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Research Article

THE SEQUENCE OF DIOPHANTINE TRIPLES INVOLVING HALF COMPANION SEQUENCE AND PELL NUMBERS

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ABSTRACT

In this paper, we search for the sequence of triples $\{a,b,c\}, \{b,c,d\}, \{c,d,e\}, \{d,e,f\}$....... involving half companion sequence with the properties $D(-8)$, $D(8)$ and $D(32)$. Also, we search for the sequence of triples involving Pell numbers with the property $D(-1)$.

INTRODUCTION

Let n be an integer. A set of positive integers $(a_1, a_2, a_3, ..., a_m)$ is said to have the property $D(n)$ if $a_i a_j + n$ is a perfect square for all $1 \le i < j \le m$; such a set is called a Diophantine $m-$tuple or a p_n set of size m. The problem of construction of such set was studied by Diophantus. He studied the following problem. Find four (positive rational) numbers such that the product of any two of them increased by 1 is a perfect square. He obtained the following solution: $\frac{1}{2}, \frac{33}{16}, \frac{17}{4}, \frac{105}{16}$ (see [1]). The first set of four positive integers with the above property was found by Fermat, and it was $\{1,3,8,120\}$. Euler gave the solution $\{a, b, a+b+2r, 4r(r+a)(r+b)\}$, where $ab+1 = r^2$ $(see\ [2])$. For an extensive review of various articles one may refer [3- 18]. In this Communication, we find the sequence of Diophantine triples consist of half companion sequence and Pell numbers satisfying various properties.

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Method of Analysis

Let $a = Q_{2k+2}$, $b = Q_{2k}$ where $Q_n = (1-\sqrt{2})^n + (1+\sqrt{2})^n$ be any two integers such that $ab-8$ is a perfect square.
Let c be the non –zero integer such that

$$ca - 8 = \alpha^2 \tag{1}$$

$$cb - 8 = \beta^2 \tag{2}$$

From (1), we get

$$c = \frac{\alpha^2 + 8}{a} \tag{3}$$

Substituting (3) in (2), we notice that

$$(\alpha^2 + 8)b - 8a = a\beta^2 \tag{4}$$

Let

$$\alpha = X + Q_{2k+2}T \tag{5}$$

$$\beta = X + Q_{2k}T \tag{6}$$

On substituting (5) and (6) in (4), we obtain

$$X^2 = DT^2 - 8 \text{ where } D = ab \tag{7}$$

Choosing the initial solution to the negative Pellian equation (7) as

$$X_0 = Q_{2k+1},\ T_0 = 1,$$

we get

$$\alpha = Q_{2k+1} + a$$

$$\beta = Q_{2k+1} + b$$

Substituting the values of α in (1), we get

$$c = Q_{2k} + 2Q_{2k+1} + Q_{2k+2}$$

Thus, $\{Q_{2k+2}, Q_{2k}, Q_{2k} + 2Q_{2k+1} + Q_{2k+2}\}$ is a Diophantine triple with the property $D(-8)$

Let d be the non –zero integer such that

$$d\,b - 8 = \alpha_1^2 \tag{8}$$

$$d\,c - 8 = \beta_1^2 \tag{9}$$

From (8), we get

$$d = \frac{\alpha_1^2 + 8}{b} \tag{10}$$

Now, let e be the non –zero integer such that

$$ec - 8 = \alpha_2^2 \tag{11}$$

$$ed - 8 = \beta_2^2 \tag{12}$$

From (11), we get

$$e = \frac{\alpha_2^2 + 8}{c} \tag{13}$$

Let f be the non –zero integer such that

$$fd - 8 = \alpha_3^2 \tag{14}$$

$$fe - 8 = \beta_3^2 \tag{15}$$

From (14), we get

$$f = \frac{\alpha_3^2 + 8}{d} \tag{16}$$

Choose

$$\alpha_1 = 2Q_{2k} + Q_{2k+1},$$

$$\beta_1 = 2Q_{2k} + 3Q_{2k+1} + Q_{2k+2}$$

$$\alpha_2 = 3Q_{2k} + 5Q_{2k+1} + 2Q_{2k+2},$$

$$\beta_2 = 6Q_{2k} + 7Q_{2k+1} + 2Q_{2k+2}$$

$$\alpha_3 = 10Q_{2k} + 11Q_{2k+1} + 3Q_{2k+2},$$

$$\beta_3 = 15Q_{2k} + 19Q_{2k+1} + 6Q_{2k+2}$$

and applying the same procedure as explained above, we evaluate

$$d = 4Q_{2k} + 4Q_{2k+1} + Q_{2k+2}$$

$$e = 9Q_{2k} + 12Q_{2k+1} + 4Q_{2k+2}$$

$$f = 25Q_{2k} + 30Q_{2k+1} + 9Q_{2k+2}$$

Thus, we note that

$$\{Q_{2k},\ Q_{2k} + 2Q_{2k+1} + Q_{2k+2},\ 4Q_{2k} + 4Q_{2k+1} + Q_{2k+2}\}$$
$$\{Q_{2k} + 2Q_{2k+1} + Q_{2k+2},\ 4Q_{2k} + 4Q_{2k+1} + Q_{2k+2},\ 9Q_{2k} + 12Q_{2k+1} + 4Q_{2k+2}\}$$
$$\{4Q_{2k} + 4Q_{2k+1} + Q_{2k+2},\ 9Q_{2k} + 12Q_{2k+1} + 4Q_{2k+2},\ 25Q_{2k} + 30Q_{2k+1} + 9Q_{2k+2}\} \ldots\ldots$$

is a sequence of Diophantine triples such the product of any two of them decreased by 8 is a perfect square.

Hence, we get the sequence of triples $\{a, b, c\}, \{b, c, d\}, \{c, d, e\}, \{d, e, f\}, \ldots\ldots\ldots$ with the property $D(-8)$.

Some numerical examples for the above sequence of Diophantine triples with the property $D(-8)$ are presented below

n	(a,b,c)	(b,c,d)	(c,d,e)	(d,e,f)
1	(34,6,68	(6,68,114)	(68,114,358)	(114,358,867)
2	(198,34,396)	(34,396,662)	(396,662,2082)	(662,2082,5092)
3	(1154,198,2308)	(198,2308,3858)	(2308,3858,12134)	(3858,12134,29676)

Following the same procedure as mentioned above, we find the following sequence of triples with various properties.

(i) $\{Q_{2k+1},\ Q_{2k+1} + 2Q_{2k} + Q_{2k-1},\ Q_{2k-1} + 4Q_{2k+1} + 4Q_{2k}\}$,
$\{Q_{2k+1} + 2Q_{2k} + Q_{2k-1},\ Q_{2k-1} + 4Q_{2k+1} + 4Q_{2k},\ 12Q_{2k} + 9Q_{2k+1} + 4Q_{2k-1}\}$,
$\{Q_{2k-1} + 4Q_{2k+1} + 4Q_{2k},\ 12Q_{2k} + 9Q_{2k+1} + 4Q_{2k-1},\ 30Q_{2k} + 25Q_{2k+1} + 9Q_{2k-1}\}$

etc is a sequence of triples with the property $D(8)$.

Some numerical examples for the above sequence of Diophantine triples with the property $D(8)$ are presented below

n	(a,b,c)	(b,c,d)	(c,d,e)	(d,e,f)
1	(2,14,28)	(14,28,82)	(28,82,206)	(82,206,548)
2	(14,82,164)	(82,164,478)	(164,478,1202)	(478,1202,3196)
3	(82,478,956)	(478,956,2786)	(956,2786,7006)	(2786,7006,18628)

(i) $\{Q_{2k-1},\ Q_{2k-1} + 2Q_{2k+1} + Q_{2k+3},\ 4Q_{2k-1} + 4Q_{2k+1} + Q_{2k+3}\}$,
$\{Q_{2k-1} + 2Q_{2k+1} + Q_{2k+3},\ 4Q_{2k-1} + 4Q_{2k+1} + Q_{2k+3},\ 9Q_{2k-1} + 12Q_{2k+1} + 4Q_{2k+3}\}$,
$\{4Q_{2k-1} + 4Q_{2k+1} + Q_{2k+3},\ 9Q_{2k-1} + 12Q_{2k+1} + 4Q_{2k+3},\ 25Q_{2k-1} + 30Q_{2k+1} + 9Q_{2k+3}\}$

etc is a sequence of triples with the property $D(32)$.

Some numerical examples for the above sequence of Diophantine triples with the property $D(32)$ are presented below

n	(a,b,c)	(b,c,d)	(c,d,e)	(d,e,f)
1	(82,2,112)	(2,112,146)	(112,146,514)	(146,514,1208)
2	(478,14,656)	(14,656,862)	(656,862,3022)	(862,3022,7112)
3	(2786,82,3824)	(82,3824,5026)	(3824,5026,17618)	(5026,17618,41464)

(iii) $\{p_{2n+1},\ p_{2n+1} + 2p_{2n} + p_{2n-1},\ p_{2n-1} + 4p_{2n+1} + 4p_{2n}\}$,
$\{p_{2n+1} + 2p_{2n} + p_{2n-1},\ p_{2n-1} + 4p_{2n+1} + 4p_{2n},\ 12p_{2n} + 9p_{2n+1} + 4p_{2n-1}\}$,
$\{p_{2n-1} + 4p_{2n+1} + 4p_{2n},\ 12p_{2n} + 9p_{2n+1} + 4p_{2n-1},\ 30p_{2n} + 25p_{2n+1} + 9p_{2n-1}\}$

etc is a sequence of triples with the property $D(-1)$.

Some numerical examples for the above sequence of Diophantine triples with the property $D(-1)$ are presented below

n	(a,b,c)	(b,c,d)	(c,d,e)	(d,e,f)
1	(1,5,10)	(5,10,29)	(10,29,73)	(29,73,194)
2	(5,29,58)	(29,58,169)	(58,169,425)	(169,425,1130)
3	(29,169,338)	(169,338,985)	(338,985,2477)	(985,2477,6586)

CONCLUSION

In this communication, we evaluate the sequence of triples concerning a half companion sequence and Pell numbers satisfying some interesting properties. In this manner, one can search for the sequence of triples, quadruples, quintuples etc with some other properties.

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