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CODEN: IJRSFP (USA)

International Journal of Recent Scientific Research
Vol. 8, Issue, 8, pp. 19499-19503, August, 2017

ISSN: 0976-3031

International Journal of Recent Scientific Research

DOI: 10.24327/IJRSR

Research Article

LMI APPROACH FOR STABILITY ANALYSIS FOR NEUTRAL DELAY-DIFFERENTIAL SYSTEMS

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DOI: http://dx.doi.org/10.24327/ijrsr.2017.0808.0706

ARTICLE INFO

Article History:

Received 15th May, 2017
Received in revised form 25th June, 2017
Accepted 23rd July, 2017
Published online 28th August, 2017

Key Words:

Stability, time delay systems, Linear Matrix Inequality, Lyapunov method.

ABSTRACT

In this Literature the problem of stability analysis for neutral delay-differential systems is investigated. Using lyapunov method, we present new sufficient conditions for the stability of the systems in terms of Linear matrix inequality (LMI) which can be easily solved by using the YALMIP MATLAB toolbox. Numerical examples are given to illustrate the improvement of our proposed method.

INTRODUCTION

Problem of Stability with delay-differential systems with an emphasis on Lyapunov methods by framing LMIs ,These LMI techniques have seen more investigation and have emerged as powerful design tools in the areas ranging from control engineering to system identification, structural design and also other engineering systems.

LMI techniques very useful because A variety of design specification and constraints can be expressed as LMIs, Once formulated in the LMIs, a problem can be solved exactly by Using MATLAB LMI tool box, YALMIP tool box and also by convex optimization problems.

While most of the engineering or control system problems with multiple constraints or objectives and it is difficult to getting analytical solutions in terms of matrix equations, they can solve efficiently in the LMI frame work. This makes LMI-based design problems a valuable alternative to remaining classical analytical method.

In this journey here we continued the analysis of stability of neutral delay differential systems by referring some of the literature as follows Ref. [11] P. Balasubramaniam and *et.al* are worked on Leakage delays in T-S fuzzy cellular neural networks systems by using LMI approach and compared results with existing literature, Ref. [3] Pin-Lin-Liu worked on further results on the exponential stability criteria for time delay singular systems with delay-dependence and shows the more conserved results than the earlier literature solutions existing in control journal. Ref. [7, 8, 9] researchers are worked on to give the improved results for the time delay systems, continuation of this Ref. [5] Liu and SU studied about the stability analysis and they given one or more choice by an Lyapunov method to get the best results for time delay systems of equation and Ref. [2] Liu continued the work by Lyapunov method and compared the results with earlier literature. On the other hand Chang-Hua Ref. [1] is improved the results which is existing Ref. [5] by using Lyapunov-karovaski method and given four numerical examples to show their improved results. Ref[4] researcher given the results for neutral delay differential systems continuation of this here we continued the work on stability of the system of equations.

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To solve the problems of neutral delay differential equation, LMI formation using Lyapunov function is a best method. It will give us more conserved results than any other methods, methodology of Lyapunov method to analyze stability of a system as follows.

Consider the linear system

$\dot{x}(t) = Ax(t)$ (1)

Where $A \in R^{n*n}$ and $x(t) \in R^n$. Assume that (1) has equilibrium X=0

Definition 1.1 Asymptotically Stable

Let $X = 0$ be an equilibrium point of $\dot{X} = f(X)$,let $V: R^n \to R$ be a continuously differentiable function such that:

1. $V(0) = 0$
2. $V(X(t)) > 0$
3. $\dot{V}(X(t)) < 0$ (2)

This leads to the celebrated theorem of Lyapunov of (1).

Theorem 1.2 (Lyapunov second Theorem on R) Given system (1) with equilibrium X=0, if there exists an Lyapunov function V, then X=0 is Lyapunov stable. Furthermore, if $\dot{V}(X(t)) < 0$, then X=0 is asymptotically stable.

The power of Theorem 1.2 is that one can make conclusions about trajectories of a system (1) without actually solving the differential equation. For the system(1), a common choice of Lyapunov function candidate is the quadratic form,

By choosing Lyapunov function $V(X) = X^T PX$, $P > 0$, Where X=x(t) (3)

Then derivative analyses are
$$\begin{aligned}\dot{V} &= \dot{X}^T PX + X^T P\dot{X} \\ &= X^T A^T PX + X^T PAX \\ &= X^T(A^T P + PA)X\end{aligned} \quad (4)$$

The quadratic form of this derivative proves, if the central quantity satisfies

$A^T P + PA < 0$ (5)

$\dot{V}(X) < 0$ (6)

It shows that the given systems of equations are asymptotically stable and the corresponding conservative results.
By continue of this methods here we continued the work on improve the stability of a neutral delay differential systems.

Remark: In this article to prove Theorem1 along with the definition of Asymptotic stability and Lyapunov theory we are considering following two Lemma’s to improve the solutions of the Neutral Delay-Differential systems.

Lemma 1.3. [18]For any positive semi-definite matrices

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X33 \end{bmatrix} \geq 0 \quad (7)$$

The following integral inequality holds

$$-\int_{t-h}^{t} \dot{x}^T(s) X_{33} \dot{x}(s) \leq \int_{t-h}^{t} [x^T(t) \quad x^T(t-h) \quad \dot{x}^T(s)] \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ {X_{12}}^T & X_{22} & X_{23} \\ {X_{13}}^T & {X_{23}}^T & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \\ \dot{x}(s) \end{bmatrix} ds \quad (8)$$

We now present a delay-dependent criterion for asymptotic stability of the systems (1).

Lemma 1.4 [19]The following matrix inequality

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} < 0 \quad (9)$$

Where $Q(x) = Q^T(x)$, $R(x) = R^T(x)$ and $S(x) depend$ $affinely$ on x, is $equivalent$ to
$R(x) < 0$
$Q(x) < 0$

And

$Q(x) - S(x)R^{-1}(x)S^T(x) < 0$ (10)

This Lemma also called as Schur-complement Lemma.

MAIN RESULT

Consider a Neutral delay-differential system of the form

$$\dot{x}(t) = Ax(t) + Bx(t-h) + C\dot{x}(t-h) \tag{11}$$

With the initial condition function

$$x(t_0+\theta) = \emptyset(\theta), for\ all\ \theta \in (-h, 0) \tag{12}$$

$x(t) \in R^n$is the state vector, A ,$Band\ C \in R^{n*n}$ are constant matrices, h is a positive constant time-delay, $\emptyset(.)$is the given continuously differentiable function on $(-h, 0)$,and the system matrix A is assumed to be a Hurwitz matrix.

The system given in (11) often appears in the theory of automatic control or population dynamics. First, we establish a delay-independent criterion, for the asymptotic stability of the delay-differential system (11) using Lyapunov method in terms of LMI.

Theorem 2. 1: For a given Scalar $h > 0$,the neutral delay differential System (11) is asymptotically stable if there exist positive-definite symmetric matrices $P = P^T > 0, R = R^T > 0$ are of appropriate dimensions and a positive semi-definite Matrix$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} \geq 0$ such that the following LMIs hold: $\begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix} < 0and[R - X_{33}] > 0$
where

$$\phi_{11} = A^TP + PA + A^TA + R + hA^TAR + hX_{11} + X_{13} + {X_{13}}^T$$
$$\phi_{12} = PB + A^TB + A^TBhR + hX_{12} + {X_{23}}^T - X_{13}$$
$$\phi_{13} = PC + A^TC + hA^TCR$$
$$\phi_{21} = B^TP + B^TA + B^TAhR + h{X_{12}}^T + X_{23} - {X_{13}}^T$$
$$\phi_{22} = B^TB - R + hB^TBR + hX_{22} - X_{23} - {X_{23}}^T$$
$$\phi_{23} = B^TC + hRB^TC$$
$$\phi_{31} = C^TP + C^TA + hC^TAR$$
$$\phi_{32} = C^TB + hC^TBR$$
$$\phi_{33} = C^TC - I + hC^TCR$$

Proof: Consider the Delay differential system (11), using the Lyapunov- Krasovskii functional candidate in the following form, we can write

$$V = v_1 + v_2 + v_3 + v_4 \tag{13}$$

Where

$$v_1 = x^T(t)Px(t) \tag{14}$$

$$v_2 = \int_{-h}^{0} \dot{x}^T(t+s)\dot{x}(t+s)ds \tag{15}$$

$$v_3 = \int_{-h}^{0} x^T(t+s)Rx(t+s)ds \tag{16}$$

$$v_4 = \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta \tag{17}$$

The time derivative of V along the solution of (11) is given by

$$\dot{V} = \dot{v_1} + \dot{v_2} + \dot{v_3} + \dot{v_4} \tag{18}$$

Where $x, x_h, \dot{x_h}$ denote $x(t), x(t-h), \dot{x}(t-h)$respectively. From 14 , 15,16 &17 we obtain

$$\dot{v_1} = x^T(A^TP + PA)x + 2x^TPBx_h + 2x^TPC\dot{x_h} \tag{19}$$

$$\dot{v_2} = \dot{x}^T\dot{x} - {\dot{x}_h}^T\dot{x}_h$$
$$= x^TA^TAx + x_h^TB^TBx_h + {\dot{x}_h}^TC^TC\dot{x}_h + 2x^TA^TBx_h + 2x^TA^TC\dot{x}_h + 2x_h^TB^TC\dot{x}_h - \dot{x}_h^T\dot{x}_h \tag{20}$$

$$\dot{v_3} = x^TRx - {x_h}^TRx_h \tag{21}$$

$$\dot{v_4} = \dot{x}^ThR\dot{x} - \int_{t-h}^{t} \dot{x}^T(s)\, X_{33}\dot{x}(s) - \int_{t-h}^{t} \dot{x}^T(s)\,(R - X_{33})\dot{x}(s) \tag{22}$$

where

$$-\int_{t-h}^{t} \dot{x}^T(s)\, X_{33}\dot{x}(s) \leq \int_{t-h}^{t} [x^T(t) \quad x^T(t-h) \quad \dot{x}^T(s)] \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ {X_{12}}^T & X_{22} & X_{23} \\ {X_{13}}^T & {X_{23}}^T & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \\ \dot{x}(s) \end{bmatrix} ds$$

Then

$$\dot{v_4} = x^T A^T AhRx + x^T A^T BhRx_h + x^T A^T ChR\dot{x_h} + x_h B^T AhRx + x_h^T B^T BhRx_h + x_h^T B^T ChR\dot{x}_h + \dot{x}_h^T C^T AhRx + \dot{x}_h^T C^T BhRx_h + \dot{x}_h^T C^T ChR\dot{x}_h + x^T\left[hX_{11} + X_{13} + {X_{13}}^T\right]x + x^T\left[hX_{12} - X_{13} + {X_{23}}^T\right]x_h + x_h^T\left[h{X_{12}}^T + X_{23} - {X_{13}}^T\right]x + x_h^T\left[hX_{22} - X_{23} - {X_{23}}^T\right]x_h - \int_{t-h}^{t} \dot{x}^T(s)\,(R - X_{33})\dot{x}(s) \quad (23)$$

Total Derivative becomes

$$\dot{V} = x^T(A^T P + PA)x + 2x^T PBx_h + 2x^T PC\dot{x_h} + x^T A^T Ax + x_h^T B^T Bx_h + \dot{x_h}^T C^T C\dot{x}_h + 2x^T A^T Bx_h + 2x^T A^T C\dot{x}_h + 2x_h^T B^T C\dot{x}_h - \dot{x}_h^T\dot{x}_h + x^T Rx - {x_h}^T Rx_h + x^T A^T AhRx + x^T A^T BhRx_h + x^T A^T ChR\dot{x_h} + x_h B^T AhRx + x_h^T B^T BhRx_h + x_h^T B^T ChR\dot{x}_h + \dot{x}_h^T C^T AhRx + \dot{x}_h^T C^T BhRx_h + \dot{x}_h^T C^T ChR\dot{x}_h + x^T\left[hX_{11} + X_{13} + {X_{13}}^T\right]x + x^T\left[hX_{12} - X_{13} + {X_{23}}^T\right]x_h + x_h^T\left[h{X_{12}}^T + X_{23} - {X_{13}}^T\right]x + x_h^T\left[hX_{22} - X_{23} - {X_{23}}^T\right]x_h - \int_{t-h}^{t} \dot{x}^T(s)\,(R - X_{33})\dot{x}(s) \quad (24)$$

Can be written as

$$\begin{bmatrix} x(t) \\ x(t-h) \\ \dot{x}(t-h) \end{bmatrix}^T \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \\ \dot{x}(t-h) \end{bmatrix} < 0 \quad (25)$$

where

$\phi_{11} = A^T P + PA + A^T A + R + hA^T AR + hX_{11} + X_{13} + {X_{13}}^T$
$\phi_{12} = PB + A^T B + A^T BhR + hX_{12} + {X_{23}}^T - X_{13}$
$\phi_{13} = PC + A^T C + hA^T CR$
$\phi_{21} = B^T P + B^T A + B^T AhR + h{X_{12}}^T + X_{23} - {X_{13}}^T$
$\phi_{22} = B^T B - R + hB^T BR + hX_{22} - X_{23} - {X_{23}}^T$
$\phi_{23} = B^T C + hRB^T C$
$\phi_{31} = C^T P + C^T A + hC^T AR$
$\phi_{32} = C^T B + hC^T BR$
$\phi_{33} = C^T C - I + hC^T CR$
and $(R - X_{33}) > 0$ (26)

Hence Proved.

Numerical Examples

To illustrate the improvement of our proposed method, we present the following two simulation examples.

Example 1

Consider the following system:

$$\dot{\boldsymbol{x}}(\boldsymbol{t}) = \boldsymbol{A}\,\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{B}\,\boldsymbol{x}(\boldsymbol{t} - \boldsymbol{\tau}) \quad (27)$$

Where

$\boldsymbol{A} = \begin{bmatrix} -2 & 1 \\ -1 & -1 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} -1 & -1 \\ 1 & -2 \end{bmatrix}$**,h=0.1**

By solving LMI proposed in the Theorem.1, a following feasible solution is obtained by using YALMIP toolbox

$\boldsymbol{P} = \begin{bmatrix} 4.8717 & 1.4161 \\ 1.4161 & 4.4256 \end{bmatrix}, \boldsymbol{R} = \begin{bmatrix} 3.4964 & -0.2506 \\ -0.2506 & 1.6330 \end{bmatrix}, \boldsymbol{X_{11}} = \begin{bmatrix} 3.4422 & -0.0664 \\ -0.0664 & 2.7230 \end{bmatrix},$

$\boldsymbol{X_{12}} = \begin{bmatrix} 3.5864 & -0.8307 \\ -0.8307 & 3.8343 \end{bmatrix}, \boldsymbol{X_{13}} = \begin{bmatrix} 4.3431 & 1.3003 \\ 1.3003 & 0.5714 \end{bmatrix}, \boldsymbol{X_{22}} = \begin{bmatrix} 3.6309 & -0.4225 \\ -0.4225 & 3.5854 \end{bmatrix}$

$\boldsymbol{X_{23}} = \begin{bmatrix} 2.0700 & 0.2117 \\ 0.2117 & 4.3046 \end{bmatrix}, \boldsymbol{X_{33}} = \begin{bmatrix} 1.7482 & -0.1253 \\ -0.1253 & 0.8165 \end{bmatrix}$

and maximum upper bounded value is 0.44223

The obtained result shows that the delayed differential system is asymptotically stable.

Example 2

Consider the following system:

$$\dot{\boldsymbol{x}}(\boldsymbol{t}) = \boldsymbol{A}\,\boldsymbol{x}(\boldsymbol{h}) + \boldsymbol{B}\,\boldsymbol{x}(\boldsymbol{t} - \boldsymbol{h}) + \boldsymbol{C}\dot{\boldsymbol{x}}(\boldsymbol{t} - \boldsymbol{h}) \quad (28)$$

Where

$\boldsymbol{A} = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}$,**h=0.1**

By solving LMI proposed in the Theorem.1, a following feasible solution is obtained by using YALMIP toolbox

$\boldsymbol{P} = 1.0\mathrm{e}+07 * \begin{bmatrix} 3.6883 & 2.1207 \\ 2.1207 & 1.6992 \end{bmatrix}, \boldsymbol{R} = \begin{bmatrix} 3.5648 & -3.6638 \\ -3.6638 & 3.5648 \end{bmatrix}$

$\boldsymbol{X_{11}} = 1.0\mathrm{e}+07 * \begin{bmatrix} 2.6137 & 0.1038 \\ 0.1038 & 2.0975 \end{bmatrix}, \quad \boldsymbol{X_{12}} = 1.0\mathrm{e}{+}07 * \begin{bmatrix} 2.7615 & 0.0384 \\ 0.0384 & 2.0614 \end{bmatrix}$

$\boldsymbol{X_{13}} = 1.0\mathrm{e}{+}07 * \begin{bmatrix} 3.6023 & 0.8448 \\ 0.8448 & 0.9788 \end{bmatrix}, \boldsymbol{X_{22}} = 1.0\mathrm{e}+07 * \begin{bmatrix} 2.5894 & -0.0763 \\ -0.0763 & 2.4098 \end{bmatrix}$

$\boldsymbol{X_{23}} = 1.0\mathrm{e}{+}07 * \begin{bmatrix} 3.1400 & 0.9276 \\ 0.9276 & 1.2090 \end{bmatrix}, \boldsymbol{X_{33}} = \begin{bmatrix} 1.7824 & -1.8319 \\ -1.8319 & 1.7824 \end{bmatrix}$

The obtained result shows that the delayed differential system is asymptotically stable.

CONCLUSION

In this paper we study about the sufficient conditions stability of the delay differential systems. The derived conditions are expressed in terms of LMI using schur-complement lemma and two simulation examples are given to demonstrate of our result with help of Yalmipmatlab tool box.

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