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Research Article

INTERPOLATION PROCESS BY g-SPLINES

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ABSTRACT

Th. Fauzi constructed special kinds of lacunary quintic g-splines and proved that for functions f∈C^((4)) the methods converges faster than that investigated by A.K. Verma and for functions ∈C^((5)) the order of approximation is the same as the best order of approximation using quintic g-splines. In this paper, we construct quintic lacunary g-splines which are solutions of (0,1,4)-Interpolation problem and obtain their local approximations with functions belonging to C^((4)) (I) and C^((5)) (I). Our methods are of lower degree having better convergence property than the earlier investigations.

INTRODUCTION

Let

$$\Delta:\ 0 = x_0 < x_1 < \dots < x_{n-1} < x_n = 1 \tag{1}$$

Be a partition of the interval I = [0,1] with $X_{k+1} - x_k = h_k$, $k = o(1)n - 1$. Th. Fauzi [3] constructed special kinds of lacunary quintile g-splines and proved that for functions $f \in C^{(4)}$ the methods converge faster than that investigated by A.K. Varma [1] and for functions $f \in C^{(5)}$ the order of approximation is the same as the best order of approximation using quintic g-splines. J. Gyorvari [4] considered local methods of degree six of class C[I] , which settles the problem of (0, 2, 3) and (0, 2, 4)-interpolations offering better approximation than the interpolants investigated by R. S. Misra and first author [2] . By varying continuity class and nature of the spline functions R.B. Saxena and H.C. Tripathi [5,6] obtained for functions $f \in C^{(6)}$ in the case of uniform partition the estimates of $|\tilde{s}^{(q)}$ - $\tilde{f}^{(q)}|$ and $|\tilde{s}^{(q)}$ - $\tilde{f}^{(q)}|$ Where $\tilde{s}_\Delta$ and $\hat{s}_\Delta$ each of degree six interpolate the data (0, 1 , 3) and (0, 2, 4), q = 0 (1) 5 choosing suitable initial and boundary conditions respectively.

In this paper, we construct quintic lacunary g-splines, which are solutions of (0, 1, 4) – Interpolation problems and obtain their local approximations with functions belonging to $C^{(4)}(I)$ and $C^{(5)}(I)$. Our methods are of lower degree having better convergence property than the earlier investigations made in [[1], [2], [4], [5], [6], [7], [8],[9]]. More over, our results have no counterpart in polynomial approximation theory. § 2. Is devoted to the study of quintic spline interpolant (0, 1, 4) for $C^{(4)}(I)$.

Spline Interpolant (0, 1, 4) for $f \in C^{(5)}(I)$

Let $s_{1,\Delta}$ be a piecewise polynomial of degree ∈ 5. The spline interpolant (0, 1, 4) for functions $\in C^{(5)}(I)$ is given by :

$$s_{1,\Delta}(x) = s_{1,k}(x) = \Sigma_{j=0}^{5} \frac{s_{k,j}^{(1)}}{j!}(x - x_k)^j, x_k \le x \le x_{k+1}, k = 0(1)n - 1, \tag{2}$$

Where $s_{k,j}^{(1)}$, s are explicitly given below in terms of the prescribed data $\{f_k^{(j)}\}$, j = 0, 1, 4; K = 0(1)n, viz for k = 0(1)n-1 ,

$$s_{k,j}^{(1)} = f_k^{(j)} \ , \ j = 0, 1, 4 . \tag{3}$$

For j = 2, 3, 5, we have

$$s_{k,5}^{(1)} = \frac{1}{h}\ [f_{k+1}^{(4)} - f_k^{(4)}] , \tag{4}$$

$$s_{k,3}^{(1)} = -\ \frac{12}{h^3}\ [(f_{k+1} - f_k - hf_k^{(1)} - \frac{h^4}{4!} f_k^{(4)}) - \frac{h}{2}\left(f_{k+1}^{(1)} - f_k^{(1)} - \frac{h^3}{3!} f_k^{(4)}\right) + \frac{h^s}{80}\, s_{k,5}^{(1)}\ ] \tag{5}$$

and

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$s_{k,2}^{(1)} = \frac{2}{h^2}\,[f_{k+1} - f_k - hf_k^{(1)} - \frac{h^4}{4!}\ f_k^{(4)} - \frac{h^3}{3!}\ s_{k,3}^{(1)} - \frac{h^5}{5!}\ s_{k,5}^{(1)}\,]$ (6)

The coefficients $s_{k,j}^{(1)}$, j = 2, 3, 5 have been so chosen
That

$D_L^{(p)}\, s_{1,k}\,(x_{k+1}) = D_R^{(p)}\, S_{1,k+1}\,(x_{k+1})$, p = 0, 1, 4; k = 0(1) n – 1

Thus
$s_{1,\Delta}\,\epsilon C^{(0.1,4)}$ [I] = $\{$ f : $f^{(p)} \in$ C(I), p = 0, 1, 4 $\}$.

Is a unique quintic piecewise polynomial satisfying interpolator conditions (2).

If f ∈ $C^{(5)}$ [I], then owing to (3) – (5) and using Taylor ' s expansion, we have

$\left|s_{k,j}^{(1)} - f_k^{(j)}\right| \le C_{k,j}^{(1)}\, h^{5-j}\ \omega\big(f^{(5)}\,;h\big)$, j = 2, 3, 5; k = 0(1)n – 1 (7)

Where the constant $C_{k,j}^{(1)}$ are given by:

$C_{k,2}^{(1)} = \frac{1}{10}$, $C_{k,3}^{(1)} = \frac{1}{4}$ and $C_{k,5}^{(1)} = 1$.

Using (1) - (7) and a little computation gives:

Theorem

Let f ∈ $C^{(5)}$ [I] and $s_{1,\Delta}\,\epsilon C^{(0.1,4)}$ (I) be the unique spline interpolant (0, 1, 4) given in (8) - (12),

then

$|\,|\,D^{(j)}$ (f- $S_{1,\Delta}$) $|\,|\,L_\infty\,[X_k, x_{k_{+1}}\,] \le c_{1,k}^{j}\ h^{5-j}\ \omega\ (f^{(5)}$, h)
j =0(1) 5; k=0 (1) n-1 (8)

Where the constants $c_{1,k}^{j}$, s are given by:

$c_{1,k}^{0} = \frac{1}{10}$, $c_{1,k}^{1} = \frac{4}{15}$, $c_{1,k}^{2} = \frac{31}{60}$, $c_{1,k}^{3} = \frac{3}{4}$, $c_{1,k}^{4} = c_{1,k}^{5} = 1$

Almost Quartic Spline Interpolant (0, 1, 4) * for f ∈ $C^{(4)}$ (I).

Almost quartic spline interpolant (0, 1, 4)* is a piecewise polynomial of degree 4 in each subinterval except in the last one, where it is a polynomial of degree 5. In this case, we have

$S_{1,\Delta}^{*}(x) = S_{1,k}^{*}(x) = \sum_{j=0}^{4} \frac{S_{k,j}^{*(1)}}{j!}(x - x_k)^j$, $x_k \le x \le x_{k+1}, k = 0(1)n - 2$ (9)

$=\sum_{j=0}^{5} \frac{S_{n-1,j}^{*(1)}}{j!}(x - x_{n-1})^j$, $x_{n-1} \le x \le x_n$, $k = n - 1$

The coefficients $S_{k,j}^{*(1)}$ are explicitly given in terms of the data. In particular, for K=O(1) n-1, we prescribe

$S_{k,j}^{*(1)} = f_k^{(j)}$, j = 0, 1, 4 . (10)

For k = 0(1)n-2 and j = 2, 3, $S_{k,j}^{*(1)}$ are given by

$S_{k,2}^{*(1)} = \frac{6}{h^2}$ [$(f_{k+1}$ - f_k - hf_k' - $\frac{h^4}{4!}\,f_k^{(4)}$) - $\frac{h}{3}\,(f_{k+1}'$ - f_k' - $\frac{h^3}{3!}\,f_k^{(4)}$)] (11)
and

$S_{k,3}^{*(1)} = -\frac{12}{h^3}$ [$(f_{k+1}$ - f_k - hf_k' - $\frac{h^4}{4!}\,f_k^{(4)}$) - $\frac{h}{2}\,(f_{k+1}'$ - f_k' - $\frac{h^3}{3!}\,f_k^{(4)}$)] (12)

For k=n-1 and j=2, 3 and 5, we have

$S_{n-1,5}^{*(1)} = \frac{1}{h}\,(f_n^{(4)}$ - $f_{n-1}^{(4)}$) (13)

$S_{n-1,3}^{*(1)}$ = - $\frac{12}{h^3}$ [$(f_n$ - f_{n-1} - hf_{n-1}' - $\frac{h^4}{4!}\,f_{n-1}^{(4)}$) - $\frac{h}{2}\,(f_n'$ - f_{n-1}' - $\frac{h^3}{3!}$ $f_{n-1}^{(4)}$) + $\frac{h^5}{80}\ S_{n-1,5}^{*(1)}$] (14)

and

$S_{n-1,2}^{*(1)}$ = - $\frac{2}{h^2}$ [$(f_n$ - f_{n-1} - hf_{n-1}' - $\frac{h^3}{3!}\ S_{n-1,3}^{*(1)}$ - $\frac{h^4}{4!}\ f_{n-1}^{(4)}$ - $\frac{h^5}{5!}$ $S_{n-1}^{*(1)}$] (15)

(11) and (12) are obtained from the condition.

$S_{1,\Delta}^{*} \in C^{(1)}$ [I], (16)

While (13)-(15) are determined from conditions (8) for k = n-1 in (8).

Analogous to (7) for ∈ $C^{(4)}$ [I] , one can establish

| $S_{k,j}^{*(1)}$ - $f_k^{(j)}$ | $\le C_{k,j}^{*(1)}\ h^{4-j}\ \omega\ (f^{(4)}$, h), (17)

Where the constants $C_{k,j}^{*(1)}$ are given by

$C_{k,j}^{*(1)} = \begin{cases}\frac{1}{3}, & j = 2\\ 1, & j = 3\end{cases}$ k=0(1)n-2

$C_{k,j}^{*(1)} = \begin{cases}\frac{2}{3}, & j = 2\\ \frac{17}{10}, & j = 3\end{cases}$ k=n-1

Finally, similar to theorem 2.1 , we have

Theorem

Let f ∈ $C^{(4)}$ [I] and $S_{1,\Delta}^{*}$ be the unique almost quartic spline interpolant (0, 1, 4)* , given by (8) , then
(18)

$\left\|D^{(j)}(f - S_{1,\Delta}^{*})\right\|\ L_\infty\,[x_k\,, x_{k+1}] \le C_{1,k}^{*(j)}\ h^{4-j}\omega\big(f^{(4)}, h\big)$,

Where the constants $C_{1,k}^{*(j)}$ are given by:

	$C_{1,k}^{*(0)}$	$C_{1,k}^{*(1)}$	$C_{1,k}^{*(2)}$	$C_{1,k}^{*(3)}$	$C_{1,k}^{*(4)}$
k=0(1)n-2	$\frac{3}{8}$	1	$\frac{11}{6}$	2	1
K=n-1	$\frac{77}{120}$	$\frac{197}{120}$	$\frac{14}{5}$	$\frac{53}{20}$	1

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