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Research Article

RELIABILITY ANALYSIS OF A MULTI-STATE SYSTEM WITH PERFECT SWITCHOVER

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ABSTRACT

The purpose of this paper is to describe the availability of plate manufacturing system of a steel industry. This system consists of six principal subsystems viz. furnace, roughing mill, tandem mill, pinch roll, down coiler and strapping machine. Failure and repair rate of these subsystems affects the overall system availability. Analysis has been calculated using Supplementary Variable Technique (SVT). Failure rates have been assumed constant whereas, repair rates are varied. Chapman-Kolmogorov differential equations have been developed from the transition diagram of the plate manufacturing system, using mnemonic rule. These equations are then solved using Lagrange's method. The transient state availability of the system and Mean Time Between Failure (MTBF) have been calculated numerically. The conclusions drawn at the end reflect the criticality of a particular subsystem and also assist the plant management in deciding maintenance priorities for optimum utilization of the resources.

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INTRODUCTION

The growth of science and technology and ever increasing needs of modern society i.e. applications of automation, embedded technology, software and hardware interfaces, application of advanced technology, multiple functions and many other features have made the engineering systems more complicated. The complexities of industrial systems as well as their products are increasing day-by-day. Safety and environment concerns, product cost and uninterrupted services also play a vital role in decision-making process. Globalization of market and availability of products in many varieties have thrown a great challenge before the industries to achieve the target. Their products should be available to consumers to their satisfaction at reasonable cost. The products should also provide satisfactory performance with minimum failures to consumers during their entire life. The improvements in effectiveness of such complex systems have therefore acquired special importance in recent years.

In past, the performance of different industrial systems has been measured using several techniques. Biswas and Sarkar (2000) studied the availability of a system maintained through several imperfect repairs before a replacement or a perfect repair. Singh *et al*. (2005) analyzed a three-unit standby system of water pumps in which two units were operative simultaneously and the third one was a cold standby for an ash handling plant. You and Chen (2005) proposed an efficient heuristic approach for series-parallel redundant reliability problems. Kumar *et al*. (2007) analyzed the reliability of a non-redundant robot using fuzzy lambda-tau methodology. Zio and Zoia (2009) applied the reversible-jump Markov chain Monte Carlo technique for identifying the parameters responsible for component degradation. Sachdeva *et al*. (2009) presented an approach based on Petri Nets for studying the behaviour of a real industrial system. Ghosh and Majumdar (2010) modeled the occurrences of successive failure types and time to failure of the two repairable machine systems. Yuan and Meng (2011) assumed the exponential distribution of working and repair time for a warm standby repairable system consisting of two dissimilar units and one repairman. Taheri and Zarei (2011) investigated the Bayesian system reliability assessment in a vague environment. Lisnianski (2012) presented a multi-state Markov model for a coal power generating unit. Shakuntla *et al*. (2011) discussed the availability analysis for a tube

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manufacturing industry using supplementary variable technique. Natarajan *et al*. (2013) proposed a model that would facilitate the infusing of quality and reliability in new products by blending the six sigma concept and the new product development process. Khalsaraei (2015) dealt with the numerical solution of initial value problems (for systems of ordinary differential equations by an explicit fourth-order Runge–Kutta method. Cekyay and Ozekici (2015) analyzed system reliability, mean time to failure, and steady-state availability as a function of the component failure rates.

For the purpose of estimating transient state availability of plate manufacturing system of a steel plant; in this paper; Runge-Kutta fourth order method has been employed. Firstly, Chapman-Kolmogorov equations of the system are developed using SVT. Repair rate has been varied while failure rate has been kept constant. The equations have also been developed keeping both, failure as well as repair rates, constant. The differential equations have been solved using Runge-Kutta fourth order method and the behaviour of system with various combinations of failure and repair rates of different subsystems has been analyzed. MTBF which is nothing but the average expected time between failures has been calculated using Simpson's 3/8 rule. In the conclusion part, performance of all the subsystems has been compared and maintenance priority has been proposed.

This paper is organized as follows. Present section consists of introduction and literature review. System description, various notations and assumptions used in the analysis have been covered in section 2. In section 3, mathematical modeling of the system has been done. In section 4, for analyzing the transient state availability, the differential equations have been solved using Runge-Kutta fourth order method and MTBF has also been calculated in each case. Section 5 gives us the conclusion of the analysis done in previous section.

SYSTEM DESCRIPTION, VARIOUS NOTATIONS AND ASSUMPTIONS

Plate manufacturing unit is an important part of the steel industry. The mill takes in bloom as input from the casting system and produces plate coil of 5 mm thickness which is further sent to plate shearing section for manufacturing of plate of required size.

System description

The plate manufacturing system consists of following subsystems:

***Furnace* (A):** It consists of a pusher which pushes blooms one by one inside the furnace. It consists of heating and soaking zones. It has two units in parallel. Failure of single unit reduces the capacity of the system. Complete system fails when both units breakdown.

***Extractor*:** It is a hydraulically operated robotic arm having finger like structure to hold the bloom for inward and outward movement. This subsystem never fails.

***Conveyors*:** These are cylindrically shaped barrels used for transporting blooms through the entire mill. This subsystem rarely fails.

***De-scaling unit*:** It removes oxide scale from the heated bloom and consists of pump, hose pipe and nozzle. It never fails.

***Roughing Mill* (B):** Meant for reducing the thickness of bloom from 200mm to 40mm; it consists of two horizontal roles rotating in opposite directions. The reduction in thickness is achieved in five to seven passes. It is a single unit; failure of which results in complete failure of the system.

***Tandem Mill* (C):** After passing through roughing mill, bloom enters tandem mill. Here, after thickness reduction, the bar is known as transfer bar. It is passed through tandem mill for final thickness reduction. It is single unit whose failure shuts down the complete system.

***Pinch Roll* (D):** The transfer bar so produced is passed through pinch roll before coiling to avoid loopy formation and to keep the transfer bar in 'tension' for proper winding. It is having two units in series. Failure of single units results in stoppage of the system.

***Down Coiler* (E):** Its main function is winding of coil. It consists of a moving mandrel on which coil is made to wind up. After complete winding, the mandrel moves inside thereby making the coil to fall outside. This coil is then carried away by the conveyors and overhead crane. It consists of two units out of which one is standby. If one unit fails, system still keeps working at full capacity. Failure of both units results in failure of the system.

***Strapping Machine* (F):** Its function is to wind straps on the coil so that coil does not open up while being carried away. Like down coiler, it also contains two units, one main and another standby. System fails only when both units fail.

Notations

A, B, C, D, E, F	indicate that the respective subsystems are working at full capacity
a, b, c, d, e, f	indicate that the respective subsystems are in failed state
E_s, F_s	indicate that one respective subsystem has failed
A'	indicate that the respective subsystem is working at reduced capacity
α_i $(i = 1\ to\ 6)$	indicate the failure rates of subsystems A, B, C, D, E and F respectively
β_i $(i = 1\ to\ 6)$	indicate the repair rates of subsystems A, B, C, D, E and F respectively
$P_0(t)$	denotes the probability that at time 't', all the units are working
$P_i(x, t)$	denotes the probability that at time 't', the system is in state i and having an elapsed repair time x

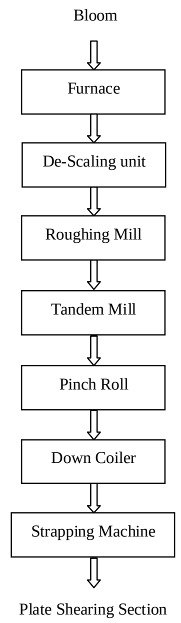

Fig 1 Schematic diagram of plate manufacturing system

Assumptions

Following assumptions have been made to develop the performance model:

1. Failure and repair rates are constant and independent of each other and their unit is taken as per day.
2. In case of assessment of availability using SVT, repair rates are considered variable and failure rates as constant.
3. Performance wise, a repaired unit is as good as new.
4. Service and repair/maintenance and replacement facilities are always available and there is no waiting time.
5. There are no simultaneous failures.
6. System may work at reduced capacity.
7. Standby/switchover units work perfectly.

PERFORMANCE MODELING OF THE SYSTEM

To determine the reliability of the plate manufacturing system, Chapman-Kolmogorov differential equations are developed by applying SVT. Probability considerations, using mnemonic rule, give us the following differential equations associated with the transition diagram (Fig. 2) of the system at time ($t+\Delta t$):

$$P_0(t+\Delta t) = [1-\alpha_1\Delta t-\alpha_2\Delta t-\alpha_3\Delta t-\alpha_4\Delta t-\alpha_5\Delta t-\alpha_6\Delta t]P_0(t) + \int\beta_1(x)P_7(x,t)dx\Delta t + \int\beta_2(x)P_{12}(x,t)dx\Delta t + \int\beta_3(x)P_{13}(x,t)dx\Delta t + \int\beta_4(x)P_{14}(x,t)dx\Delta t + \int\beta_5(x)P_5(x,t)dx\Delta t + \int\beta_6(x)P_1(x,t)dx\Delta t$$

$$P_0(t+\Delta t) - P_0(t) = -[\alpha_1\Delta t+\alpha_2\Delta t+\alpha_3\Delta t+\alpha_4\Delta t+\alpha_5\Delta t+\alpha_6\Delta t]P_0(t) + \int\beta_1(x)P_7(x,t)dx\Delta t + \int\beta_2(x)P_{12}(x,t)dx\Delta t + \int\beta_3(x)P_{13}(x,t)dx\Delta t + \int\beta_4(x)P_{14}(x,t)dx\Delta t + \int\beta_5(x)P_5(x,t)dx\Delta t + \int\beta_6(x)P_1(x,t)dx\Delta t$$

Dividing both sides by Δt, we get

$$\frac{P_0(t+\Delta t)-P_0(t)}{\Delta t} = -[\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6]P_0(t) + \int\beta_1(x)P_7(x,t)dx + \int\beta_2(x)P_{12}(x,t)dx + \int\beta_3(x)P_{13}(x,t)dx + \int\beta_4(x)P_{14}(x,t)dx + \int\beta_5(x)P_5(x,t)dx + \int\beta_6(x)P_1(x,t)dx +$$

$$\left[\frac{\partial}{\partial t} + L_0\right]P_0(t) = M_0(t) \quad (1)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + L_1(x)\right]P_1(x,t) = M_1(x,t) \quad (2)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + L_2(x)\right]P_2(x,t) = M_2(x,t) \quad (3)$$

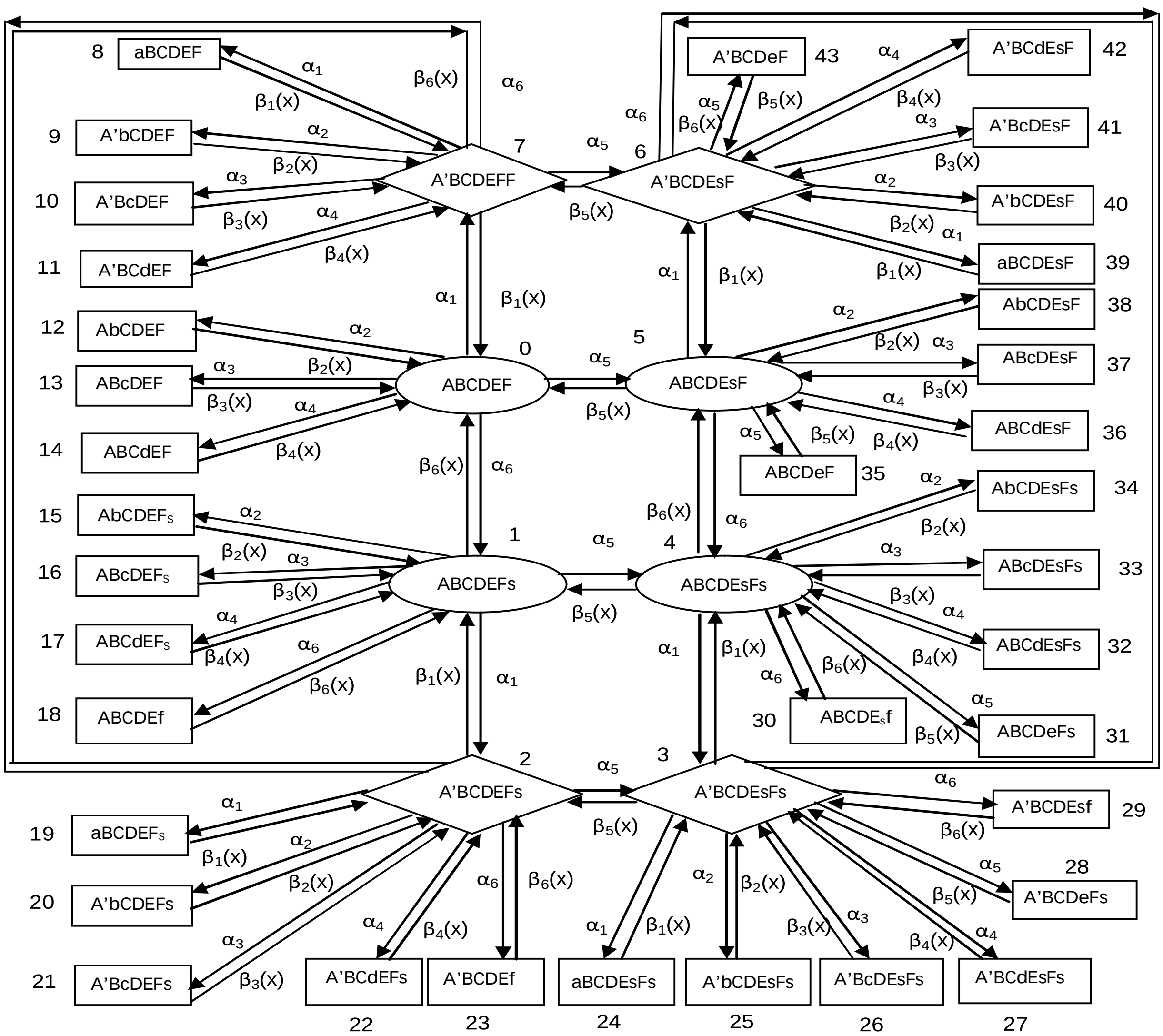

Fig 2 Transition diagram of plate manufacturing system

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+L_3(x)\right]P_3(x,t)=M_3(x,t) \quad (4)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+L_4(x)\right]P_4(x,t)=M_4(x,t) \quad (5)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+L_5(x)\right]P_5(x,t)=M_5(x,t) \quad (6)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+L_6(x)\right]P_6(x,t)=M_6(x,t) \quad (7)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+L_7(x)\right]P_7(x,t)=M_7(x,t) \quad (8)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\beta_1(x)\right]P_j(x,t)=0; \quad j=8,19,24,39 \quad (9)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\beta_2(x)\right]P_k(x,t)=0; \quad k=9,12,15,20,25,34,38,40 \quad (10)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\beta_3(x)\right]P_l(x,t)=0; \quad l=10,13,16,21,26,33,37,41 \quad (11)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\beta_4(x)\right]P_m(x,t)=0; \quad m=11,14,17,22,27,32,36,42 \quad (12)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\beta_5(x)\right]P_n(x,t)=0; \quad n=28,31,35,43 \quad (13)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\beta_6(x)\right]P_r(x,t)=0; \quad r=18,23,29,30 \quad (14)$$

Where,

$L_0=\sum_{i=1}^{6}\alpha_i$
$L_1(x)=\sum_{i=1}^{6}\alpha_i+\beta_6(x)$
$L_2(x)=\sum_{i=1}^{6}\alpha_i+\beta_1(x)+\beta_6(x)$
$L_3(x)=\sum_{i=1}^{6}\alpha_i+\beta_1(x)+\beta_5(x)+\beta_6(x)$
$L_4(x)=\sum_{i=1}^{6}\alpha_i+\beta_5(x)+\beta_6(x)$
$L_5(x)=\sum_{i=1}^{6}\alpha_i+\beta_5(x)$
$L_6(x)=\sum_{i=1}^{6}\alpha_i+\beta_1(x)+\beta_5(x)$
$L_7(x)=\sum_{i=1}^{6}\alpha_i+\beta_1(x)$

$M_0(t)=\int\beta_1(x)P_7(x,t)dx+\int\beta_2(x)P_{12}(x,t)dx+\int\beta_3(x)P_{13}(x,t)dx+\int\beta_4(x)P_{14}(x,t)+\int\beta_5(x)P_5(x,t)dx+\int\beta_6(x)P_1(x,t)dx$

$M_1(x,t)=\alpha_6P_0(t)+\int\beta_1(x)P_2(x,t)dx+\int\beta_2(x)P_{15}(x,t)dx+\int\beta_3(x)P_{16}(x,t)dx+\int\beta_4(x)P_{17}(x,t)+\int\beta_5(x)P_4(x,t)dx+\int\beta_6(x)P_{18}(x,t)dx$

$M_2(x,t)=\alpha_1P_1(t)+\alpha_6P_7(t)+\int\beta_1(x)P_{19}(x,t)dx+\int\beta_2(x)P_{20}(x,t)dx+\int\beta_3(x)P_{21}(x,t)dx+\int\beta_4(x)P_{22}(x,t)dx+\int\beta_5(x)P_3(x,t)dx+\int\beta_6(x)P_{23}(x,t)dx$

$M_3(x,t)=\alpha_1P_4(t)+\alpha_5P_2(t)+\alpha_6P_6(t)+\int\beta_1(x)P_{24}(x,t)dx+\int\beta_2(x)P_{25}(x,t)dx+\int\beta_3(x)P_{26}(x,t)dx+\int\beta_4(x)P_{27}(x,t)dx+\int\beta_5(x)P_{28}(x,t)dx+\int\beta_6(x)P_{29}(x,t)dx$

$M_4(x,t)=\alpha_5P_1(t)+\alpha_6P_5(t)+\int\beta_1(x)P_3(x,t)dx+\int\beta_2(x)P_{34}(x,t)dx+\int\beta_3(x)P_{33}(x,t)dx+\int\beta_4(x)P_{32}(x,t)dx+\int\beta_5(x)P_{31}(x,t)dx+\int\beta_6(x)P_{30}(x,t)dx$

$M_5(x,t)=\alpha_5P_0(t)+\int\beta_1(x)P_6(x,t)dx+\int\beta_2(x)P_{38}(x,t)dx+\int\beta_3(x)P_{37}(x,t)dx+\int\beta_4(x)P_{36}(x,t)dx+\int\beta_5(x)P_{35}(x,t)dx+\int\beta_6(x)P_4(x,t)dx$

$M_6(x,t)=\alpha_1P_5(t)+\alpha_5P_7(t)+\int\beta_1(x)P_{39}(x,t)dx+\int\beta_2(x)P_{40}(x,t)dx+\int\beta_3(x)P_{41}(x,t)dx+\int\beta_4(x)P_{42}(x,t)dx+\int\beta_5(x)P_{43}(x,t)dx+\int\beta_6(x)P_3(x,t)dx$

$M_7(x,t)=\alpha_1P_0(t)+\int\beta_1(x)P_8(x,t)dx+\int\beta_2(x)P_9(x,t)dx+\int\beta_3(x)P_{10}(x,t)dx+\int\beta_4(x)P_{11}(x,t)dx+\int\beta_5(x)P_6(x,t)dx+\int\beta_6(x)P_2(x,t)dx$

Initial Conditions

$P_0(0)=1$
$P_i(x,0)=0 \qquad (i=1,2,3\ldots\ldots\ldots\ldots..43)$

Boundary Conditions

$P_1(0,t)=\alpha_6P_0(t)$
$P_2(0,t)=\int\alpha_1P_1(x,t)dx+\int\alpha_6P_7(x,t)dx$
$P_3(0,t)=\int\alpha_1P_4(x,t)dx+\int\alpha_5P_2(x,t)dx+\int\alpha_6P_6(x,t)dx$
$P_4(0,t)=\int\alpha_5P_1(x,t)dx+\int\alpha_6P_5(x,t)dx$
$P_5(0,t)=\alpha_5P_0(t)$
$P_6(0,t)=\int\alpha_1P_5(x,t)dx+\int\alpha_5P_7(x,t)dx$
$P_7(0,t)=\alpha_1P_0(t)$
$P_8(0,t)=\int\alpha_1P_7(x,t)dx$
$P_9(0,t)=\int\alpha_2P_7(x,t)dx$
$P_{10}(0,t)=\int\alpha_3P_7(x,t)dx$
$P_{11}(0,t)=\int\alpha_4P_7(x,t)dx$
$P_{12}(0,t)=\alpha_2P_0(t)$
$P_{13}(0,t)=\alpha_3P_0(t)$
$P_{14}(0,t)=\alpha_4P_0(t)$
$P_{15}(0,t)=\int\alpha_2P_1(x,t)dx$
$P_{16}(0,t)=\int\alpha_3P_1(x,t)dx$
$P_{17}(0,t)=\int\alpha_4P_1(x,t)dx$
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$P_{21}(0,t)=\int\alpha_3P_2(x,t)dx$
$P_{22}(0,t)=\int\alpha_4P_2(x,t)dx$
$P_{23}(0,t)=\int\alpha_6P_2(x,t)dx$
$P_{24}(0,t)=\int\alpha_1P_3(x,t)dx$
$P_{25}(0,t)=\int\alpha_2P_3(x,t)dx$
$P_{26}(0,t)=\int\alpha_3P_3(x,t)dx$
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$P_{30}(0,t)=\int\alpha_6P_4(x,t)dx$
$P_{31}(0,t)=\int\alpha_5P_4(x,t)dx$
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$P_{33}(0,t)=\int\alpha_3P_4(x,t)dx$
$P_{34}(0,t)=\int\alpha_2P_4(x,t)dx$
$P_{35}(0,t)=\int\alpha_5P_5(x,t)dx$
$P_{36}(0,t)=\int\alpha_4P_5(x,t)dx$
$P_{37}(0,t)=\int\alpha_3P_5(x,t)dx$
$P_{38}(0,t)=\int\alpha_2P_5(x,t)dx$
$P_{39}(0,t)=\int\alpha_1P_6(x,t)dx$
$P_{40}(0,t)=\int\alpha_2P_6(x,t)dx$
$P_{41}(0,t)=\int\alpha_3P_6(x,t)dx$
$P_{42}(0,t)=\int\alpha_4P_6(x,t)dx$
$P_{43}(0,t)=\int\alpha_5P_6(x,t)dx$

Set of differential equations from (1) to (14) along with initial conditions and boundary conditions is called Chapman-Kolmogorov differential difference equations. Equation (1) is a linear differential equation of first order and equations (2) to (14) are linear partial differential equations of first order (Lagrange's type). All these equations have been solved using Lagrange's method. The probabilities of each state and expression of availability has been derived as follows:

$P_0(t) = e^{-L_0 t}[1 + \int M_0(t)e^{L_0 t}\,dt]$
$P_1(x,t) = e^{-\int L_1(x)dx}\left[\int M_1(x,t)e^{\int L_1(x)dx}\,dx + \alpha_6 P_0(t-x)\right]$
$P_2(x,t) = e^{-\int L_2(x)dx}\left[\int M_2(x,t)e^{\int L_2(x)dx}\,dx + \int \alpha_1 P_1(x,t-x)dx + \int \alpha_6 P_7(x,t-x)dx\right]$
$P_3(x,t) = e^{-\int L_3(x)dx}\left[\int M_3(x,t)e^{\int L_3(x)dx}\,dx + \alpha_1 P_4(x,t-x) + \int \alpha_5 P_2(x,t-x)dx + \int \alpha_6 P_6(x,t-x)dx\right]$
$P_4(x,t) = e^{-\int L_4(x)dx}\left[\int M_4(x,t)e^{\int L_4(x)dx}\,dx + \int \alpha_5 P_1(x,t-x)dx + \int \alpha_6 P_5(x,t-x)dx\right]$
$P_5(x,t) = e^{-\int L_5(x)dx}\left[\int M_5(x,t)e^{\int L_5(x)dx}\,dx + \alpha_5 P_0(t-x)\right]$
$P_6(x,t) = e^{-\int L_6(x)dx}\left[\int M_6(x,t)e^{\int L_6(x)dx}\,dx + \int \alpha_1 P_5(x,t-x)dx + \int \alpha_5 P_7(x,t-x)dx\right]$
$P_7(x,t) = e^{-\int L_7(x)dx}\left[\int M_7(x,t)e^{\int L_7(x)dx}\,dx + \alpha_1 P_0(t-x)\right]$
$P_8(x,t) = e^{-\int \beta_1(x)dx}\int \alpha_1 P_7(x,t-x)dx$
$P_9(x,t) = e^{-\int \beta_2(x)dx}\int \alpha_2 P_7(x,t-x)dx$
$P_{10}(x,t) = e^{-\int \beta_3(x)dx}\int \alpha_3 P_7(x,t-x)dx$
$P_{11}(x,t) = e^{-\int \beta_4(x)dx}\int \alpha_4 P_7(x,t-x)dx$
$P_{12}(x,t) = e^{-\int \beta_2(x)dx}\alpha_2 P_0(t-x)$
$P_{13}(x,t) = e^{-\int \beta_3(x)dx}\alpha_3 P_0(t-x)$
$P_{14}(x,t) = e^{-\int \beta_4(x)dx}\alpha_4 P_0(t-x)$
$P_{15}(x,t) = e^{-\int \beta_2(x)dx}\int \alpha_2 P_1(x,t-x)dx$
$P_{16}(x,t) = e^{-\int \beta_3(x)dx}\int \alpha_3 P_1(x,t-x)dx$
$P_{17}(x,t) = e^{-\int \beta_4(x)dx}\int \alpha_4 P_1(x,t-x)dx$
$P_{18}(x,t) = e^{-\int \beta_6(x)dx}\int \alpha_6 P_1(x,t-x)dx$
$P_{19}(x,t) = e^{-\int \beta_1(x)dx}\int \alpha_1 P_2(x,t-x)dx$
$P_{20}(x,t) = e^{-\int \beta_2(x)dx}\int \alpha_2 P_2(x,t-x)dx$
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$P_{29}(x,t) = e^{-\int \beta_6(x)dx}\int \alpha_6 P_3(x,t-x)dx$
$P_{30}(x,t) = e^{-\int \beta_6(x)dx}\int \alpha_6 P_4(x,t-x)dx$
$P_{31}(x,t) = e^{-\int \beta_5(x)dx}\int \alpha_5 P_4(x,t-x)dx$
$P_{32}(x,t) = e^{-\int \beta_4(x)dx}\int \alpha_4 P_4(x,t-x)dx$
$P_{33}(x,t) = e^{-\int \beta_3(x)dx}\int \alpha_3 P_4(x,t-x)dx$
$P_{34}(x,t) = e^{-\int \beta_2(x)dx}\int \alpha_2 P_4(x,t-x)dx$
$P_{35}(x,t) = e^{-\int \beta_5(x)dx}\int \alpha_5 P_5(x,t-x)dx$
$P_{36}(x,t) = e^{-\int \beta_4(x)dx}\int \alpha_4 P_5(x,t-x)dx$
$P_{37}(x,t) = e^{-\int \beta_3(x)dx}\int \alpha_3 P_5(x,t-x)dx$
$P_{38}(x,t) = e^{-\int \beta_2(x)dx}\int \alpha_2 P_5(x,t-x)dx$
$P_{39}(x,t) = e^{-\int \beta_1(x)dx}\int \alpha_1 P_6(x,t-x)dx$
$P_{40}(x,t) = e^{-\int \beta_2(x)dx}\int \alpha_2 P_6(x,t-x)dx$
$P_{41}(x,t) = e^{-\int \beta_3(x)dx}\int \alpha_3 P_6(x,t-x)dx$
$P_{42}(x,t) = e^{-\int \beta_4(x)dx}\int \alpha_4 P_6(x,t-x)dx$
$P_{43}(x,t) = e^{-\int \beta_5(x)dx}\int \alpha_5 P_6(x,t-x)dx$

Finally, the expression of time dependent availability A(t) is obtained by summation of probabilities of all the working states and reduced capacity states, i.e.

$$A(t) = P_0(t) + \int \sum_{i=1}^{7} P_i(x,t)dx \tag{15}$$

Availability expression of the plate manufacturing system as given by equation (15) can be solved using constant failure rates and variable repair rates from the concerned plant.

Availability of the system when failure and repair rates are constant

As is clear from the above analysis how difficult it is to solve the problem if either failure rate or repair rate are varied. In order to simplify the problem, failure and repair rates are considered constant. In this case, the system of equations (1) to (14) can be represented as follows:

$$P_0(t)\left[\frac{\partial}{\partial t} + \sum_{i=1}^{6}\alpha_i\right] = P_7(t)\beta_1 + P_{12}(t)\beta_2 + P_{13}(t)\beta_3 + P_{14}(t)\beta_4 + P_5(t)\beta_5 + P_1(t)\beta_6 \tag{16}$$

$$P_1(t)\left[\frac{\partial}{\partial t} + \sum_{i=1}^{6}\alpha_i + \beta_6\right] = P_2(t)\beta_1 + P_{15}(t)\beta_2 + P_{16}(t)\beta_3 + P_{17}(t)\beta_4 + P_4(t)\beta_5 + P_{18}(t)\beta_6 + P_0(t)\alpha_6 \tag{17}$$

$$P_2(t)\left[\frac{\partial}{\partial t} + \sum_{i=1}^{6}\alpha_i + \beta_1 + \beta_6\right] = P_{19}(t)\beta_1 + P_{20}(t)\beta_2 + P_{21}(t)\beta_3 + P_{22}(t)\beta_4 + P_3(t)\beta_5 + P_{23}(t)\beta_6 + P_1(t)\alpha_1 + P_7(t)\alpha_6 \tag{18}$$

$$P_3(t)\left[\frac{\partial}{\partial t} + \sum_{i=1}^{6}\alpha_i + \beta_1 + \beta_5 + \beta_6\right] = P_{24}(t)\beta_1 + P_{25}(t)\beta_2 + P_{26}(t)\beta_3 + P_{27}(t)\beta_4 + P_{28}(t)\beta_5 + P_{29}(t)\beta_6 + P_4(t)\alpha_1 + P_2(t)\alpha_5 + P_6(t)\alpha_6 \tag{19}$$

$$P_4(t)\left[\frac{\partial}{\partial t} + \sum_{i=1}^{6}\alpha_i + \beta_5 + \beta_6\right] = P_3(t)\beta_1 + P_{34}(t)\beta_2 + P_{33}(t)\beta_3 + P_{32}(t)\beta_4 + P_{31}(t)\beta_5 + P_{30}(t)\beta_6 + P_1(t)\alpha_5 + P_5(t)\alpha_6 \tag{20}$$

$$P_5(t)\left[\frac{\partial}{\partial t} + \sum_{i=1}^{6}\alpha_i + \beta_5\right] = P_6(t)\beta_1 + P_{38}(t)\beta_2 + P_{37}(t)\beta_3 + P_{36}(t)\beta_4 + P_{35}(t)\beta_5 + P_4(t)\beta_6 + P_0(t)\alpha_5 \tag{21}$$

$$P_6(t)\left[\frac{\partial}{\partial t} + \sum_{i=1}^{6}\alpha_i + \beta_1 + \beta_5\right] = P_{39}(t)\beta_1 + P_{40}(t)\beta_2 + P_{41}(t)\beta_3 + P_{42}(t)\beta_4 + P_{43}(t)\beta_5 + P_3(t)\beta_6 + P_5(t)\alpha_1 + P_7(t)\alpha_5 \tag{22}$$

$$P_7(t)\left[\frac{\partial}{\partial t} + \sum_{i=1}^{6}\alpha_i + \beta_1\right] = P_8(t)\beta_1 + P_9(t)\beta_2 + P_{10}(t)\beta_3 + P_{11}(t)\beta_4 + P_6(t)\beta_5 + P_2(t)\beta_6 + P_0(t)\alpha_1 \tag{23}$$

$$P_i(t)\left[\frac{\partial}{\partial t} + \beta_1\right] = P_j(t)\alpha_1 \tag{24}$$

$for\ i = 8, j = 7; i = 19, j = 2; i = 24, j = 3; i = 39, j = 6$

$$P_i(t)\left[\frac{\partial}{\partial t} + \beta_2\right] = P_j(t)\alpha_2 \tag{25}$$

$for\ i = 9, j = 7; i = 12, j = 0; i = 15, j = 1; i = 20, j = 2; i = 25, j = 3; i = 34, j = 4; i = 38, j = 5; i = 40, j = 6$

$$P_i(t)\left[\frac{\partial}{\partial t} + \beta_3\right] = P_j(t)\alpha_3 \tag{26}$$

$for\ i = 10, j = 7; i = 13, j = 0; i = 16, j = 1; i = 21, j = 2; i = 26, j = 3; i = 33, j = 4; i = 37, j = 5; i = 41, j = 6$

$$P_i(t)\left[\frac{\partial}{\partial t} + \beta_4\right] = P_j(t)\alpha_4 \tag{27}$$

$for\ i = 11, j = 7; i = 14, j = 0; i = 17, j = 1; i = 22, j = 2; i = 27, j = 3; i = 32, j = 4; i = 36, j = 5; i = 42, j = 6$

$$P_i(t)\left[\frac{\partial}{\partial t}+\beta_5\right] = P_j(t)\alpha_5 \quad (28)$$

$for\ i = 28, j = 3; i = 31, j = 4; i = 35, j = 5; i = 43, j = 6$

$$P_i(t)\left[\frac{\partial}{\partial t}+\beta_6\right] = P_j(t)\alpha_6 \quad (29)$$

$for\ i = 18, j = 1; i = 23, j = 2; i = 29, j = 3; i = 30, j = 4$

Initial Conditions

$$P_i(t) = 1 \quad for\ i = 0$$
$$= 0 \quad for\ i \neq 0$$

To examine the effect of failure and repair rates on the availability in transient state, the system of differential equations (16) to (29) with initial conditions has been solved numerically using Runge-Kutta fourth order method. Analysis has been done for a period of 360 days divided over an interval of 30 days and the data has been presented in tables 1-12. These tables show the effect of failure and repair rates of various subsystems on the overall system availability. MTBF, which has been computed using Simpson's 3/8 rule, with corresponding failure and repair rates, has been given in the last row of each table.

RESULTS AND ANALYSIS

Effect of failure rate of furnace (α_1) on system availability

By varying failure rate α_1 from 0.0003472, 0.0007812, 0.0012153, 0.0016493 and 0.0020833 and keeping α_2 = 0.0016667, α_3 = 0.000833, α_4 = 0.0020833, α_5 = 0.0020833, α_6 = 0.0010417, β_1 = 0.0138891, β_2 = 0.06666667, β_3 = 0.04, β_4 = 0.10, β_5 = 0.0645161 and β_6 = 0.025, the availability of the system has been computed and compiled in Table 1, which shows that there is a decrease in availability upto 1.54 percent. Also availability decreases by upto 2.51 percent with the increase in time from 30 to 360 days. MTBF shows a decline of around 4 days with the increase in failure rate from 0.0003472 to 0.0020833.

Table 1 Effect of failure rate of furnace (α_1) on system availability

Time (days) ↓ / α_1 →	0.0003472	0.0007812	0.0012153	0.0016493	0.0020833
30	0.9456	0.9455	0.9452	0.9448	0.9443
60	0.9378	0.9374	0.9366	0.9355	0.9342
90	0.9360	0.9352	0.9339	0.9321	0.9298
120	0.9353	0.9343	0.9325	0.9300	0.9270
150	0.9350	0.9338	0.9316	0.9286	0.9248
180	0.9349	0.9334	0.9309	0.9275	0.9232
210	0.9348	0.9332	0.9304	0.9267	0.9220
240	0.9347	0.9330	0.9301	0.9260	0.9211
270	0.9347	0.9329	0.9298	0.9256	0.9204
300	0.9347	0.9328	0.9296	0.9252	0.9198
330	0.9347	0.9327	0.9294	0.9250	0.9194
360	0.9346	0.9327	0.9293	0.9248	0.9192
MTBF	337.77	337.32	336.54	335.48	334.17

Effect of failure rate of roughing mill (α_2) on system availability

As presented in Table 2, as failure rate α_2 increases from 0.0016667 to 0.0083333 and the values of α_1, α_3, α_4, α_5, α_6, β_1, β_2, β_3, β_4, β_5 and β_6 are kept at 0.0003472, 0.000833, 0.0020833, 0.0020833, 0.0010417, 0.0138891, 0.06666667, 0.04, 0.10, 0.0645161 and 0.025, respectively, availability shows a downward trend of maximum 8.01 percent. However it decreases by upto 1.69 percent as time increases from 30 to 360 days. It is also seen that MTBF also decreases by approximately 28 days as failure rate increases.

Effect of failure rate of tandem mill (α_3) on system availability

Next, we have studied the effect of failure rate of tandem mill on the availability of plate manufacturing system. The results shown in Table 3 indicate that by varying failure rate α_3 = 0.0008333, 0.0016666, 0.0025, 0.00333333 and 0.0041667 and taking α_1 = 0.0003472, α_2 = 0.0016667, α_4 = 0.0020833, α_5 = 0.0020833, α_6 = 0.0010417, β_1 = 0.0138891, β_2 = 0.06666667, β_3 = 0.04, β_4 = 0.10, β_5 = 0.0645161 and β_6 = 0.025, the availability decreases by 6.76 percent. It is also observed that there is a decrease of 2.71 percent in availability with the increase in time from 30 to 360 days. In this case, MTBF decreases by 23 days with the increase in failure rate.

Table 2 Effect of failure rate of roughing mill (α_2) on system availability

Time (days) ↓ / α_2 →	0.0016667	0.0033335	0.005	0.0066667	0.0083333
30	0.9456	0.9262	0.9074	0.8892	0.8717
60	0.9378	0.9166	0.8962	0.8767	0.8580
90	0.9360	0.9146	0.8941	0.8746	0.8559
120	0.9353	0.9140	0.8936	0.8740	0.8553
150	0.9350	0.9137	0.8933	0.8738	0.8551
180	0.9349	0.9135	0.8932	0.8737	0.8550
210	0.9348	0.9135	0.8931	0.8736	0.8549
240	0.9347	0.9134	0.8930	0.8735	0.8549
270	0.9347	0.9134	0.8930	0.8735	0.8548
300	0.9347	0.9133	0.8930	0.8735	0.8548
330	0.9347	0.9133	0.8929	0.8734	0.8548
360	0.9346	0.9133	0.8929	0.8734	0.8548
MTBF	337.77	330.39	323.34	316.58	310.10

Table 3 Effect of failure rate of tandem mill (α_3) on system availability

Time (days) ↓ / α_3 →	0.0008333	0.0016666	0.0025	0.0033333	0.0041667
30	0.9456	0.9324	0.9194	0.9066	0.8942
60	0.9378	0.9212	0.9051	0.8894	0.8742
90	0.9360	0.9184	0.9015	0.8852	0.8694
120	0.9353	0.9175	0.9004	0.8839	0.8680
150	0.9350	0.9172	0.9000	0.8835	0.8675
180	0.9349	0.9170	0.8999	0.8833	0.8673
210	0.9348	0.9169	0.8998	0.8832	0.8673
240	0.9347	0.9169	0.8997	0.8832	0.8672
270	0.9347	0.9169	0.8997	0.8831	0.8672
300	0.9347	0.9168	0.8996	0.8831	0.8671
330	0.9347	0.9168	0.8996	0.8831	0.8671
360	0.9346	0.9168	0.8996	0.8831	0.8671
MTBF	337.77	331.74	325.94	320.34	314.92

Effect of failure rate of pinch roll (α_4) on system availability

The results presented in Table 4 indicate the availability of the system when failure rate α_4 increases from 0.0020833 to 0.020833 and the values of α_1, α_2, α_3, α_5, α_6, β_1, β_2, β_3, β_4, β_5 and β_6 are kept at 0.0003472, 0.0016667, 0.0008333, 0.0020833, 0.0010417, 0.0138891, 0.06666667, 0.04, 0.10, 0.0645161 and 0.025 respectively. It is seen that availability decreases by 14.04 percent. However availability decreases by upto 1.12 percent as time increases from 30 to 360 days. It is observed that MTBF also decreases by 49 days as failure rate increases.

Table 4 Effect of failure rate of pinch roll (α_4) on system availability

Time (days) / α_4	0.0020833	0.0067708	0.0114583	0.0161458	0.0208333
30	0.9456	0.9066	0.8705	0.8369	0.8057
60	0.9378	0.8983	0.8619	0.8284	0.7974
90	0.9360	0.8966	0.8604	0.8271	0.7962
120	0.9353	0.8960	0.8599	0.8266	0.7958
150	0.9350	0.8958	0.8597	0.8264	0.7956
180	0.9349	0.8957	0.8596	0.8263	0.7955
210	0.9348	0.8956	0.8595	0.8262	0.7954
240	0.9347	0.8955	0.8594	0.8262	0.7954
270	0.9347	0.8955	0.8594	0.8261	0.7953
300	0.9347	0.8955	0.8594	0.8261	0.7953
330	0.9347	0.8954	0.8594	0.8261	0.7953
360	0.9346	0.8954	0.8594	0.8261	0.7953
MTBF	337.77	324.08	311.48	299.85	289.09

Effect of failure rate of down coiler (α_5) on system availability

Now, the effect of failure rate of down coiler on the system availability has been studied. It is noted that as failure rate α_5 increases from 0.0020833 to 0.0416667 and the values of α_1, α_2, α_3, α_4, α_6, β_1, β_2, β_3, β_4, β_5 and β_6 are taken as 0.0003472, 0.0016667, 0.0008333, 0.0020833, 0.0010417, 0.0138891, 0.06666667, 0.04, 0.10, 0.0645161 and 0.025 respectively, availability declines by maximum 17.85 percent. However it decreases by upto 5.90 percent with the increase in time from 30 to 360 days. MTBF decreases by around 60 days with the increase in failure rate.

Effect of failure rate of strapping machine (α_6) on system availability

Table 6 shows the effect of failure rate of strapping machine on overall system availability. It is seen that as failure rate α_6 increases from 0.00104167 to 0.0104167 and the values of α_1, α_2, α_3, α_4, α_5, β_1, β_2, β_3, β_4, β_5 and β_6 are considered as 0.0003472, 0.0016667, 0.0008333, 0.0020833, 0.0010417, 0.0138891, 0.06666667, 0.04, 0.10, 0.0645161 and 0.025 respectively, availability decreases upto 9.47 percent. However availability decreases upto 8.28 percent as time increases from 30 to 360 days. It is observed that MTBF also decreases by around 27 days as failure rate increases.

Table 5 Effect of failure rate of down coiler (α_5) on system availability

Time (days) / α_5	0.0020833	0.0119792	0.0218750	0.0317708	0.0416667
30	0.9456	0.9306	0.9002	0.8625	0.8152
60	0.9378	0.9160	0.8744	0.8257	0.7683
90	0.9360	0.9125	0.8684	0.8177	0.7591
120	0.9353	0.9115	0.8669	0.8158	0.7571
150	0.9350	0.9112	0.8664	0.8153	0.7566
180	0.9349	0.9110	0.8662	0.8151	0.7564
210	0.9348	0.9109	0.8661	0.8150	0.7564
240	0.9347	0.9108	0.8661	0.8149	0.7563
270	0.9347	0.9108	0.8660	0.8149	0.7563
300	0.9347	0.9108	0.8660	0.8149	0.7563
330	0.9347	0.9108	0.8660	0.8149	0.7563
360	0.9346	0.9108	0.8660	0.8149	0.7562
MTBF	337.77	329.82	314.81	297.53	277.50

Table 6 Effect of failure rate of *strapping machine* (α_6) on system availability

Time (days) / α_6	0.00104167	0.00338542	0.00572918	0.00807294	0.0104167
30	0.9456	0.9431	0.9383	0.9314	0.9227
60	0.9378	0.9319	0.9210	0.9058	0.8875
90	0.9360	0.9276	0.9122	0.8916	0.8673
120	0.9353	0.9253	0.9072	0.8834	0.8556
150	0.9350	0.9240	0.9043	0.8785	0.8488
180	0.9349	0.9232	0.9025	0.8757	0.8450
210	0.9348	0.9228	0.9015	0.8741	0.8428
240	0.9347	0.9225	0.9009	0.8731	0.8415
270	0.9347	0.9224	0.9006	0.8726	0.8408
300	0.9347	0.9223	0.9004	0.8722	0.8403
330	0.9347	0.9222	0.9003	0.8720	0.8401
360	0.9346	0.9222	0.9002	0.8719	0.8399
MTBF	337.77	334.25	327.96	319.76	310.32

Effect of repair rate of furnace (β_1) on system availability

Effect of repair rate of furnace subsystem has been presented in Table 7. It is seen that when repair rate β_1 of the furnace subsystem is varied from 0.0138891 to 0.6666667 and values of α_1 = 0.0003472, α_2 = 0.0016667, α_3 = 0.0008333, α_4 = 0.0020833, α_5 = 0.0020833, α_6 = 0.0010417, β_2 = 0.06666667, β_3 = 0.04, β_4 = 0.10, β_5 = 0.0645161 and β_6 = 0.025 are considered, there is almost no change in availability. Whereas, there is a decrease of only 1.05-1.10 percent in availability as number of days increase from 30 to 360. MTBF also remains almost constant.

Table 7 Effect of repair rate of furnace (β_1) on system availability

Time (days) / β_1	0.0138891	0.1770834	0.3402778	0.5034727	0.6666667
30	0.9456	0.9457	0.9457	0.9457	0.9457
60	0.9378	0.9379	0.9379	0.9379	0.9379
90	0.9360	0.9361	0.9362	0.9362	0.9362
120	0.9353	0.9356	0.9356	0.9356	0.9356
150	0.9350	0.9354	0.9354	0.9354	0.9354
180	0.9349	0.9353	0.9353	0.9353	0.9353
210	0.9348	0.9352	0.9352	0.9352	0.9352
240	0.9347	0.9352	0.9352	0.9352	0.9352
270	0.9347	0.9352	0.9352	0.9352	0.9352
300	0.9347	0.9352	0.9352	0.9352	0.9352
330	0.9347	0.9352	0.9352	0.9352	0.9352
360	0.9346	0.9352	0.9352	0.9352	0.9352
MTBF	337.77	337.89	337.89	337.89	337.89

Effect of repair rate of roughing mill (β_2) on system availability

Next, the effect of repair rate of roughing mill on the system availability has been computed. As β_2 is varied from 0.06666667 to 0.5 in five steps and the values of failure and repair rates of other subsystems i.e. α_1, α_2, α_3, α_4, α_5, α_6, β_1, β_3, β_4, β_5, and β_6 are taken as 0.0.0003472, 0.0016667, 0.0008333, 0.0020833, 0.0020833, 0.0010417, 0.0138891, 0.04, 0.10, 0.0645161 and 0.025 respectively, it is observed that availability of the system decreases by 0.86-1.10 percent with the increase in time from 30 to 360 days. But, it increases by 1.94 percent as repair rate increases from 0.06666667 to 0.50. Improvement in repair rate results in increase in MTBF of around 7 days as shown in the Table 8.

Effect of repair rate of tandem mill (β_3) on system availability

Table 9 shows the effect of improvement of repair rate of tandem mill on the system availability. It is noted that as β_3 increases from 0.04 to 0.10 and the value of failure and repair rates of other subsystems are kept at α_1 = 0.0003472, α_2 = 0.0016667, α_3 = 0.0008333, α_4 = 0.0020833, α_5 = 0.0020833, α_6 = 0.0010417, β_1 = 0.0138891, β_2 = 0.06666667, β_4 = 0.10, β_5 = 0.0645161 and β_6 = 0.025, availability shows an increase of 1.11 percent. But as the number of days increase from 30 to 360, there is a decrease of around 0.61-1.10 percent in the value of availability. MTBF increases by around 4 days with the increase in repair rate.

Table 8 Effect of repair rate of roughing mill (β_2) on system availability

Time (days) ↓ / β_2 →	0.06666667	0.175	0.2833333	0.3916667	0.50
30	0.9456	0.9569	0.9602	0.9617	0.9626
60	0.9378	0.9513	0.9547	0.9561	0.9570
90	0.9360	0.9497	0.9530	0.9545	0.9553
120	0.9353	0.9491	0.9524	0.9538	0.9547
150	0.9350	0.9488	0.9521	0.9535	0.9544
180	0.9349	0.9486	0.9519	0.9534	0.9542
210	0.9348	0.9485	0.9518	0.9533	0.9541
240	0.9347	0.9485	0.9517	0.9532	0.9541
270	0.9347	0.9484	0.9517	0.9532	0.9540
300	0.9347	0.9484	0.9517	0.9532	0.9540
330	0.9347	0.9484	0.9517	0.9531	0.9540
360	0.9346	0.9484	0.9517	0.9531	0.9540
MTBF	337.77	342.47	343.62	344.13	344.43

Table 9 Effect of repair rate of tandem mill (β_3) on system availability

Time (days) ↓ / β_3 →	0.04	0.055	0.07	0.085	0.10
30	0.9456	0.9478	0.9495	0.9508	0.9518
60	0.9378	0.9418	0.9443	0.9461	0.9474
90	0.9360	0.9406	0.9434	0.9453	0.9466
120	0.9353	0.9402	0.9431	0.9449	0.9463
150	0.9350	0.9400	0.9429	0.9447	0.9461
180	0.9349	0.9399	0.9427	0.9446	0.9459
210	0.9348	0.9398	0.9427	0.9445	0.9458
240	0.9347	0.9397	0.9426	0.9445	0.9458
270	0.9347	0.9397	0.9426	0.9444	0.9458
300	0.9347	0.9397	0.9425	0.9444	0.9457
330	0.9346	0.9397	0.9425	0.9444	0.9457
360	0.9347	0.9396	0.9425	0.9444	0.9457
MTBF	337.77	339.37	340.31	340.94	341.39

Effect of repair rate of pinch roll (β_4) on system availability

Effect of improvement of repair rate of pinch roll on the overall system availability has been presented in Table 10. As β_4 increases from 0.10 to 2.0 and the value of failure and repair rates of other subsystems are kept at α_1 = 0.0003472, α_2 = 0.0016667, α_3 = 0.0008333, α_4 = 0.0020833, α_5 = 0.0020833, α_6 = 0.0010417, β_1 = 0.0138891, β_2 = 0.06666667, β_3 = 0.04, β_5 = 0.0645161 and β_6 = 0.025, availability increases by 1.78 percent. But it decreases by 1.07-1.10 percent as the number of days increase from 30 to 360. In this case, MTBF increases by around 6 days.

Table 10 Effect of repair rate of pinch roll (β_4) on system availability

Time (days) ↓ / β_4 →	0.10	0.575	1.05	1.525	2.0
30	0.9456	0.9606	0.9621	0.9627	0.9630
60	0.9378	0.9532	0.9547	0.9553	0.9556
90	0.9360	0.9513	0.9528	0.9533	0.9536
120	0.9353	0.9506	0.9521	0.9527	0.9530
150	0.9350	0.9503	0.9518	0.9524	0.9527
180	0.9349	0.9502	0.9517	0.9522	0.9525
210	0.9348	0.9501	0.9516	0.9521	0.9524
240	0.9347	0.9500	0.9515	0.9521	0.9524
270	0.9347	0.9500	0.9515	0.9520	0.9523
300	0.9347	0.9500	0.9514	0.9520	0.9523
330	0.9347	0.9499	0.9514	0.9520	0.9523
360	0.9346	0.9499	0.9514	0.9520	0.9523
MTBF	337.77	343.09	343.61	343.81	343.91

Effect of repair rate of down coiler (β_5) on system availability

Table 11 shows the effect of improvement of repair rate of down coiler on the system availability. We see that as β_5 increases from 0.0645161 to 0.2857143 and the value of failure and repair rates of other subsystems are kept at α_1 = 0.0003472, α_2 = 0.0016667, α_3 = 0.0008333, α_4 = 0.0020833, α_5 = 0.0020833, α_6 = 0.0010417, β_1 = 0.0138891, β_2 = 0.06666667, β_3 = 0.04, β_4 = 0.10 and β_6 = 0.025, availability increases upto just 0.10 percent. But as the number of days increase from 30 to 360, there is a decrease of around 1.06-1.10 percent in the value of availability. MTBF also does not show much variation.

Table 11 Effect of repair rate of down coiler (β_5) on system availability

Time (days) ↓ / β_5 →	0.0645161	0.1198157	0.1751152	0.2304148	0.2857143
30	0.9456	0.9459	0.9460	0.9461	0.9461
60	0.9378	0.9384	0.9385	0.9385	0.9386
90	0.9360	0.9366	0.9367	0.9368	0.9368
120	0.9353	0.9359	0.9361	0.9361	0.9362
150	0.9350	0.9357	0.9358	0.9359	0.9359
180	0.9349	0.9355	0.9356	0.9357	0.9357
210	0.9348	0.9354	0.9356	0.9356	0.9356
240	0.9347	0.9354	0.9355	0.9356	0.9356
270	0.9347	0.9353	0.9355	0.9355	0.9355
300	0.9347	0.9353	0.9354	0.9355	0.9355
330	0.9347	0.9353	0.9354	0.9355	0.9355
360	0.9346	0.9353	0.9354	0.9355	0.9355
MTBF	337.77	337.97	338.02	338.04	338.04

Effect of repair rate of strapping machine (β_6) on system availability

At last, the effect of improvement of repair rate of strapping machine on the overall system availability has been computed and presented in Table 12. It is observed that as β_6 increases from 0.025 to 0.125 and the value of failure and repair rates of other subsystems are considered as α_1 = 0.0003472, α_2 = 0.0016667, α_3 = 0.0008333, α_4 = 0.0020833, α_5 = 0.0020833, α_6 = 0.0010417, β_1 = 0.0138891, β_2 = 0.06666667, β_3 = 0.04, β_4 = 0.10 and β_5 = 0.0645161, availability increases by only 0.14 percent. But, availability decreases by 0.98-1.10 percent as the number of days increase from 30 to 360. MTBF increases by less than half day with the increase in repair rate.

Table 12 Effect of repair rate of strapping machine (β_6) on system availability

Time (days) ↓ / β_6 →	0.025	0.05	0.075	0.10	0.125
30	0.9456	0.9457	0.9458	0.9458	0.9458
60	0.9378	0.9382	0.9383	0.9384	0.9384
90	0.9360	0.9366	0.9368	0.9368	0.9369
120	0.9353	0.9361	0.9363	0.9364	0.9364
150	0.9350	0.9360	0.9362	0.9362	0.9363
180	0.9349	0.9359	0.9361	0.9362	0.9362
210	0.9348	0.9358	0.9360	0.9361	0.9361
240	0.9347	0.9358	0.9360	0.9361	0.9361
270	0.9347	0.9358	0.9360	0.9361	0.9361
300	0.9347	0.9358	0.9360	0.9360	0.9361
330	0.9347	0.9357	0.9359	0.9360	0.9361
360	0.9346	0.9357	0.9359	0.9360	0.9360
MTBF	337.77	338.06	338.12	338.14	338.15

CONCLUSION

By comparing the results computed in of Tables 1-12, it reveals that improvement in repair rate of furnace, down coiler and strapping machine subsystems does not make any notable change on the overall system availability. Hence the plant management does not need to emphasise much on their maintenance. Based on the above analysis, the maintenance priority on the basis of repair rate can be as follows:

1. Roughing mill (maximum effect of repair rate on this subsystem)
2. Pinch Roll
3. Tandem Mill

Fig. 3 shows the effect of repair rate of roughing mill subsystem on the system availability.

However, it is observed that variation in failure rate makes huge difference on the system availability.

On the basis of variation in failure rate, it is seen that subsystem E (down coiler) has maximum impact on the availability as well as on MTBF of the system. This phenomenon has been depicted in the Fig. 4. Second and third most important subsystems are D (pinch roll) & F (strapping machine) respectively. However, subsystem A i.e. furnace has least effect on the availability and MTBF of the system. Hence, we infer that as far as maintenance planning and scheduling on the basis of failure rate is concerned, the maintenance priority should be given as per the following order:

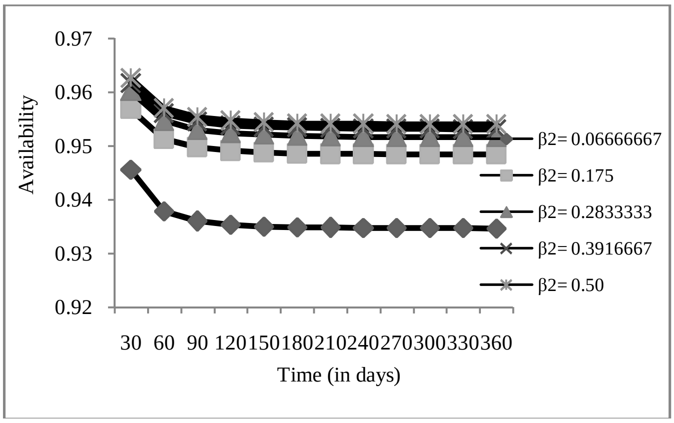

Figure 3 Effect of repair rate of roughing mill on system availability

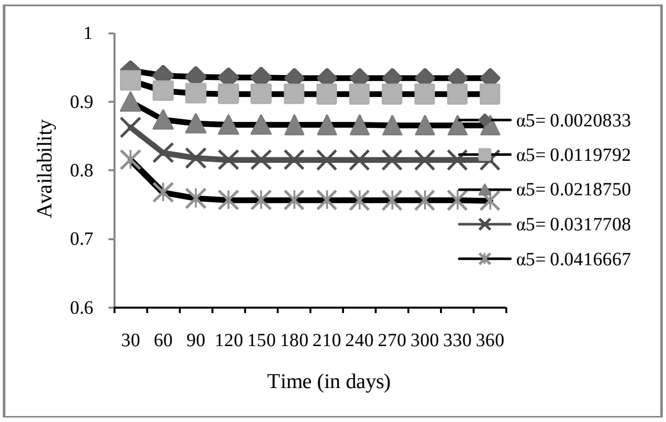

Figure 4 Effect of failure rate of down coiler on system availability

1. Down coiler
2. Pinch Roll
3. Strapping machine
4. Roughing mill
5. Tandem mill
6. Furnace (least priority for maintenance)

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