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Research Article

TAX EVASION CONTROL VIA MULTI-AGENT-BASED EQUILIBRIUM AND NON-EQUILIBRIUM MODEL ON VARIOUS TOPOLOGIES

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ABSTRACT

We study tax evasion dynamics in both an equilibrium model of statistical mechanics as the Ising model, and the non-equilibrium Majority-Vote model (MVM). These models of multi-agent-based are evolved via Monte Carlo simulations. Here, we also study the effect of various topologies on tax evasion dynamics. We found out that the reduction or not of tax morale also depends on the applied topology.

INTRODUCTION

The evolution of computers provides the simulation of several systems as differential equations and a system that has discrete entities. The origin of agent-based modeling can be traced back to the 1940s [1], to the introduction by von Neumann [2] and Ulam [3] of the notion of a cellular automaton. Agent-based models were primarily used for social systems by Reynolds [4], who tried to model the reality of living biological agents, know as artificial life. Reynolds introduced the notion of individual-based models, in which one investigates the global consequences of local interactions of members of a population.

In economics, the problem of tax evasion from a multi-agent-based per-spective has recently attracted attention from mathematicians and physicists. Experimental evidence provided by Gšachter [5] suggests that taxpayers tend to condition their decision regarding whether to pay taxes or not on the tax evasion decision of the members of their group or conditional cooperators. The conditional cooperators are thus more likely to behave honestly, if they have the impression that many others pay their taxes. However, if most others evade taxes, an individual is more likely to cheat on her taxes. Frey and Togler [6] also provide empirical evidence on the relevance of conditional cooperation for tax morale. They find a positive correlation between peoples tax morale, which is measured by asking whether tax evasion is justified if the chance arises, and their perception regarding how many others evade paying tax. Conditional cooperation from the viewpoint of the standard economic theory may be explained by changes in risk aversion due to changes in equity [7]. Therefore, realistic models of tax evasion appear to be necessary because tax evasion remains to be a major predicament facing governments [8, 9, 10,11]. The goal of this chapter is to show that substantial fluctuations in tax evasion may be caused by local interactions or herding behaviour among heterogeneous agents and that already minimal audit rates of a tax authority may suffice to alleviate this problem. Therefore, we use the Ising model (IM) [12] and Majority-Vote model (MVM)[13] model to study the implications of conditional cooperation in a multi-agent-based framework. Our setup allows us to consider a large number of heterogeneous agents who interact locally with each other and base their decision whether to evade taxes or not on the behavior of their neighbors (and thus they display some kind of imitation/herding behavior). The Ising model was first used in an economic context by Föllmer [14]. To be precise, we incorporate the behavior of tax evasion into the standard two-dimensional square lattice Ising spin model and furthermore add a policy maker's tax enforcement mechanism. Zaklan *et al*. [15] were the first to apply Statistical Mechanics in the study of the problem of tax evasion using the multi-agent-based concept. Following Pickhardt and Seibold [16] the model

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proposed by Zaklan *et al*. [15] is considered a standard agent-based tax evasion model that falls into the econophysics domain. Such models are based on the Ising model [15, 17, 18] and on non-equilibrium extensions by Lima [19, 20] differ fundamentally from other models of the economics domain. We aim to extend the study of Zaklan *et al.*[15], which illustrates how in the world where agents are conditionally cooperative different levels of enforcement affect aggregate tax evasion over time. Enforcement consists of two components: a probability of an audit each person is subject to in every period and a length of time detected tax evaders remain honest for. We also embed our tax evasion model in different network structures and find that substantial fluctuations in aggregate tax evasion behavior may arise if no enforcement is used. For our simulations, we make use of the fact that a dominant strategy exists at temperatures below the critical level and that the states, which are predominant, may change over time if the temperature is chosen to be slightly below its critical value. This process has been used for decades in the field of physics, and also by Hohnisch *et al.* [21] To replicate the evolution of the IFO business climate index. We also observe a second important and maybe less obvious effect of enforcement: numerical evidence suggests that even minimal levels of enforcement may help to reduce the presence of fluctuations in tax evasion.

Such fluctuations may, for instance, be mostly prevented in the considered networks by setting the audit rate to sufficiently high but still realistic levels. Everybody then remains compliant for most of the time. In this paper, we present two evolution dynamics for the model proposed by Zaklan *et al.* [15], where one is via equilibrium Ising model and other is via nonequilibrium MVM.

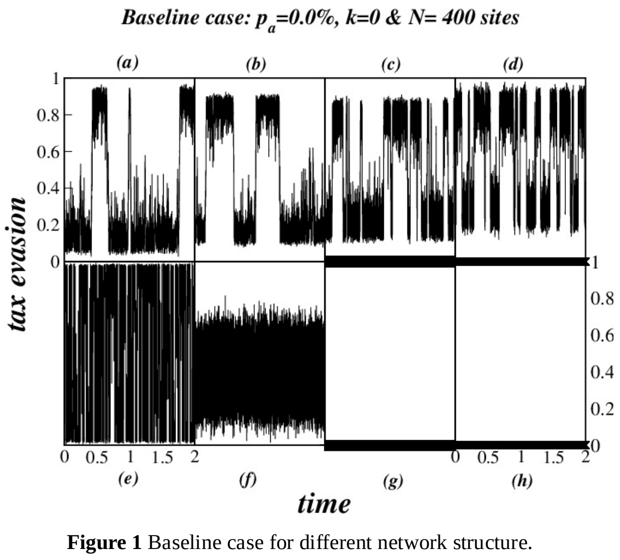

Figure 1 Baseline case for different network structure.

In Fig1. (a), we use for SL q = 0.95qc, in (b) for VD lattice at q = qc = 0.117 (critical noise for MVM model on VD lattice), in (c)for AN q = 0.8qc = 0.178, in (d) for SHP q = 0.8qc = 0.178, in (e)for DBA q = 0.8qc = 0.431, in (f) UBA q = 0.8qc = 0.306, in (g) for DER q = 0.9qc = 0.175 and in (h) for UER q = 0.9qc = 0.181. All simulation are performed over 20, 000 time steps.

Here, $time = 20{,}000/10{,}000$.

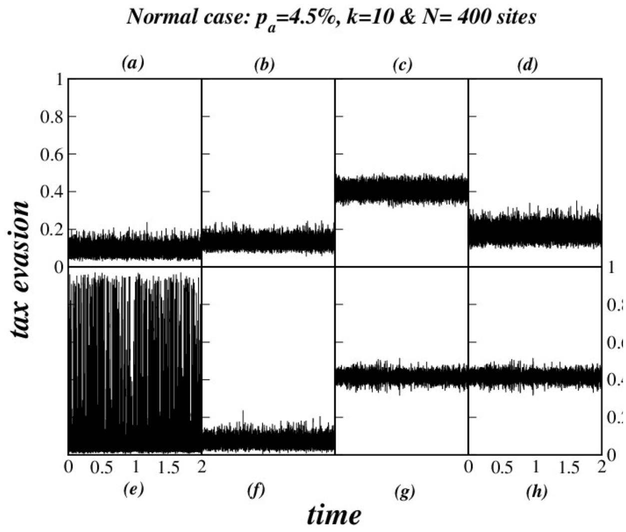

Figure 2 The same that Fig. 1, but now for $k = 10$ and $p_a = 4.5\%$

Model and evolution dynamics

Here, we study the model proposed by Lima [18] that is a version of Zaklan *et al.* [15] where the temporal evolution is via a non-equilibrium model as MVM. The model is given as follows:

1. It consists of agents located on nodes or sites of a regular or irregular structure and each agent is represented by an individual spin ; $\sigma_i = \pm 1$ who can either be an honest taxpayer +1 or a cheater -1;
2. Initially, everybody is assumed honest and in each iteration individual can reconsider their behaviour to become the opposite type of agent they were in the previous period. Each agent';s environment may prefer tax evasion or reject it.
3. The agent depends on two factors: First, the agent';s environment experts influence on the agent in the next period. Second, people's decisions are partly autonomous, independent of their environment, this autonomous part is responsible for the emergence of tax evasion, because some initially honest taxpayers decide to evade taxes and then exert influence on others to do so as well. Taxpayers have the greatest influence to turn honest citizens into tax cheaters if cheaters constitute a majority in the respective neighborhood. On the other hand, if most people in the vicinity are honest, the respective individual is likely to become a taxpayer if (s)he was a tax cheater before.
4. The model also presents an enforcement mechanism that consists of two components: a probability of an efficient audit pa and a punishment time k. If tax evasion is detected, the individual must remain honest for a number k of periods to be specified. One iteration is one sweep through the entire lattice. The temporal evolution of this model can be performed by using an equilibrium or by nonequilibrium dynamics.

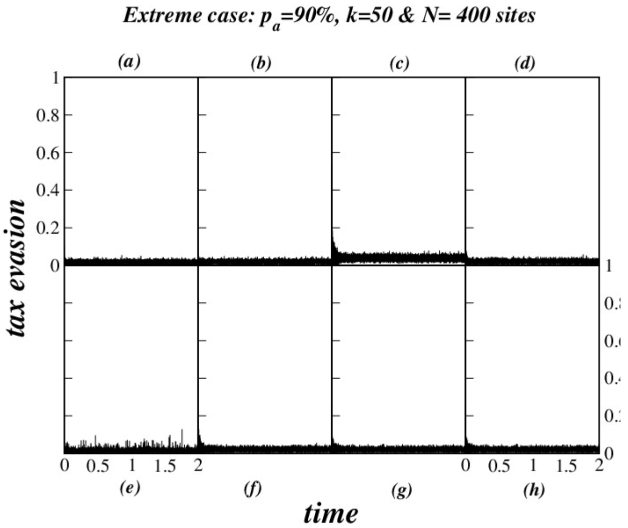

Figure 3 The same that Fig. 1, but now for $k=50$ and $p_a = 90$.

Model via equilibrium dynamics of Ising model

Zaklan *et al*. [15] performed Monte Carlo simulations using the Ising model as an equilibrium dynamics temporal evolution. The Hamiltonian or Energy (E) of Ising models is given by

$$H=-J\sum_{i,j} S_i S_j$$

where J = 1 is the factor of interaction between a central site and its neighbor and the sum runs over all its neighbors. Zaklan *et al*. [15] used the single spin-flip heat-bath algorithm to simulate the Ising model on SL with periodic boundary conditions. In this algorithm, irrespective of the current state of the chosen spin, its new state is chosen with probabilities

$$P_{\pm}=\frac{e^{-\beta H_{\pm}}}{e^{-\beta H_{+}}+e^{-\beta H_{-}}},$$

where P± are the probabilities of choosing the up (+) and down (-) states, respectively [27], and β is defined as 1/(kBT). Then, in every period each lattice site is occupied by a taxpayer who can either be honest ($S_i=+1$) or a tax evader ($S_i=-1$). We assume that everybody is honest, initially.

In consecutive time periods agents have the opportunity to change their agent-type, enabling cheating agents to become honest and honest citizens to become tax evaders. Each agent's social network is made up of four nearest neighbors, and the neighborhood may either prefer tax evasion, reject it or be indifferent. Tax evaders have the greatest influence to turn honest citizens into tax evaders if they constitute unanimity in the given neighborhood or contrariwise if a the whole neighborhood is formed by honest citizen. The strength of neighborhood influence can be controlled by adjusting the temperature T.

Model via non-equilibrium dynamics of MVM

Different from Zaklan *et al*. [15], Lima [18] performed Monte Carlo simula-tions using the MVM as a nonequilibrium dynamics temporal evolution. Therefore, on a square lattice or complex networks each site of the structure is inhabited, at each time step, by an agent with 'voters' or spin variables taking the values +1 representing an honest taxpayer, or -1 trying to at least partially escape her tax duty. Here is assumed that initially everybody is honest. Each period individuals can rethink their behavior and have the opportunity to become the opposite type of agent they were in previous pe-riod. In each period the system evolves by a single spin-flip dynamics

with a probability w_i given by

$$w_i=\frac{1}{2}[1-(1-2q)\sigma_i S(\sum_{\delta=1}^{k_i}\sigma_i+1)]$$

where $S(x)$ is the sign ± 1 of x if $x\neq 0$, $S(x)=0$ if $x=0$, and the summation runs over all k_i nearest-neighbor sites $\sigma_{i+\delta}$ of σ_i. This model an agent assumes the value ± 1 depending on the opinion of the majority of its neighbors. The control noise parameter q plays the role of the temperature in equilibrium systems and measures the probability of aligning spins antiparallel to the majority of neighbors. Then various degrees of homogeneity regarding either position is possible. An extremely homogenous group is entirely made of honest people or tax evaders, depending on the sign $S(x)$ of the majority of neighbors. If $S(x)$ of the neighbors is zero the agent σ_i will be honest or evader in the next time period with probability $\frac{1}{2}$. We further introduce a probability of an efficient audit (p_a). Therefore, if tax evasion is detected, the agent must remain honest for a number k of time steps. Here, one time step is one sweep through the entire lattice.

Lattices, graphs and networks

Directed and undirected Apollonian network

The Apollonian network (AN) is composed of $N=3+(3^n-1)/2$ nodes, where n is the generation number and N the node number [25, 26]. On these AN structures we can introduce a disorder, in such a way that we redirect a fraction p of the links. This redirecting results in a directed network, preserving the outgoing node of the redirected link but changing the incoming node. When $p=0$ we have the standard AN networks, while for $p=1$ we have something similar to random networks [22]. In this procedure of the redirecting links, the number of outgoing links of each node is preserved even when p = 1 and the network still have hubs that are the most influent nodes.

These networks display a scale-free degree distribution and a hierarchical structure. In the undirected case, there exists the reciprocity of redirected link.

Directed and undirected small-world network

To generate the directed SW networks [23] we use a square and triangular grids. The disorder introduced here is identical to the procedure used on AN network cited above for both cases directed and undirected network.

Directed and undirected Erdös-Rényi random graphs

An ER random graph is formed by a set of N vertices (sites) connected by K links (bonds) [24] . With a probability p a given pair of sites is connected by a bond type $p=\frac{2K}{N(N-1)}$
The connectivity of a site is defined as the total number of bonds connected to it, like $k_j = l_{ij}$, where $l_{ij}=1$ if there is a link between the sites i and j and $l_{ij}=0$ otherwise.

Random graphs

are completely characterized by the mean number of bonds per site, or the average connectivity $z=p(N-1)$. These links can be directed or undirected as well.

Directed and undirected Barabási-Albert Network

In the directed BA network, each new site added to the network selects (a preferential attachment proportional to the number of previous selections), with connectivity z, already existing sites or nodes as neighbors influencing it; the newly added site does not influence these neighbors. In the case of the undirected BA network, the newly added spin does influence these neighbors.

Directed and undirected Voronoi-Delaunay random lattices

In the undirected VD random lattice the construction of the lattice obey the following procedure: for each point in a given set of points in a plane, we determine the polygonal cell that contains the region of space nearest to that point than any other. Two cells are considered neighbors when they possess an extremity in common (Voronoi tessellation). From this Voronoi tessellation, we can obtain the dual lattice, or triangulation of Delaunay, by the following procedure:

1. When we have two neighbor cells, a link is placed between the two points located in the cells;
2. From the links, we obtain the triangulation of space that is called the Delaunay lattice.
3. The Delaunay lattice is dual to the Voronoi tessellation in the sense that its points correspond to cells, links to edges and triangles to the vertices of the Voronoi tessellation. The directed VD random lattices is constructed in the same way as the directed SW network.

Stauffer-Hohnisch-Pittnauer (SHP) networks

Hohnisch bonds of SHP networks [28, 29, 30] are links connecting nodes with different values (spins, opinions, etc.) on them; they are at each time step with a low probability 0.0001 replaced by a link to another randomly se- lected node. Links connecting agreeing nodes are not replaced. In the present work, we start with each node having links to four randomly selected neigh-bors. Thus our SHP networks are similar to Small-World (Watts-Strogatz) networks but start from a random network instead of a square lattice and use opinion-dependent (instead of random) rewiring. All links are directed [28, 29].

RESULTS AND DISCUSSIONS

In order to calculate the rate of tax evaders, we use the equation below,

$$tax\,evasion=\frac{(N-N_{honest})}{N}$$

where N is the total number and N_{honest} the honest number of agents. The tax evasion is calculated at every time step t of system evolution; one time step is one sweep through the entire network. Here, we first will present the baseline case $k=0$ and $p_a=0$, i.e., no use of enforcement, with N = 400 sites for SL, VD, AN, SHP, DBA, UBA, DER and UER network. All simulation are performed over 20, 000 time steps, as shown in Fig. 1. For very low noises the part of autonomous decisions almost completely disappears.

The individuals then base their decision solely on what most of their neighbors do. A rising noise has the opposite effect. Individuals then decide more autonomously.

In Fig. 2, we show the tax evasion versus time for all structures the Fig. 1, but now for case $k=10$ and $p_a=4.5\%$, which is normal for European countries like Germany. Now honesty occurs more often than cheating, but in amounts strongly depending on noise and network type. Fig.2 shows that the largest reduction in tax evasion occurs in the UBA network (Fig. 2(f)) followed by SL Fig. 2(a) via nonequilibrium dynamics (MVM)In Fig. 3, we also show the tax evasion versus time for all structures the Fig. 1, but now for case $k=50$ and $p_a=90\%$, that is considered an extreme case of audit and punishment at the same time. Here, we can observe that even for the case where fluctuation of the tax evasion is high (Fig.1 and 2 (g and h)), this can be reduced to a very small value as it happens in all other structures studied here.

CONCLUSION

To study of the fluctuations of tax evasion in a country (Germany), Zaklan et al. [15] proposed a model using Monte Carlo simulations and an equilibrium dynamics (Ising model) on square lattices. Their results are in good agree-ment with analytical and experimental results obtained by [8, 9, 10, 11, 5, 6].

We showed that tax evasion is diminished drastically if the audit probability p_a and/or the honesty period k are enlarged. In summary, in this work we show that the model proposed by Zaklan *et al*. [15] is very robust for analysis and control of tax evasion, because we use a nonequilibrium dynamics (MVM) to simulate it, that is the opposite of the study done by [15] equilib rium dynamics (Ising model), and also on various topologies. Our results are similar to the results obtained by Zaklan *et al*. [15] regardless of dynamic or topology. Therefore, as this model is a sociophysics and econophysics model, we also believe that besttopology used until now for simulations of this model is UBA networks. We, believe that this is due to the fact that these network have highly connected hubs.

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