\ is\ odd. \\ 1 \le i \le \dfrac{n-2}{2}, when\ n\ is\ even \end{cases}$$

$$f(u) = 2m+4n+2$$

$$f(u_{2i+1}) = 8+2i, \quad \begin{cases} 0 \le i \le \dfrac{n-3}{2}, when\ n\ is\ odd. \\ 0 \le i \le \dfrac{n-2}{2}, when\ n\ is\ even \end{cases}$$

$$f(u_{2i}) = 2m+4n+1-2i, \quad \begin{cases} 1 \le i \le \dfrac{n-1}{2}, when\ n\ is\ odd. \\ 1 \le i \le \dfrac{n-2}{2}, when\ n\ is\ even \end{cases}$$

Let f^* be the induced edge labeling of f. Then,

$f^*(uv_{2i+1}) = m+2n+1-i, \quad 0 \le i \le 1$

$f^*(uv_{2i}) = m+2n-3+i, \quad 1 \le i \le 2$

$f^*(v_iv_{i+1}) = \begin{cases} 4-i, & 1 \le i \le 3 \\ m+n+2-i, & 4 \le i \le m-1 \end{cases}.$

$f^*(v_mw_1) = n+2$

$f^*(w_jw_{j+1}) = n+2-j, \quad 1 \le j \le n-2$

$f^*(uu_1) = m+2n-3$

$f^*(u_ju_{j+1}) = m+2n-3-j, \quad 1 \le j \le n-2$

The induced edge labels are all distinct and are $\{1,2,\dots,m+2n+1\}$.

Example: The Near skolem difference mean labeling of the edge deleted graph G^* obtained from $(F_9\ @2P_6), (F_5@2P_9), (F_8\ @2P_5)$ and $(F_8@\ 2P_6)$ are given fig 13, fig 14, fig 15 and fig 16 respectively.

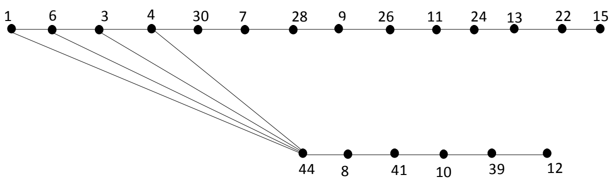


Fig 13

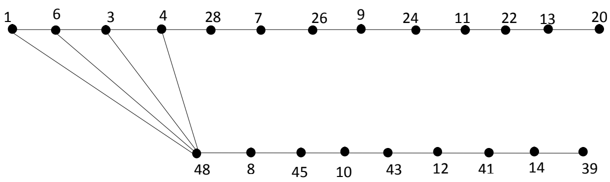


Fig 14

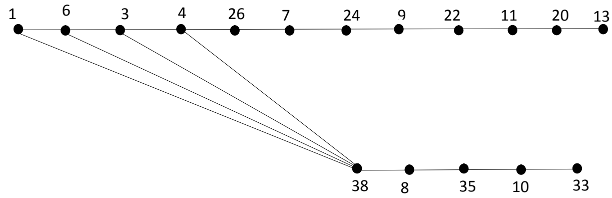


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Fig 15

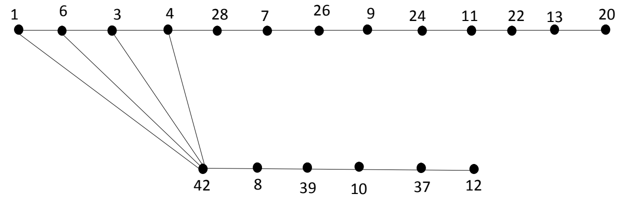

Fig 16

CONCLUSION

In this paper, we investigated a non-Near skolem difference mean graph G and introduced a new parameter to check whether removal of minimum number of edges from G converts it into a Near skolem difference mean graph. We have planned to investigate this property for some more cases of graphs in our next paper.

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