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## Research Article

### TRUNCATED NUCLEON STRUCTURE FUNCTION MOMENT USING THERMODYNAMICAL BAG MODEL

Ganesamurthy K\*, Rajagopal V., Ramesh C and Moses Raj A

Research Department of Physics, Urumu Dhanalakshmi College, Trichy 620 019, Tamil Nadu, India

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#### ABSTRACT

We estimate the neutron second truncated structure function moment and ratio of neutron and proton truncated moments using Thermodynamical Bag Model (TBM). We evaluate the valence quark moments in the kinematic region of  $0 < x < 1$  at  $Q^2 = 4 \text{ GeV}^2$ . Our evaluated theoretical results are in good agreement with available experimental data.

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#### INTRODUCTION

The quark's spin contribution for the proton was determined by EMC[1] which is not fulfill determination. After this measurement, the investigation of unpolarized and polarized parton distribution functions have been much interested both theoretical and experimental aspect. Over quarter century of years, our knowledge of the spin structure of the nucleon is still incomplete and the internal structure of the nucleon as measured in the polarized lepton-nucleon deep inelastic scattering experiments[2-5].

The truncated moments of the parton distributions and structure functions which are essential in testing sum rules obtained as integrals of the distribution or structure functions over the Bjorken variable  $x$ . The truncated moments introduced by Forte [6] to study the structure function moments for which small  $x$  data were not available. By restricting or truncating the integration region to some minimum value of the Bjorken variable, one could avoid the problem of extrapolating parton distribution into unmeasured region at small  $x$ . Later Kotlorz [7] developed an alternative formulation of the evolution equations which avoids the problem of minimizing of higher truncated moments when evolving in  $Q^2$ . In this present work, we estimate the truncated neutron structure function moment and ratio of neutron to proton truncated moments for three

invariant mass regions which are compared with BONUS experimental data [8].

##### Thermodynamical Bag Model

Thermodynamical Bag Model (TBM) developed by Ganesamurthy *et.al* [9-13] considering the nucleon to be in the Infinite Momentum Frame (IMF), where the quarks and gluons are treated as fermions and bosons respectively. The invariant mass ( $W$ ) of the final hadron and the equation of states are:

$$W^2 = M^2 + 2M\nu - Q^2 \quad (1)$$

$$6(n_u - n_{\bar{u}}) = \frac{2}{V} = \mu_u T^2 + \frac{\mu_u^3}{\pi^2} \quad (2)$$

$$6(n_d - n_{\bar{d}}) = \frac{2}{V} = \mu_d T^2 + \frac{\mu_d^3}{\pi^2} \quad (3)$$

Where  $V$  is the volume of bag,  $B$  is the bag constant,  $W$  is the invariant mass of excited nucleon at  $T$ ,  $\nu$  is the energy transfer,  $Q^2$  is square of four momentum transfer,  $M$  is the mass of the nucleon at ground state,  $6(n_u - n_{\bar{u}})$  is number density of up quark,  $6(n_d - n_{\bar{d}})$  is the number density of down quark,  $\mu_u$  is the chemical potential of up quark and  $\mu_d$

\*Corresponding author: Ganesamurthy K

Research Department of Physics, Urumu Dhanalakshmi College, Trichy 620 019, Tamil Nadu, India

is the chemical potential of down quark. The invariant mass in TBM is obtained by considering the energy transfer to the nucleon results in heating up the constituents of the nucleon. The temperature and chemical potential are not free parameters. They are evaluated either fixed  $Q^2$  or  $x$ .

The total energy density  $\epsilon(T)$  of the bag can be obtained by the sum of energy densities of up quark, down quark and gluon as

$$\epsilon_u + \epsilon_{\bar{u}} = \left(\frac{1}{8\pi^2}\right)\mu_u^4 + \left(\frac{1}{8}\right)\mu_u^2 T^2 + \left(\frac{7\pi^2}{120}\right)T^4 \quad (4)$$

$$\epsilon_d + \epsilon_{\bar{d}} = \left(\frac{1}{8\pi^2}\right)\mu_d^4 + \left(\frac{1}{8}\right)\mu_d^2 T^2 + \left(\frac{7\pi^2}{120}\right)T^4 \quad (5)$$

$$\epsilon_g = \frac{\pi^2 T^4}{30} \quad (6)$$

$$\epsilon(T) = d_q(\epsilon_u + \epsilon_{\bar{u}}) + d_q(\epsilon_d + \epsilon_{\bar{d}}) + d_g \epsilon_g \quad (7)$$

Where  $d_q = 6$  and  $d_g = 16$  denotes the degeneracy of quarks and gluon orderly.

The statistical Parton Distribution Functions are expressed as[11]

$$q_i(x, Q^2) = \left(\frac{6V}{4\pi^2}\right) M^2 T x \ln \left\{ 1 + \exp \left[ \left(\frac{1}{T}\right) \left(\mu_i - \frac{Mx}{T}\right) \right] \right\} \quad (8)$$

$$\bar{q}_i(x, Q^2) = \left(\frac{6V}{4\pi^2}\right) M^2 T x \ln \left\{ 1 + \exp \left[ \left(\frac{1}{T}\right) \left(-\mu_i - \frac{Mx}{T}\right) \right] \right\} \quad (9)$$

$\mu_i$  is the chemical potential of quark with the flavour 'i'.

Here 'i' denotes u or d quark. In order to relate the PDF's with  $\Lambda_{QCD}$ , which is quark gluon coupling parameter, we introduce the strong quark gluon coupling constant. The experimental fit could be made by considering only with the QCD corrections. The quark and anti-quark distributions are modified by including QCD parameters as,

$$q_i'(x, Q^2) = q_i(x, Q^2) \left( 1 - \frac{\alpha_s(Q^2)}{2\pi} \right) \quad (10)$$

$$\bar{q}_i'(x, Q^2) = \bar{q}_i(x, Q^2) \left( 1 - \frac{\alpha_s(Q^2)}{2\pi} \right) \quad (11)$$

The strong running coupling constant ( $\alpha_s$ ) for various  $Q^2$  is evaluated using the Next to Leading Order (NLO) solution.

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)} \left[ 1 - \frac{\beta_1 \ln(\ln(Q^2/\Lambda^2))}{\beta_0 \ln(Q^2/\Lambda^2)} \right] \quad (12)$$

Where  $\beta_0 = 11 - (2 N_f / 3)$  and  $\beta_1 = 102 - (38 N_f / 3)$ .

### Evaluation of truncated structure function moments

The sum rules relating the parton with Generalized Parton Distributions (GPDs) are given as,

$$F_1(t) = \int_0^1 dx \left[ \frac{2}{3} H^u(x, t) - \frac{1}{3} H^d(x, t) \right] \quad (13)$$

Where  $H^q(x, t) = q(x) x^{Q^2/4k^2}$

Here  $q(x)$  is quark distribution and  $k$  is the magnetic moment of quarks which are determined explicitly in our previous work[15]. In the Quark Parton Model (QPM), the nucleon is composed of three quarks are called valence quarks and the sum of these quantum number of quarks should be equal to proton spin. The structure function  $F_1$  and  $F_2$  are related Callon-Gross equation[16] is given by

$$2x F_1(x) = F_2(x) \quad (14)$$

We obtained the general form of truncated nucleon structure function moment and it can be written as,

$$M_2^n(x_{\min}, x_{\max}, Q^2) = \int_0^1 dx x^{N-2} F_2(x, Q^2) \quad (15)$$

In this work, we calculate second truncated neutron moments with three regions  $1.3 \leq W^2 \leq 1.9 \text{ GeV}^2$ ,  $1.9 \leq W^2 \leq 2.5 \text{ GeV}^2$  and  $2.5 \leq W^2 \leq 3.1 \text{ GeV}^2$ . Therefore with  $N=2$  equation (15) modified as,

$$M_2^n(x_{\min}, x_{\max}, Q^2) = \int_0^1 dx x F_2(x, Q^2) \quad (16)$$

The equation of truncated moments are universal and it is used to determine the unpolarized and polarized parton distribution in the various approximations like LO, NLO, NNLO etc. Since the experimental data cover only a limited range of  $x$  except very small  $x(x \rightarrow 0)$  as well as large  $x(x \rightarrow 1)$ . It is very natural and convenient to deal with the double truncated moments.

## RESULTS AND DISCUSSION

We evaluate the truncated neutron structure function and ratio of  $M_2^n/M_2^p$  based on quark distribution using Thermodynamical Bag Model.

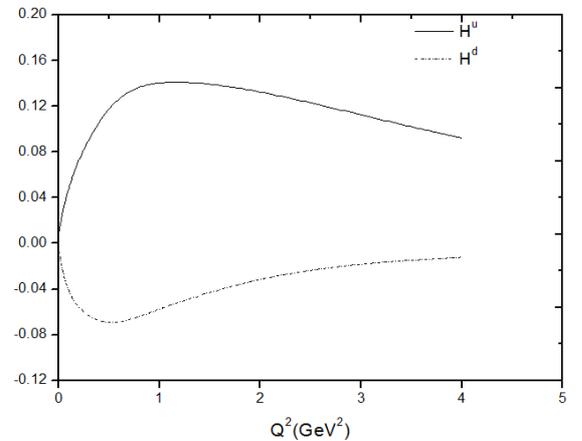


Figure 1 The variation of Generalized Parton Distribution as function of  $Q^2(\text{GeV}^2)$ .

Figure 1 shows that the variation of  $H^u$  and  $H^d$  as a function of  $Q^2$ . As  $Q^2$  increases  $H^u$  increases to the maximum value up to  $Q^2 = 1\text{GeV}^2$  and increasing  $Q^2$ ,  $H^u$  becomes decrease. The evaluated  $H^u$  values are positive due to the up quark has the positive magnetic moment. As  $Q^2$  increases,  $H^d$  decreases up to  $Q^2 = 0.5\text{GeV}^2$ . Further increasing  $Q^2$ ,  $H^d$  increases. The evaluated  $H^d$  is negative distribution which is due to the down quark has negative charge contribution to the nucleon.

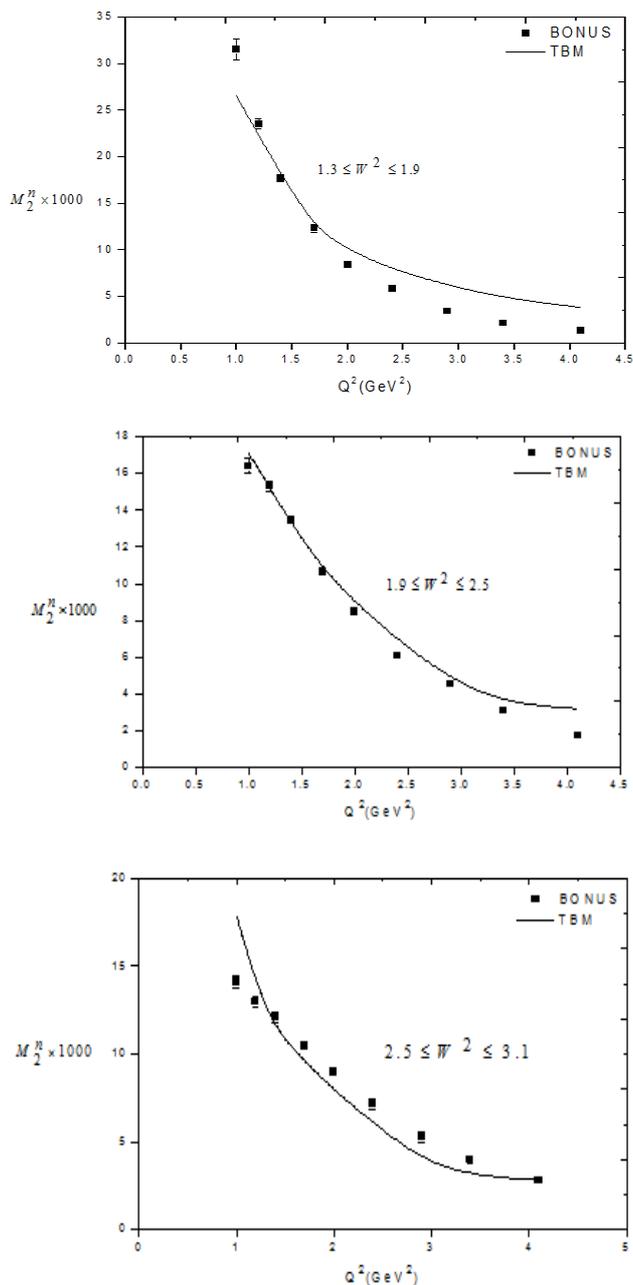


Figure 2 TBM results (solid line) of  $M_2^n$  is compared with BONUS experimental data (shaded squares) for three different  $W^2$  regions.

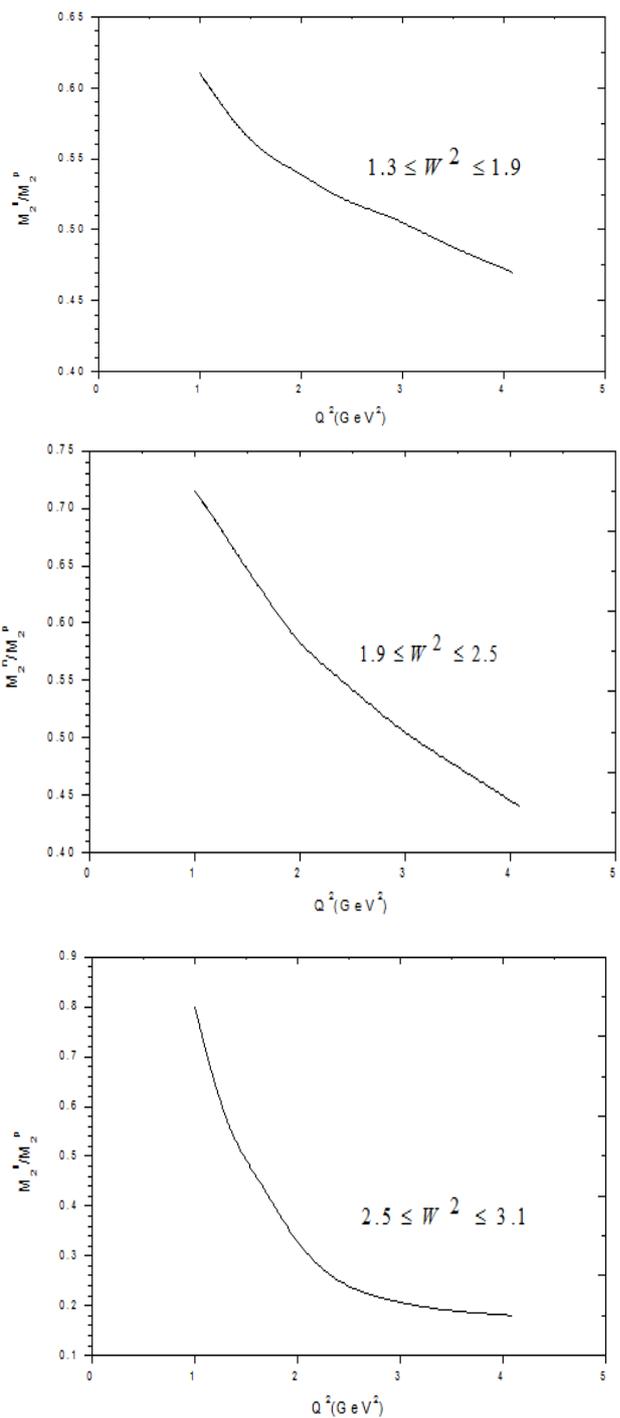


Figure 3 Variation of  $M_2^n / M_2^p$  with  $Q^2$  ( $\text{GeV}^2$ ) for three different  $W^2$  regions.

Figure 2 shows the truncated neutron structure function moment as a function of  $Q^2$  with three  $W^2$  regions. The decreasing the value of Bjorken variable  $x$  produces the increasing the invariant mass of the final hadronic system which leads to the production of sea quarks and gluons. At low  $x$  region, the energy transfer is greater than the momentum transfer which represents the excited state of the target nucleon[17] and at high  $x$  region, only the valence quarks are dominated which is the natural consequence of this model. The evaluated results of  $M_2^n$  have similar behavior up to

$Q^2 = 1.7\text{GeV}^2$  with experimental data particularly in the  $1.9 \leq W^2 \leq 2.5\text{GeV}^2$ . This is due to the value of  $H^d$  is merely equal to the value of  $H^u$ . Above  $Q^2 = 1.7\text{GeV}^2$ , there is deviation between theoretical and experimental results because of  $H^u$  is more than the  $H^d$ . Figure 3 shows that the variation of  $M_2^n / M_2^p$  for the same three  $w_2$  regions. In the regions of  $1.3 \leq W^2 \leq 1.9$  and  $2.5 \leq W^2 \leq 3.1\text{GeV}^2$ , similar behavior is observed with experimental data. This is due to  $M_2^n$  values are equal to  $M_2^p$  values whereas in the region  $1.9 \leq W^2 \leq 2.5\text{GeV}^2$ ,  $M_2^p$  is more than  $M_2^n$ . The valence quark moment is also obtained for the Bjorken value  $0 < x < 1$  at  $Q^2 = 4\text{GeV}^2$  and theoretical prediction is good agreement with the experimental results of N3LO and ABKM.

**Table 1** Comparison of valence quark distribution at  $Q^2 = 4\text{GeV}^2$ .

	ABKM[18]	N3LO[19]	Present model
$xV$	$0.0072 \pm 0.0007$	$0.1610 \pm 0.0043$	0.0074

Where

$$xV = x \{ [u(x, Q^2) + \bar{u}(x, Q^2)] - [d(x, Q^2) + \bar{d}(x, Q^2)] \}$$

**Table 2** Comparison of flavor asymmetry at  $Q^2 = 4\text{GeV}^2$

	ABKM[18]	Present model
$x(\bar{d} - \bar{u})$	$0.0072 \pm 0.0007$	0.0074

**Table 3** Comparison of second moment of valence quark distribution at  $Q^2 = 4\text{GeV}^2$

	ABKM[18]	BBG[19]	JR[20]	AMP06[21]	BBG[19] (N3LO)	Present model
$xu_v$	$0.2981 \pm 0.0025$	$0.2986 \pm 0.0029$	$0.29 \pm 0.003$	$0.2947 \pm 0.003$	$0.3006 \pm 0.0031$	0.2801
$xd_v$	$0.1191 \pm 0.0023$	$0.1239 \pm 0.0026$	$0.125 \pm 0.0056$	$0.1129 \pm 0.0031$	$0.1129 \pm 0.0031$	0.1370
$x(u_v - d_v)$	$0.1790 \pm 0.0023$	$0.1747 \pm 0.0030$	$0.1640 \pm 0.0060$	$0.1820 \pm 0.0056$	$0.1754 \pm 0.0041$	0.1437

## CONCLUSION

The present model prediction of truncated neutron structure function moment and ratio  $M_2^n / M_2^p$  are similar with the

region  $1.3 \leq W^2 \leq 1.9$  and  $1.9 \leq W^2 \leq 2.5\text{GeV}^2$  and  $2.5 \leq W^2 \leq 3.1\text{GeV}^2$  the valence quark moment is also obtained for the Bjorken value  $0 < x < 1$  at  $Q^2 = 4\text{GeV}^2$  and antidown quark dominates over the antiup quark in the evaluated  $x$  region. The momentum carried by the up valence quark is greater than down valence quark. But in the case of antiquarks, the momentum carried by the antidown quarks is greater than antiup quarks. Theoretical results are good agreement with available experimental results.

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