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**Research Article**

# ACCELERATED LIFE TESTING PLANS FOR THE STOCHASTIC DEGRADATION MODELS FOR CENSORED DATA USING LOG LOCATION SCALE DISTRIBUTION

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**ABSTRACT**

Accelerated Life tests expose the products to greater environmental stress levels so that we can obtain lifetime and degradation measurements in a more timely techniques for performing an ALT include constant stress. In this paper presents methods for planning accelerated life tests for models in which the logarithm of time-to-failure follows a location-scale distribution and the location parameter is a function of stress. Different choices of test-stress levels and test length can result in different precision of the estimate of the reliability of the product at normal-use conditions. A test plan that gives minimum variance of the maximum likelihood estimates (MLEs) of the unknown location and scale parameters of the log-location-scale family of distributions at specified stress levels by suitably determining the test length. Under this model, the determination of the optimal choice of $\tau$ for lognormal distributions are addressed using the asymptotic variance optimality criterion. Numerical illustration is also provided.



## INTRODUCTION

Accelerated life tests (ALT) are commonly used in product design processes. Because there is limited time to launch new products, engineers use accelerated tests to obtain needed information on the reliability by raising the levels of certain acceleration variables like temperature, voltage, humidity, stress, and pressure. For highly reliable modern products, it often takes much more time to obtain lifetime and degradation data under usual use conditions, and this requires one to use accelerated tests. Accelerated tests expose the products to greater environmental stress levels so that we can obtain lifetime and degradation measurements in a more timely Techniques for performing an ALT include constant stress, step stress, and ramp stress, among others. Evaluation of the variance of an estimator of a log location scale distribution quantile with varying stress has many practical applications, refer to Meeker, and Escobar (1998). It is of interest to develop useful, accurate probability models for inferences on the lifetime of the devices or systems under study. Such models should realistically incorporate the acceleration variables and measurements of degradation as well as any actual failures observed. Thus, in many engineering reliability experiments, measures of degradation or wear toward failure can often be observed over a period of time before failure occurs. Because the degradation values provide additional information beyond that provided by the failure observations, both sets of observations need to be considered when doing inference on the statistical parameters of the product or system lifetime distributions as discussed by Nelson (1990).

The objective of the present paper is to extend existing results by developing general failure models based on Stochastic processes for degradation which incorporate several accelerating variables, and use both degradation measurements, and different choices of test-stress levels and test length can result in different precision of the estimate of the reliability of the product at normal-use conditions. Our goal is to find a test plan that gives minimum variance of the maximum likelihood estimates (MLEs) of the unknown location and scale parameters of the log-location-scale family of distributions at specified stress levels by suitably determining the test length.

***Early and Recent Developments in Stochastic Degradation models***

Within the framework of cumulative exposure model, Miller and Nelson (1990), Bai *et al*. (1989), Khamis and Higgins (1996), Khamis (1997), Yeo and Tang (1999), Gouno *et al*. (2004), Han *et al*. (2006), Wu *et al*. (2006), Balakrishnan *et al*. (1997),Wu *et al*.

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(2008), and Balakrishnan and Han (2009) have all discussed the problems of designing optimal two-level, three-level, and general k-equal-length-level step-stress ALT based on the complete or censored lifetime data from an exponential distribution. Their work was further extended to the Weibull, lognormal, and log-logistic distributions by Bai and Kim (1993), and Balakrishnan *et.al*.,(2009) Optimal step-stress testing for progressively for censored data from exponential distribution and discuss more recent work, Chung and Bai (1998), Optimal designs of simple step-stress accelerated life tests for lognormal lifetime distributions, Ma and Meeker, Optimum step-stress accelerated life test plans for log-location-scale distributions, (2008).

***Model description***

At stress level $x_i, (i = 1, 2 \ldots k,)$ the lifetime Y of a test unit is assumed to follow a log-location scale distribution with cumulative distribution function (CDF)

$$F_i(y;\, \mu_i, \sigma) = \Phi\left(\frac{\ln y - \mu_i}{\sigma}\right), y \geq 0,$$

Where $\Phi(\cdot)$ is the standard log-location-scale CDF, and the location parameter is

$$\mu_i = \mu(x_i) = \beta_0 + \beta_1 x_i\,,$$

and $\sigma$ is the unknown scale parameter. Here, the regression parameters $\beta_0$ and $\beta_1 (< 0)$ are unknown and need to be estimated, and the scale parameter $\sigma$ is assumed to be free of the stress levels. The CDF of the lifetime of a test unit under the k-level step-stress ALT is given by

$$G(y) = F_i(y + s_{i-1} - \tau_{i-1};\, \mu_i, \sigma) \text{ for } \begin{cases} \tau_{i-1} \leq y \leq \tau_i, & \text{for } i = 1,2, \cdots k-1 \\ \tau_{i-1} \leq y \leq \infty, & \text{for } i = k, \end{cases}$$

Where $s_0=0$ and $s_{i-1} = \tau_{i-1} + s_{i-2} - \tau_{i-2} \exp(\mu_i - \mu_{i-1})$ is the solution of the equation $F_i(s_{i-1}; \mu_i, \sigma) = F_{i-1}(\tau_{i-1} + s_{i-2} - \tau_{i-2}; \mu_{i-1}, \sigma), i = 2,3, \cdots, k$

$$G(y) = \begin{cases} G_1(y) = \Phi\left(\frac{\ln y - \mu_i}{\sigma}\right), & \text{for } 0 < y \leq \tau_1 \\ G_1(y) = \Phi\left[\frac{\ln(y + s_{i-1} - \tau_{i-1}) - \mu_i}{\sigma}\right], & \text{for } \tau_{i-1} \leq y \leq \tau_i \end{cases} \quad \ldots\ldots\ldots(1)$$

where $i = 2, \cdots k-1$, and $\tau_{i-1} \leq y \leq \infty$ where $i = k$

and the corresponding probability density function (PDF) of the lifetime of a test unit is given by

$$g(y) = \begin{cases} g_1(y) = \frac{1}{\sigma y}\Phi\left(\frac{\ln y - \mu_1}{\sigma}\right), & \text{for } 0 < y \leq \tau_1 \\ g_i(y) = \frac{1}{\sigma(y + s_{i-1} - \tau_{i-1})}\Phi\left[\frac{\ln(y + s_{i-1} - \tau_{i-1}) - \mu_i}{\sigma}\right], & \text{for } \tau_{i-1} \leq y \leq \tau_i\,, \end{cases} \quad \ldots\ldots\ldots(2)$$

where $i = 2, \cdots k-1$, and $\tau_{i-1} \leq y \leq \infty$ where $i = k$

***Maximum Likelihood Estimation***

From Equations (1) and (2), the joint PDF of observed data are

$n = (n_1, n_2, \ldots, n_k)$ and $y = (y_1, y_2, \ldots, y_k)$ with $y_i = (y_{i,1}, y_{i,2}, \ldots, y_{i,n_i})$ is given by

$$f(y,n) = \frac{n!}{(n - \sum_{i=1}^{k} n_i)!}\left\{\prod_{i=1}^{k}\left[\prod_{j=1}^{n_i} g_i(y_{i,j})\right]\right\}[1 - G_k(\tau_k)]^{n - \sum_{i=1}^{k} n_i} \quad \ldots\ldots\ldots(2)$$

and so the log-likelihood function of $(\beta_0, \beta_1, \sigma)$ is given by

$$n \propto -\left(\sum_{i=1}^{k} n_i\right)\ln\sigma - \sum_{i=1}^{k}\sum_{j=1}^{n_i}\ln(y_{i,j} + s_{i-1} - \tau_{i-1}) + \sum_{i=1}^{k}\sum_{j=1}^{n_i}\ln\phi(Z_{i,j}) + \left(n - \sum_{i=1}^{k} n_i\right)\ln[1 - \phi(\eta_k)]$$

Where

$$z_{i,j} = \frac{\ln(y_{i,j} + s_{i-1} - \tau_{i-1}) - (\beta_0 + \beta_1 x_i)}{\sigma}, i = 1,2, \cdots, k \text{ and } j = 1,2, \cdots n_i,$$

$$\eta_k = \frac{\ln(\tau_k + s_{k-1} - \tau_{k-1}) - (\beta_0 + \beta_1 x_k)}{\sigma}$$

Note that the MLEs of $\beta_0, \beta_1$, and $\sigma$ exist only if $n_i > 0, i = 2, \cdots, k$, in Equation (3). By using following expressions

$$\frac{\partial s_{i-1}}{\partial_{\beta_1}} = \sum_{h=2}^{i}(x_h - x_{h-1})s_{h-1}e^{\beta_1(x_i - x_h)}$$

$$\frac{\partial \mathrm{z_{i,j}}}{\partial_{\beta_0}} = -\frac{1}{\sigma}\,, \qquad \frac{\partial \mathrm{z_{i,j}}}{\partial_{\beta_1}} = \frac{1}{\sigma}\left[\sum_{h=2}^{i}(x_h - x_{h-1})\frac{s_{h-1}e^{\beta_1(x_i - x_h)}}{\mathrm{y_{i,j}} + s_{i-1} - \tau_{i-1}} - x_i\right], \frac{\partial \mathrm{z_{i,j}}}{\partial_\sigma} = -\frac{\mathrm{z_{i,j}}}{\sigma}$$

$for\ i = 1,2,\cdots,k\ and\ j = 1,2,\cdots,n_i$

$$\frac{\partial \eta_k}{\partial_{\beta_0}} = \frac{1}{\sigma}\,, \frac{\partial \eta_k}{\partial_{\beta_1}} = \frac{1}{\sigma}\left[\sum_{h=2}^{k}(x_h - x_{h-1})\frac{s_{h-1}e^{\beta_1(x_k - x_h)}}{\mathrm{y_{i,j}} + s_{k-1} - \tau_{k-1}} - x_k\right], \frac{\partial \eta_k}{\partial_\sigma} = -\frac{\eta_k}{\sigma}$$

$$\frac{\partial \Phi(\eta_k)}{\partial_{\beta_0}} = -\frac{\Phi(\eta_k)}{\sigma}, \frac{\partial \Phi(\eta_k)}{\partial_{\beta_1}} = \frac{\Phi(\eta_k)}{\sigma}\left[\sum_{h=2}^{k}(x_h - x_{h-1})\frac{s_{h-1}e^{\beta_1(x_k - x_h)}}{\tau_k + s_{k-1} - \tau_{k-1}} - x_k\right],$$

$$\frac{\partial \Phi(\eta_k)}{\partial_\sigma} = \frac{\eta_k \Phi(\eta_k)}{\sigma},$$

The MLE is $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\sigma}$ can be obtained by solving the following likelihood equations:

$$\frac{\partial \mathrm{In\ L}}{\partial_{\beta_0}} = \frac{1}{\sigma}\left[-\sum_{i=1}^{k}\sum_{j=1}^{n_i}\frac{1}{\phi(z_{i,j})}\cdot\frac{d\phi(z_{i,j})}{dz} + \left(n - \sum_{i=1}^{k}n_i\right)\frac{\phi(\eta_k)}{1 - \Phi(\eta_k)}\right] = 0,$$

$$\begin{aligned}\frac{\partial \mathrm{In\ L}}{\partial_{\beta_1}} = \frac{1}{\sigma}\Bigg\{&-\sum_{i=1}^{k}x_i\sum_{j=1}^{n_i}\frac{1}{\phi(z_{i,j})}\cdot\frac{d\phi(z_{i,j})}{dz} - \sigma\sum_{i=2}^{k}\sum_{j=1}^{n_i}\sum_{h=2}^{i}(x_h - x_{h-1})\frac{s_{h-1}e^{\beta_1(x_i - x_h)}}{\mathrm{y_{i,j}} + s_{i-1} - \tau_{i-1}} + \sum_{i=2}^{k}\sum_{j=1}^{n_i}\frac{1}{\phi(z_{i,j})}\\ &\cdot\frac{d\phi(z_{i,j})}{dz}\sum_{\mathrm{h=2}}^{\mathrm{i}}(x_h - x_{h-1})\frac{s_{h-1}e^{\beta_1(x_i - x_h)}}{\mathrm{y_{i,j}} + s_{i-1} - \tau_{i-1}} + \left(n - \sum_{i=1}^{k}n_i\right)\frac{\phi(\eta_k)}{1 - \Phi(\eta_k)}\Bigg[x_k\\ &- (x_h - x_{h-1})\sum_{h=2}^{k}x_i\frac{s_{h-1}e^{\beta_1(x_k - x_h)}}{\tau_k + s_{k-1} - \tau_{k-1}}\Bigg]\Bigg\} = 0,\end{aligned}$$

$$\frac{\partial \mathrm{In\ L}}{\partial_{\beta_0}} = \frac{1}{\sigma}\left[-\sum_{i=1}^{k}n_i - \sum_{i=2}^{k}\sum_{j=1}^{n_i}\frac{z_{i,j}}{\phi(z_{i,j})}\cdot\frac{d\phi(z_{i,j})}{dz} + \left(n - \sum_{i=1}^{k}n_i\right)\frac{\eta_k\phi(\eta_k)}{1 - \Phi(\eta_k)}\right] = 0$$

The second derivatives of the log-likelihood function

$$\frac{\partial^2 \mathrm{In\ L}}{\partial_{\beta_0}^2} = -\frac{1}{\sigma^2}\left\{\sum_{i=2}^{k}\sum_{j=1}^{n_i}\frac{1}{\phi(z_{i,j})}\left[\left(\frac{d\phi(z_{i,j})}{dz}\right)^2 - \frac{d^2\phi(z_{i,j})}{dz^2}\right] + \left(n - \sum_{i=1}^{k}n_i\right)\frac{1}{1 - \Phi(\eta_k)}\left[\frac{d\phi(\eta_k)}{\mathrm{d}\eta} + \frac{\phi^2(\eta_k)}{1 - \Phi(\eta_k)}\right]\right\},$$

$$\begin{aligned}\frac{\partial^2 \mathrm{In\ L}}{\partial_{\beta_0}\partial_{\beta_1}} = -\frac{1}{\sigma^2}\Bigg\{&\sum_{i=1}^{k}x_i\sum_{j=1}^{n_i}\frac{1}{\phi(z_{i,j})}\left[\frac{1}{\phi(z_{i,j})}\left(\frac{d\phi(z_{i,j})}{dz}\right)^2 - \frac{d^2\phi(z_{i,j})}{dz^2}\right]\\ &-\sum_{i=2}^{k}\sum_{j=1}^{n_i}\frac{1}{\phi(z_{i,j})}\left[\frac{1}{\phi(z_{i,j})}\left(\frac{d\phi(z_{i,j})}{dz}\right)^2 - \frac{d^2\phi(z_{i,j})}{dz^2}\right]\sum_{\mathrm{h=2}}^{\mathrm{i}}(x_h - x_{h-1})\frac{s_{h-1}e^{\beta_1(x_i - x_h)}}{\mathrm{y_{i,j}} + s_{i-1} - \tau_{i-1}}\\ &+\left(n - \sum_{i=1}^{k}n_i\right)\frac{x_k}{1 - \Phi(\eta_k)}\left[\frac{d\phi(\eta_k)}{\mathrm{d}\eta} + \frac{\phi^2(\eta_k)}{1 - \Phi(\eta_k)}\right] - \left(n - \sum_{i=1}^{k}n_i\right)\frac{x_k}{1 - \Phi(\eta_k)}\Bigg[\frac{d\phi(\eta_k)}{\mathrm{d}\eta}\\ &+\frac{\phi^2(\eta_k)}{1 - \Phi(\eta_k)}\Bigg]\sum_{h=2}^{k}(x_h - x_{h-1})\frac{s_{h-1}e^{\beta_1(x_k - x_h)}}{\tau_k + s_{k-1} - \tau_{k-1}}\Bigg\}\end{aligned}$$

$$\frac{\partial^2 \text{In L}}{\partial_{\beta_0}\partial_\sigma} = -\frac{1}{\sigma}\left(\frac{\partial^2 \text{In L}}{\partial_{\beta_0}}\right) - \frac{1}{\sigma^2}\left\{\sum_{i=1}^{k}\sum_{j=1}^{n_i}\frac{z_{i,j}}{\phi(z_{i,j})}\left[\frac{1}{\phi(z_{i,j})}\left(\frac{d\phi(z_{i,j})}{dz}\right)^2 - \frac{d^2\phi(z_{i,j})}{dz^2}\right] + \left(n - \sum_{i=1}^{k} n_i\right)\frac{\eta_k}{1-\Phi(\eta_k)}\left[\frac{d\phi(\eta_k)}{d\eta} + \frac{\phi^2(\eta_k)}{1-\Phi(\eta_k)}\right]\right\}$$

$$-2\left(n - \sum_{i=1}^{k} n_i\right)\frac{x_k}{1-\Phi\eta_k}\left[\frac{d\phi(\eta_k)}{d\eta} + \frac{\phi^2(\eta_k)}{1-\Phi\eta_k}\right]\sum_{h=2}^{k}(x_h - x_{h-1})\frac{s_{h-1}e^{\beta_{1(x_k - x_h)}}}{\tau_k + s_{k-1} - \tau_{k-1}}$$

$$+\left(n - \sum_{i=1}^{k} n_i\right)\frac{1}{1-\Phi\eta_k}\left[\frac{d\phi(\eta_k)}{d\eta} + \frac{\phi^2(\eta_k)}{1-\Phi\eta_k}\right]\left[\sum_{h=2}^{k}(x_h - x_{h-1})\frac{s_{h-1}e^{\beta_{1(x_k - x_h)}}}{\tau_k + s_{k-1} - \tau_{k-1}}\right]^2$$

$$-\frac{d^2\phi(z_{i,j})}{dz^2}\Bigg]\left[\sum_{h=2}^{i}(x_h - x_{h-1})(x_h - x_{h-1})\frac{s_{h-1}e^{\beta_1(x_i - x_h)}}{\text{y}_{\text{i,j}} + s_{i-1} - \tau_{i-1}}\right]^2 - \sigma\sum_{i=2}^{k}\sum_{j=1}^{n_i}\frac{1}{\phi(z_{i,j})}$$

$$\cdot\frac{d\phi(z_{i,j})}{dz}\sum_{h=2}^{i}\sum_{\ell=2}^{h}(x_h - x_{h-1})(x_\ell - x_{\ell-1})\frac{s_{i-1}e^{\beta_1(x_i - x_\ell)}}{\text{y}_{\text{i,j}} + s_{i-1} - \tau_{i-1}} - \sigma\sum_{i=2}^{k}\sum_{j=1}^{n_i}\frac{1}{\phi(z_{i,j})}$$

$$\cdot\frac{d\phi(z_{i,j})}{dz}\sum_{h=2}^{i}(x_h - x_{h-1})(x_i - x_h)\frac{s_{h-1}e^{\beta_1(x_i - x_h)}}{\text{y}_{\text{i,j}} + s_{i-1} - \tau_{i-1}} + \sigma\sum_{i=2}^{k}\sum_{j=1}^{n_i}\frac{1}{\phi(z_{i,j})}$$

$$\cdot\frac{d\phi(z_{i,j})}{dz}\left[\sum_{h=2}^{i}(x_h - x_{h-1})(x_i - x_h)\frac{s_{h-1}e^{\beta_1(x_i - x_h)}}{\text{y}_{\text{i,j}} + s_{i-1} - \tau_{i-1}}\right]^2$$

$$+\sigma^2\sum_{1=2}^{k}\sum_{j=1}^{n_i}\sum_{h=2}^{i}\sum_{\ell=1}^{h}(x_h - x_\ell)(x_i - x_{\ell-1})\frac{s_{h-1}e^{\beta_1(x_i - x_h)}}{\text{y}_{\text{i,j}} + s_{i-1} - \tau_{i-1}}$$

$$+\sum_{i=2}^{k}\sum_{j=1}^{n_i}\sum_{h=2}^{h}(x_h - x_{h-1})(x_i - x_h)\frac{s_{h-1}e^{\beta_1(x_i - x_h)}}{\text{y}_{\text{i,j}} + s_{i-1} - \tau_{i-1}}$$

$$-\sigma^2\sum_{i=2}^{k}\sum_{j=1}^{n_i}\left[(x_h - x_{h-1})(x_i - x_h)\frac{s_{h-1}e^{\beta_1(x_i - x_h)}}{\text{y}_{\text{i,j}} + s_{i-1} - \tau_{i-1}}\right] + \sigma\Bigg(n$$

$$-\sum_{i=1}^{k} n_i\Bigg)\frac{\phi(\eta_k)}{1-\Phi(\eta_k)}^2\sum_{h=2}^{i}\sum_{\ell=2}^{h}(x_i - x_{\ell-1}) + \sigma\left(n - \sum_{i=1}^{k} n_i\right)\frac{\phi(\eta_k)}{1-\Phi(\eta_k)}\sum_{h=2}^{k}(x_h$$

$$-x_{h-1})(x_k - x_{k-1})\frac{s_{h-1}e^{\beta_1(x_k - x_h)}}{\tau_k + s_{k-1} - \tau_{k-1}} - \sigma\left(n - \sum_{i=1}^{k} n_i\right)\frac{\phi(\eta_k)}{\Phi(\eta_k)}\Bigg[\sum_{h=2}^{k}(x_h$$

$$-x_{h-1})\frac{S_{h-1}e^{\beta_{1(x_k - x_h)}}}{\tau_k + S_{k-1} - \tau_{k-1}}\Bigg]^2 + \left(n - \sum_{i=1}^{k} n_i\right)\frac{x_k^2}{\Phi(\eta_k)}\left[\frac{d\phi(\eta_k)}{d\eta} + \frac{\phi^2(\eta_k)}{1-\Phi(\eta_k)}\right]\Bigg\}$$

$$2\left(n-\sum_{i=1}^{k} n_i\right)\frac{x_k}{1-\Phi\eta_k}\left[\frac{d\phi(\eta_k)}{d\eta}+\frac{\phi^2(\eta_k)}{1-\Phi\eta_k}\right]\sum_{h=2}^{k}(x_h-x_{h-1})\frac{s_{h-1}e^{\beta_1(x_k-x_h)}}{\tau_k+s_{k-1}-\tau_{k-1}}$$
$$+\left(n-\sum_{i=1}^{k} n_i\right)\frac{1}{1-\Phi\eta_k}\left[\frac{d\phi(\eta_k)}{d\eta}+\frac{\phi^2(\eta_k)}{1-\Phi\eta_k}\right]\left[\sum_{h=2}^{k}(x_h-x_{h-1})\frac{s_{h-1}e^{\beta_1(x_k-x_h)}}{\tau_k+s_{k-1}-\tau_{k-1}}\right]^2$$

$$\frac{\partial^2 InL}{\partial\beta_1\partial\sigma}=-\frac{1}{\sigma}\left(\frac{\partial InL}{\partial\beta_1}\right)-\frac{1}{\sigma^2}\left\{\sum_{i=1}^{k}x_i\sum_{j=1}^{n_i}\frac{z_{ij}}{\phi(z_{ij})}\left[\frac{1}{\phi(z_{ij})}\left(\frac{d\phi(z_{ij})}{dz}\right)^2-\frac{d^2\phi(z_{ij})}{dz^2}\right]\sum_{i=2}^{k}\sum_{j=1}^{n_i}\frac{z_{ij}}{\phi(z_{ij})}\left[\frac{1}{\phi(z_{ij})}\left(\frac{d\phi(z_{ij})}{dz}\right)^2-\frac{d^2\phi(z_{ij})}{dz^2}\right]\sum_{h=2}^{i}(x_h-x_{h-1})\frac{s_{h-1}e^{\beta_1(x_i-x_h)}}{y_{i,j}+s_{i-1}-\tau_{i-1}}+\sigma\sum_{i=2}^{k}\sum_{j=1}^{n_i}\sum_{h=2}^{i}(x_h-x_{h-1})\frac{s_{h-1}e^{\beta_1(x_i-x_h)}}{y_{i,j}+s_{i-1}-\tau_{i-1}}+\left(n-\sum_{i=1}^{k}n_i\right)\frac{x_k}{1-\Phi\eta_k}\left[\frac{d\phi(\eta_k)}{d\eta}+\frac{\phi^2(\eta_k)}{1-\Phi\eta_k}\right]-\left(n-\sum_{i=1}^{k}n_i\right)\frac{1}{1-\Phi\eta_k}\left[\frac{d\phi(\eta_k)}{d\eta}+\frac{\phi^2(\eta_k)}{1-\Phi\eta_k}\right]\sum_{h=2}^{k}(x_h-x_{h-1})\frac{s_{h-1}e^{\beta_1(x_k-x_h)}}{\tau_k+s_{k-1}-\tau_{k-1}}\right\}$$

$$\frac{\partial^2 InL}{\partial\sigma^2}=-\frac{1}{\sigma}\left(\frac{\partial InL}{\partial\sigma}\right)$$
$$-\frac{1}{\sigma^2}\left\{\sum_{i=2}^{k}\sum_{j=1}^{n_i}\frac{z_{ij}}{\phi(z_{ij})}\left[\frac{1}{\phi(z_{ij})}\left(\frac{d\phi(z_{ij})}{dz}\right)^2-\frac{d^2\phi(z_{ij})}{dz^2}\right]\sum_{i=1}^{k}x_i\sum_{j=1}^{n_i}\frac{z_{ij}}{\phi(z_{ij})}\cdot\left(\frac{d\phi(z_{ij})}{dz}\right)\right.$$
$$\left.+\left(n-\sum_{i=1}^{k}n_i\right)\frac{\eta k}{1-\Phi\eta_k}\phi(\eta_k)+\eta k\frac{d\phi(\eta_k)}{d\eta}+\frac{\phi^2(\eta_k)}{1-\Phi\eta_k}\right\}$$

Since these equations cannot be solved analytically, numerical methods such as simulated annealing algorithm or some other iterative procedure must be employed for this estimation problem. An advantage of using the simulated annealing algorithm is that it allows us to find a global optimum without depending on the choice of the initial values, which is one of the main drawbacks of the commonly used numerical methods such as Newton–Raphson

***Numerical Illustration***

The optimal solutions were obtained by the simulated annealing algorithm as suggested by Corana *et al* (1987). It can be easily seen that the evaluated asymptotic variances based on complete data are the smallest, followed by those based on censored data within the searching range (0, 50], and then the ones with inspection intervals being chosen according to certain ratios. To determine the optimal unequal time points $(\tau_1<\tau_2<\cdots<\tau_k)$ that minimize the large-sample approximate variance of the MLE of the 200pth quantile $(0<p<1)$ of the log-lifetime distribution at the normal-use stress $x_0$. The MLE of the 200p quantile at the normal-use stress $x_0$ can be expressed as $\hat{y}_p=\hat{\beta}_0+\hat{\beta}_1x_0+z_p\hat{\sigma}$, where $z_p$ is the 100pth percentile of the standard log-location-scale distribution. Thus, If $x_0=0$, the asymptotic variance of the estimator $\hat{y}_p$ at the normal-use stress $x_0$ is given by

$$\mathrm{A\ Var}(y_p)=\mathrm{A\ Var}(\hat{\beta}_1+z_p\hat{\sigma})=V_{11}+z_p^2V_{33}+2_{zp}V_{13}$$

Where

$V_{ij}'$ It is verified under the C- optimality Criterion based 2-level step-stress ALT plan is preferable, whenever we optimize the general 2-level step-stress ALT plan the second K-level step – ALT plan under a censoring scheme under the considered and the results are shown in Table -1. $\tau=2:1$, and the inspection interval for the first stage is twice as long as that for the second stage.

**Table 1** The Variance – Optimality under the Step-stress setting Based on Complete data in the Log location Scale Family Distribution

| Complete data | | | | Censored data | | | |
|---|---|---|---|---|---|---|---|
| **p** | **$X_1$** | **$X_2$** | $\tau_1^c$ | $\tau_1^c,\tau_2^c$ | $\Delta_\tau=2:1$ | $\Delta_\tau=2:1$ | $\Delta_\tau=1:2$ |
| 0.5 | 0.2 | 0.5 | 10.2895 | 10.0470, 20.0000 | 10.0312(3.48) | 10.0000(3.35) | 6.6667(3.99) |
| | | 1.0 | 11.0011 | 10.7887, 20,0000 | 10.8267(2.45) | 10.8267(2.50) | 6.6667(3.10) |
| | 0.4 | 0.5 | 6.8724 | 6.8408, 20,0000 | 6.5817(15.81) | 6.5817(16.02) | 6.6667(15.83) |
| | | 1.0 | 7.3327 | 7.3107, 20.0000 | 7.1133(8.71) | 7.1133(8.32) | 0.6667(8.38) |
| 0.95 | 0.2 | 0.5 | 13.6090 | 12.6071, 20,0000 | 13.3333(7.48) | 10.0000(7.87) | 6.6667(11.03) |
| | | 1.0 | 14.9424 | 14.0111, 20,0000 | 13.3333(5.87) | 10.0000(6.56) | 6.6667(9.42) |
| | 0.4 | 0.5 | 8.1548 | 8.0935, 20,0000 | 7.8435(26.76) | 8.0765(25.03) | 6.6667(25.91) |
| | | 1.0 | 9.0548 | 9.0018, 20,0000 | 8.8341(14.17) | 9.0146(14.08) | 6.6667(16.19) |

The stress levels $x_i$, $i = 1, \ldots, k$, when $\beta_0 = 2.5$, $\beta_1 = -1.0$, $\sigma = 0.5$. to identify the optimal change points leading to variance–optimality, the optimal change points and associated asymptotic variance based on the censored data when the lengths of the inspection intervals were chosen according to certain ratio $\tau$.,

## CONCLUSION

The proposed methodology consists of large-sample approximate variance of ML estimators of quantiles of the widely-used log-location-scale family of distributions with continuous time-varying stress accelerated life tests based on censoring. Has been discussed the determination of the optimal choice of $(\tau_1, \ldots, \tau_k)$ for Log location Scale Family Distributions the asymptotic variance using optimality criterion. Considering k-level step stress ALT with unequal duration steps $(\tau_1, \ldots, \tau_k)$ and allowing the data to be censored, under a general log-location-scale lifetime distribution with mean life varying as a linear function of stress along with a cumulative exposure model,

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