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Case Report

HERON MEAN LABELING OF GRAPHS

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ABSTRACT

A (p, q) graph G is said to be a Heron mean graph if there exist a bijection $f: V \rightarrow \{1, 2, 3, \dots, p\}$ such that induced function $f^*: E(G) \rightarrow N$ given by $f^*(uv) = \left\lfloor \frac{u+v+\sqrt{uv}}{3} \right\rfloor$ or $f^*(uv) = \left\lceil \frac{u+v+\sqrt{uv}}{3} \right\rceil$ for every $uv \in E(G)$ are all distinct. In this paper the Heron mean labeling of path, cycle C_n , $K_{1,n}$ if and only if $n < 3$, $C_m \cup P_n$, $C_m \cup C_n$, nK_3 , $nK_3 \cup P_m$, $nK_3 \cup C_m$, mC_4 , crown $C_n \odot K_1$, Dragons $C_n @ P_m$, Square graph of path P_n^2 polygonal chain $G_{m,n}$ are discussed.

Key Words:

Graph, Heron mean graph, Union of graphs, square of a graph, crown.

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INTRODUCTION

Abundant literature exists as of today, concerning the structure of graphs admitting a variety of functions assigning real numbers to their elements so that given conditions are satisfied. Here we are interested the study of vertex functions $f: V(G) \rightarrow A$, $A \subseteq N$ for which the induced edge function $f^*: E(G) \rightarrow N$ is defined as $f^*(uv) = \left\lfloor \frac{u+v+\sqrt{uv}}{3} \right\rfloor$ or $f^*(uv) = \left\lceil \frac{u+v+\sqrt{uv}}{3} \right\rceil$ for every $uv \in E(G)$ are all distinct. As we know that the notion of mean labeling was introduced in a paper by Somasundaram and Ponraj [1].

A graph G with p vertices and q edges is called a mean graph if there is an injective function f from the vertices of G to $\{0, 1, 2, 3, 4, \dots, q\}$ such that when each edge uv is labeled with $(f(u) + f(v))/2$, if $f(u) + f(v)$ is even, and $(f(u) + f(v) + 1)/2$ if $f(u) + f(v)$ is odd, then the resulting edge labels are distinct. We introduce Heron mean labeling of some standard graphs.

We begin with simple, finite and undirected graph $G = (V, E)$ with p vertices and q edges. For all other terminology and notations we follow Harary [2]. The Cardinality of its edge set is called the size of G . The graph $G - e$ is obtained from G by deleting an edge e . The Sum $G_1 + G_2$ of two graphs G_1 and G_2 has vertex set $V(G_1) \cup V(G_2)$ and the edge set $E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{u \in V(G_1) \text{ and } v \in V(G_2)\}$. The Union of two graphs G_1 and G_2 is a graph $G_1 \cup G_2$ with vertex set $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$. The square G^2 of the graph G has $V(G^2)$ with u, v adjacent in G^2 Whenever $d(u, v) \leq 2$ in the graph G . For detailed survey on graph labeling we refer to Gallian [3]. According to Beineke and Hegde[1],[4],[8] graph labeling serves as a frontier between number theory and structure of graphs. The definitions which are useful for the present investigation are given below.

Definition: If the vertices are assigned values subject to certain conditions then it is known as graph labeling.

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Definition: A Graph G with p vertices and q edges is called a Heron mean graph if it possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1, 2, 3, 4, \dots, p$ in such a way that when each edge $e = \{u, v\}$ is labeled with $f^*(uv) = \left\lfloor \frac{u+v+\sqrt{uv}}{3} \right\rfloor$ or $f^*(uv) = \left\lceil \frac{u+v+\sqrt{uv}}{3} \right\rceil$ are distinct. In this case f is called Heron Mean labeling of G .

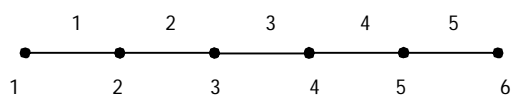
MAIN RESULTS

Theorem: Any Path P_n is Heron mean graph.

Proof: Let $v_1, v_2, v_3, \dots, v_n$ be the vertices of the path P_n of length $n - 1$.

Let us define $f: V \rightarrow \{1, 2, 3, \dots, p\}$ by $f(v_i) = i, 1 \leq i \leq n$, such that the induced function $f^*: E(G) \rightarrow \{1, 2, 3, \dots, q\}$ given by $f^*(uv) = \left\lfloor \frac{u+v+\sqrt{uv}}{3} \right\rfloor$ for every $u, v \in V(G)$ are all distinct. Hence path P_n is Heron mean graph.

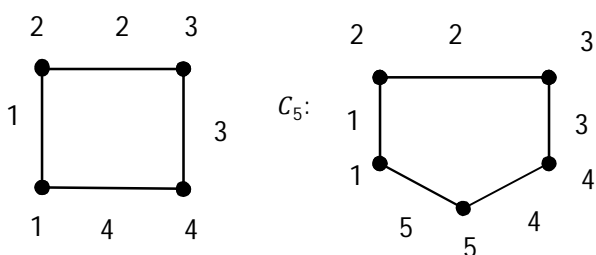
Illustration: Consider the path of length 5. The labeling is as shown in figure.



Theorem: Any cycle $C_n, n \geq 3$, is a Heron mean graph.

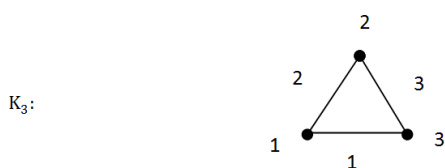
Proof: Let C_n be the cycle with vertices $v_1, v_2, v_3, \dots, v_n$. Define a function $f: V(C_n) \rightarrow \{1, 2, 3, \dots, p\}$ by $f(v_i) = i, 1 \leq i \leq n$, here f is an increasing function on $V(C_n)$, so f^* is also an increasing function on $E(C_n) - \{v_n v_1\}$. for every edge in $E(C_n) - \{v_n v_1\}$ we assign the label $f^*(v_i v_j) = \left\lfloor \frac{v_i+v_j+\sqrt{v_i v_j}}{3} \right\rfloor$ where $v_i v_j \in V(C_n)$ and $f^*(v_n v_1) = 1$. Hence $f^*(e_i) \neq f^*(e_j), i \neq j$ there fore f^* is injective and f is a Heron mean labeling on C_n . So C_n is a Heron mean graph.

Illustration: Consider the cycle of length 4 & 5. The labeling is as shown in figure.



Theorem: The Complete graph K_n is a Heron mean graph for $n \leq 3$.

Proof: The Heron mean labeling of the complete graph K_n for $n \leq 3$ is given in figure below.



Let $K_n, n \leq 3$ and $f: V(G) \rightarrow \{1, 2, 3, \dots, n\}$ be a vertex function which induces a function

$$f^* \text{ given by } f^*(v_i v_j) = \left\lfloor \frac{f(v_i)+f(v_j)+\sqrt{f(v_i)f(v_j)}}{3} \right\rfloor \text{ for every}$$

$e = v_i v_j \in E(G) - \{v_3 v_1\}$ and $f^*(v_3 v_1) = 1$. Assume $n \geq 4$ and since the graph is a complete graph we get two edges e_1 and e_2 such that $f^*(e_1 = 34) = \left\lfloor \frac{3+4+\sqrt{12}}{3} \right\rfloor = \lfloor 3.48803 \rfloor = 3$ and $f^*(e_2 = 24) = \left\lfloor \frac{2+4+\sqrt{8}}{3} \right\rfloor = \lfloor 3.33 \rfloor = 3$ are same, so we conclude that when $n \geq 4, f^*$ is not injective. Hence $K_n, n \geq 4$ is not a Heron mean graph.

Theorem: Star graph $K_{1,n}$ is Heron mean graph for $n \leq 4$.

Proof: If we label any one pendent vertex with 1 and the common vertex by 2 remaining vertices of unit degree in a one to one manner, we get a Heron mean labeling. If $n = 5$, for two different edges joining the vertices (2, 5) and (2,6) the Heron mean is $\lfloor 3.38 \rfloor$ and $\lfloor 3.82 \rfloor$ respectively which are same. Hence $K_{1,n}$ for $n \geq 5$ is not a Heron mean graph.

Theorem: The graph mK_2 consists of m pair wise disjoint edges is a Heron mean graph.

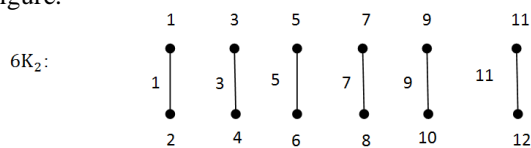
Proof: Let $V(mK_2) = \{a_1, b_1, a_2, b_2, \dots, a_m, b_m\}, E(mK_2) = \{a_i b_i; 1 \leq i \leq m\}$.

Define $f: V(mK_2) \rightarrow \{1, 2, 3, \dots, p\}$ as follows $f(a_i) = 2i - 1, 1 \leq i \leq m$ and

$$f(b_i) = 2i, 1 \leq i \leq m$$

So that $f^*(a_i b_i) = \left\lfloor \frac{(2i-1)+(2i)+\sqrt{(2i-1)(2i)}}{3} \right\rfloor, 1 \leq i \leq m$, moreover, f is an increasing function on $V(G)$ and so $f^*(a_i b_i) \neq f^*(a_j b_j), i \neq j, 1 \leq i, j \leq m$. Since these numbers forms an increasing sequence of natural numbers. Hence f^* is injective and mK_2 is a Heron mean graph.

Illustration: consider a graph $6K_2$ The labeling is as shown in figure.



Theorem: mK_3 consists of m disjoint triangles is a Contra Harmonic mean graph.

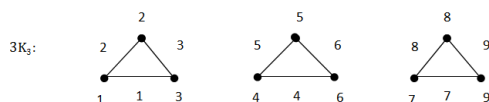
Proof: Let the vertex set of mK_3 be $V = V_1 \cup V_2 \cup \dots \cup V_m$, where $V_i = \{V_i^1, V_i^2, V_i^3\}$.

we define $f: V(mK_3) \rightarrow \{1, 2, 3, \dots, p\}$ by $f(V_i^j) = 3(i - 1) + j, 1 \leq i \leq m, 1 \leq j \leq 3$

are injective function. So that the graph with vertices labels 1, 2, 3 of first copy of K_3 the edge joining the vertices 1 and 2 assign 2, For the edge joining the vertices 2 and 3 assign 3. and for the edge joining the vertices 1 and 3 assign 1. Similarly second copy of K_3 the vertices are labeled by 4, 5, 6, for edge joining the vertices 4 and 5 assign 5, For the edge joining the vertices 5 and 6 assign 6 and for the edge joining the vertices 4 and 6 assign 4. Similarly in the i^{th} copy of K_3 the vertices are labeled with $3i - 2, 3i - 1$ and $3i$. For the edge joining the

vertices $3i - 2, 3i - 1$ and $3i - 1, 3i$ assign the label $f^*(3i - 2, 3i - 1) = \left\lfloor \frac{(3i-2)+(3i-1)+\sqrt{(3i-2)+(3i-1)}}{3} \right\rfloor$ and $f^*(3i - 1, 3i) = \left\lfloor \frac{(3i-1)+3i+\sqrt{(3i-1)(3i)}}{3} \right\rfloor$ respectively and since $3i - 2 < \frac{(3i-2)+3i+\sqrt{(3i-2)(3i)}}{3} < 3i$ we may assign the edge label $3i - 1$ for the edge joining the vertices $3i - 2$ and $3i$. Hence $f^*(e_i) \neq f^*(e_j)$, $i \neq j$ this implies f^* is injective and f is a Heron mean labeling on mK_3 . There fore mK_3 is a Heron mean graph.

Illustration: consider a graph $3K_3$ The labeling is as shown in figure.

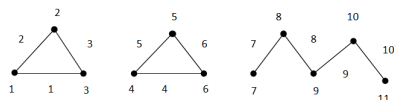


Theorem: $nK_3 \cup P_m$ consists of n disjoint triangles and a path with m vertices is a

Heron mean graph.

Proof: Let the vertices of nK_3 be $V = V_1 \cup V_2 \cup \dots \cup V_n$, where $V_i = \{v_1^i, v_2^i, v_3^i\}$ and P_m be the path with vertices $u_1, u_2, u_3, \dots, u_m$. Define a function $f: V(nK_3 \cup P_m) \rightarrow \{1, 2, 3, \dots, p\}$ by $f(v_j^i) = 3(i - 1) + j$ $1 \leq i \leq n$, $1 \leq j \leq 3$, and $f(u_i) = 3n + i$, $1 \leq i \leq m$. So that For K_3 as explained in theorem 2.6 label the vertices. The edges of a path are labeled by $f^*(uv) = \left\lfloor \frac{u+v+\sqrt{uv}}{3} \right\rfloor$ Hence $f^*(e_i) \neq f^*(e_j)$, $i \neq j$ there fore f^* is injective and f is a Heron mean labeling on $nK_3 \cup P_m$. There fore $nK_3 \cup P_m$ is a Heron mean graph.

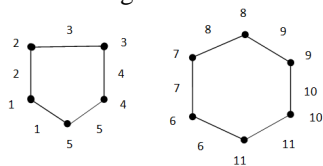
Illustration: Consider a graph $2K_3 \cup P_5$ consists of n disjoint triangles and a path of length 4. The labeling is as shown in figure.



Theorem: The graph $G = C_m \cup C_n$ is a Heron mean graph for $m \geq 3$, $n \geq 3$.

Proof: Let C_m be the cycle with vertices $v_1, v_2, v_3, \dots, v_m$ and C_n be the cycle with vertices $u_1, u_2, u_3, \dots, u_n$. Define a function $f: V(C_m \cup C_n) \rightarrow \{1, 2, 3, \dots, p\}$ by $f(v_i) = i$, $1 \leq i \leq m$ and $f(u_i) = m + i$, $1 \leq i \leq n$. Then the set of labels of the edges C_m is $\{1, 2, 3, \dots, m\}$. The set of labels of edges of C_n is $\{m + 1, m + 2, m + 3, \dots, m + n\}$ as explained in the theorem 2.2 and are all distinct. Therefore $C_m \cup C_n$ is a Heron mean graph.

Illustration: Consider a graph $C_5 \cup C_6$ consists of cycle with 5 vertices and another cycle with 6 vertices. The labeling is as shown in figure.

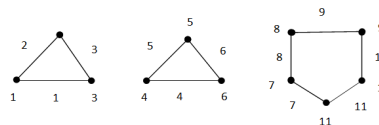


Theorem: The $G = nK_3 \cup C_m$ for $n \geq 1$, $m \geq 3$ is a Heron mean graph.

Proof: Let the vertices of nK_3 be $V = V_1 \cup V_2 \cup \dots \cup V_n$, where $V_i = \{v_1^i, v_2^i, v_3^i\}$ and C_m be the cycle with vertices

$u_1, u_2, u_3, \dots, u_n$. we define $f: V(nK_3 \cup C_m) \rightarrow \{1, 2, 3, \dots, p\}$ by $f(v_j^i) = 3(i - 1) + j$ $1 \leq i \leq m$, $1 \leq j \leq 3$ the labeling of the edges are done as explained in theorem 2.6 and the vertices of C_m are labeled by $f(u_i) = 3n + i$, $1 \leq i \leq m$ and the edges are labeled as explained in theorem 2.2 are all distinct. Hence $nK_3 \cup C_m$ is Heron mean graph.

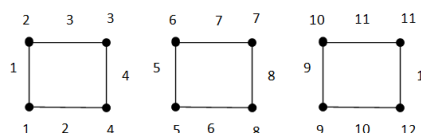
Illustration: Consider a graph $2K_3 \cup C_5$ consists of two copies of K_3 and a cycle with 5 vertices. The labeling is as shown in figure.



Theorem: The graph mC_4 is a Heron mean graph.

Proof: Let the vertex set of mC_4 be $V = V_1 \cup V_2 \cup \dots \cup V_m$, where $V_i = \{v_1^i, v_2^i, v_3^i, v_4^i\}$ we define a function $f: V(mC_4) \rightarrow \{1, 2, 3, \dots, p\}$ by $f(v_j^i) = 4(i - 1) + j$ $1 \leq i \leq m$, $1 \leq j \leq 4$. Consider a graph with vertices $4i - 3, 4i - 2, 4i - 1$ and $4i$ for the edges joining the vertices $4i - 3$ and $4i - 2$, $4i - 2$ and $4i - 1$ and $4i - 1$ and $4i$ we may assign the labels $\left\lfloor \frac{(4i-3)+(4i-2)+\sqrt{(4i-3)(4i-2)}}{3} \right\rfloor$, $\left\lfloor \frac{(4i-2)+(4i-1)+\sqrt{(4i-2)(4i-1)}}{3} \right\rfloor$ and $\left\lfloor \frac{(4i-1)+(4i)+\sqrt{(4i-1)(4i)}}{3} \right\rfloor$ respectively. Since $4i - 3 < \frac{(4i-3)+(4i)+\sqrt{(4i-3)(4i)}}{3} < 4i - 1$ we assign the edge label $4i - 2$ for the edge joining the vertices $4i - 3$ and $4i$. So mC_4 has distinct edge labels. Hence mC_4 is a Heron mean graph.

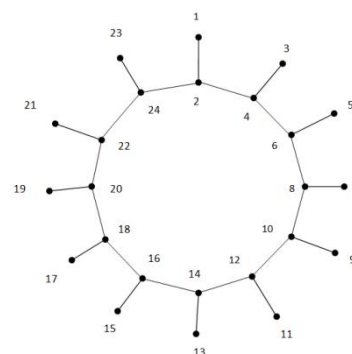
Illustration: Consider a graph $3C_4$ consists of 3 copies of C_4 . The labeling is as shown in figure.



Theorem: The crown (cycle with pendent edge attached at each vertex) $C_n @ K_1$ $n \geq 3$ is a Heron mean graph.

Proof: let $u_1, u_2, u_3, \dots, u_n, u_1$ be the cycle and V_i be the pendent vertices adjacent to each of u_i , $1 \leq i \leq n$. Define $f: V(C_n @ K_1) \rightarrow \{1, 2, 3, \dots, p\}$ by $f(u_i) = 2i$ and $f(v_i) = 2i - 1$, $1 \leq i \leq n$ we may assign the edges of the cycle as explained in the theorem 2.2 and the edges joining the vertices of cycle with the pendent vertices we use $f^*(uv) = \left\lfloor \frac{u+v+\sqrt{uv}}{3} \right\rfloor$ such that $f^*(e_i) \neq f^*(e_j)$, $i \neq j$. Hence $C_n @ K_1$ is a Heron mean graph.

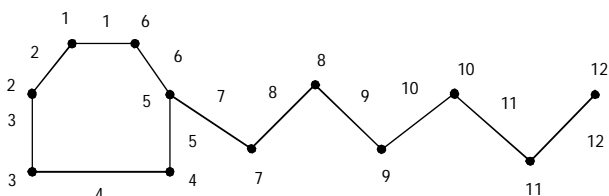
Illustration: Consider a graph $C_{12} @ K_1$ consists of a cycle C_{12} with 12 vertices and a pendent vertices. The labeling is as shown in figure.



Theorem: Dragons $C_n @ P_m$ is a Heron mean graphs.

Proof: Let $u_1, u_2, u_3, \dots, u_n$ be the cycle C_n and $v_1, v_2, v_3, \dots, v_m$ be the path P_m . Identify u_{n-1} with v_1 . Define a function $f: V(C_n @ P_m) \rightarrow \{1, 2, 3, \dots, p\}$ by $f(u_i) = i, 1 \leq i \leq n$ and $f(v_{i+1}) = n + i, 1 \leq i \leq m - 1$. The labeling of the edges are done as explained in theorem 2.2 and the labeling of edges of P_m are done as explained in the theorem 2.1 are all distinct. Hence $C_n @ P_m$ is Heron mean graph.

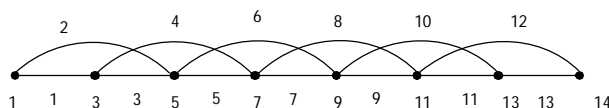
Illustration: Consider a graph $C_6 @ P_7$ consists of C_6 a cycle with 6 vertices and a path of length 6. The labeling is as shown in figure.



Theorem: The square graph P_n^2 is a Heron mean graph.

Proof: Let P_n be the path with n vertices $u_1, u_2, u_3, \dots, u_n$ clearly P_n^2 has n vertices and $2n - 3$ edges. Define $f: V(P_n^2) \rightarrow \{1, 2, 3, \dots, p\}$ by $f(u_i) = 2i - 1, 1 \leq i \leq n - 1$ and $f(u_n) = 2n - 2$. The edges are labeled by $f^*(uv) = \lfloor \frac{u+v+\sqrt{uv}}{3} \rfloor$ for all $uv = e \in E(P_n^2)$ such that $f^*(e_i) \neq f^*(e_j), i \neq j$ there fore f^* is injective. Hence P_n^2 is a Heron mean graph.

Illustration: Consider a path P_8 with 8 vertices and P_8^2 is a graph obtained by joining the vertices whenever $d(u, v) \leq 2$. The labeling is as shown in the figure.



CONCLUSION

Through this paper, new functions are defined using Heron mean. Further the results on few disconnected graphs are proved that they admit Heron mean labeling. All the results are demonstrated with suitable examples. Similar investigations are to be made to other suitable graphs.

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