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# COMMON FIXED POINTS FOR EXPANSIVE MAÞPPINGS IN CONE B-METRIC SPACES

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**ABSTRACT**

In this paper, we establish some common fixed point and coincidence point theorems for expansive type mappings in the framework of cone *b*-metric spaces without assumption of normality. Our results in this paper extends and improves upon, the corresponding results of Zoran and Murthy[13]. *MSC*: primary 47H09, 47H10, 54H25.



## INTRODUCTION

Banach contraction theorem is one of the most widely used fixed point theorems in all analysis. It has been generalised in many different directions by Mathematicians over the years. Bakhtin [2] introduced *b*-metric spaces as generalisation of metric spaces and proved contraction mapping principles in *b*-metric spaces that generalised the famous Banach contraction theorem. In contemporary time, fixed point theory has evolved in cone metric spaces equipped with partial ordering. The concept of cone metric space was introduced by Huang and Xian [2] where the set of real numbers is replaced by an ordered Banach space in the definition of metric. They introduced the basic definitions and some properties of convergence of sequences in cone metric spaces. They have proved some fixed point theorems of contracting mappings on complete cone metric spaces with assumption of normality of a cone. Thereafter various authors have generalised the result of Huang and Zhang and have studied fixed point theorems for normal and non normal cones [1,6,11,]. In [5], Hussin and Shah introduced cone *b*-metric spaces as a generalisation of *b*-metric spaces and cone metric spaces. Since then, several interesting fixed point results have been appeared in cone *b*-metric spaces.[8].

Expansive mappings in metric spaces were treated and respective fixed point results were obtained in [7,9,10,12]. Several authors have proved fixed point and common fixed point theorems for expansion mappings in the setting of cone metric spaces. Motivated by that we prove some common fixed point and common coincidence point theorems for expansive type mappings in the setting of cone *b*-metric spaces without the assumption of normality.

Consistent with Huang and Zhang [2], the following definitions and results will be needed in the sequel.

***Definition*** *Let E be a real Banach space. A subset P of E is called a cone if and only if*

1. *P is closed, nonempty and* $P\neq \{0\}$ ;
2. $a,b\in R, a,b\geq 0, x,y\in P$ *imply that* $ax+by\in P$;
3. $P\cap(-P)= \{0\}$.

Given a cone $P\subset E$, we define a partial ordering $\leq$ with respect to $P$ by $x\leq y$ if and only if $y-x\in P$. A cone $P$ is called normal if there is a number $K>0$ such that for all $x,y\in E$,

$$0\leq x \leq y \text{ implies } \|x\| \leq K\|y\| \quad (1)$$

The least positive number satisfying the above inequality is called the normal constant of $P$. We shall write $x<y$ to indicate

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that $x\leq y$ but $x\neq y$, while $x\ll y$ stands for $y-x\in$ int $P$ (interior of $P$).

In the following, we always suppose that $E$ is a Banach space, $P$ is a cone in $E$ with int$P\neq\varphi$ and $\leq$ is partial ordering with respect to $P$.

***Definition*** *Let X be a nonempty set. Suppose that the mapping d:X×X→E satisfies:*

1. *$0\leq d(x,y)$ for all $x,y\in X$ and $d(x,y)=0$ if and only if $x=y$;*
2. *$d(x,y)=d(y,x)$ for all $x,y\in X$;*
3. *$d(x,y)\leq d(x,z)+d(z,y)$ for all $x,y,z\in X$.*

Then d is called a cone metric on X and (X,d) is called a cone metric space. The concept of cone metric space is more general than that of metric space.

***Definition*** *Let X be a nonempty set and $s\geq 1$ be a given real number. Suppose that the mapping d:X×X→E satisfies:*

1. *$0\leq d(x,y)$ for all $x,y\in X$ and $d(x,y)=0$ if and only if $x=y$;*
2. *$d(x,y)=d(y,x)$ for all $x,y\in X$;*
3. *$d(x,y)\leq s[d(x,z)+d(z,y)]$ for all $x,y,z\in X$.*

Then (X,d) is called a cone b-metric space. The class of cone b-metric space is larger than the class of cone metric space since any cone metric space be a cone b- metric space. Therefore, it is obvious that cone b-metric spaces generalize b-metric spaces and cone metric spaces [4].

***Definition*** *Let (X,d) be a cone b- metric space, $\{x_n\}$ a sequence in X and $x\in X$. For every $c\in E$ with $0\ll c$, we say that$\{x_n\}$is :*

1. *a Cauchy sequence if there is an N such that, for $n,m> N, d(x_n,x_m)\ll c$*
2. *a convergent sequence if there is an N such that, for all $n,> N, d(x_n,x)\ll c$for some x in X.*

A cone b- metric space $X$ is said to be complete if every Cauchy sequence in $X$ is convergent in $X$.

***Definition*** *Let f and g be self maps of a set X. If w=fx=gx for some $x\in X$,then x is called a coincidence point of f and g, and w is called a point of coincidence f and g.*

***Definition*** *A pair of self mappings to be weakly compatible if they commute at their coincidence points.*

***Proposition*** *Let f and g be weakly compatible self maps of a set X. If f and g have a unique point of coincidence w=fx=gx, then w is the unique common fixed point of f and g .*

The following lemma is needed to prove the result.

***Lemma*** [3, lemma 1.8]. *Let P be a cone and $\{a_n\}$be a sequence in E. If $c\in$ int P and $0\leq a_n\to 0$ (as$n\to\infty$), then there exist N such that for all n>N, we have $a_n\ll c$.*

## MAIN RESULTS

***Theorem 1***

Let $(X,d)$ be a complete cone b-metric space. Suppose that the commuting mappings $f,g:X\to X$ are such that for some constant $\lambda>1$ and for every $x,y\in X$,

$$d(fx,fy)\geq\lambda d(gx,gy). \tag{2}$$

If the range of $f$ contains the range of $g$ and $f$ is continuous then $f$ and $g$ have a unique common fixed point.

***Proof***

Let $x_0$ be arbitrary. Since the range of $f$ contains the range of $g$, we can choose $x_1\in X$ such that $y_0=g(x_0)=f(x_1)$. Let $x_2\in X$ be such that $y_1=g(x_1)=f(x_2)$. Continuing this process, having chosen $x_n\in X$, we choose $x_{n+1}$ in $X$ such that

$y_n=g(x_n)=f(x_{n+1})$

Now,

$$\begin{aligned} d(y_n,y_{n-1}) &= d(fx_{n+1},fx_n)\\ &\geq \lambda d(gx_{n+1},gx_n) \text{ from (2)}\\ &\geq \lambda d(y_{n+1},y_n)\\ d(y_{n+1},y_n) &\leq \frac{1}{\lambda}d(y_n,y_{n-1})\\ &= hd(y_n,y_{n-1}) \text{ where} h=\frac{1}{\lambda}<1\\ &\leq h^2 d(y_{n-1},y_{n-2})\\ &\cdots\cdots\cdots\\ &\leq h^n d(y_1,y_0). \end{aligned}$$

Now we shall show that $\{y_n\}$ is a Cauchy sequence.

By the triangle inequality for $p\geq 1$ we have

$$\begin{aligned} d(y_n,y_{n+p}) &\leq ad(y_n,y_{n+1})+a^2d(y_{n+1},y_{n+2})+\cdots\cdots a^p d(y_{n+p-1},y_{n+p})\\ &\leq [ah^n+a^2h^{n+1}+\cdots+a^p h^{n+p-1}]\, d(y_1,y_0)\\ &\leq \frac{ah^n}{1-ah}d(y_1,y_0)\to 0 \quad as n\to\infty \end{aligned}$$

Let $0\ll c$. By using the lemma 8 we get

$d(y_n,y_{n+p})\leq\frac{ah^n}{1-ah}d(y_1,y_0)\ll c$ by setting $a_n=\frac{ah^n}{1-ah}d(y_1,y_0)$

Hence we get $\{y_n\}$ is a Cauchy sequence. Therefore $\{y_n\}=\{gx_n\}=\{fx_{n+1}\}$ is Cauchy sequence.

Since $X$ is complete, there exist some $q$ in $X$ such that

$$\lim_{n\to\infty}gx_n=\lim_{n\to\infty}fx_{n+1}=q. \tag{3}$$

Since $f$ is continuous and $g$ and $f$ commute we get

$$\begin{aligned} fq=f(\lim_{n\to\infty}fx_n) &= \lim_{n\to\infty}f^2x_n.\\ fq=f(\lim_{n\to\infty}gx_n) &= \lim_{n\to\infty}fgx_n\\ &= \lim_{n\to\infty}gfx_n. \end{aligned} \tag{4}$$

From (2) we get

$$d(f(gx_n),f(fx_n)) \geq \lambda d(ggx_n, gfx_n)$$
$$d(ggx_n, gfx_n) \leq \frac{1}{\lambda} d(fgx_n, ffx_n)$$

When n→ ∞ and using equations (3) and (4)

$$d(gq,fq) \leq \frac{1}{\lambda}(d(fq,fq))$$
$$\Rightarrow fq = gq. \quad \text{From (5)}$$

Again from (2) it follows that

$$d(fx_n,fq) \geq \lambda d(gx_n,gq).$$
$$i.e,\ d(q,fq) \geq \lambda d(q,gq)\ as\ n\to\infty \quad \text{from(3)}$$
$$= \lambda d(q,fq) \quad \text{from (5)}$$
$$\Rightarrow fq = gq.$$

Since $\lambda > 1$, $d(q,fq) = 0$, which implies that $fq = q$ .Thus using equation (5), it gives $fq= gq =q$

Then we get $gq=fq=q$

Now we prove the uniqueness of the common fixed point. For this, assume that there exists another common fixed point $q_1$ in $X$ such that $gq_1=fq_1=q_1$.

From (2) we get

$$d(q_1,q)=d(fq_1,fq) \geq \lambda d(gq_1,gq)$$
$$\geq \lambda d(q_1,q).$$

As $\lambda>1$, we get $q_1=q$. Hence theorem is proved.

***Theorem 2***

Let $(X,d)$ be a cone b-metric space. Suppose that two mappings $f,g:X\to X$ satisfy the following condition

$$d(fx,fy)\geq\lambda d(gx,gy), \quad \forall x,y\in X \tag{6}$$

where $\lambda>1$. If the range of $f$ contains the range of $g$ and one of the subsets $f(X)$ and $g(X)$ is complete, then $f$ and $g$ have a unique point of coincidence in $X$. More over, if $f$ and $g$ are weakly compatible, $f$ and $g$ have unique fixed point.

***Proof.*** Proceeding as the proof of Theorem 1 we get $\{y_n\}$ is a Cauchy sequence.

Since $f(X)$ is complete there exist $q$ in $f(X)$ such that $y_n\to q$ as $n\to\infty$ Consequently we can find $p$ in $X$ such that $f(p)=q$.

From (6) we get

$$d(fx_n,fp) \geq\lambda d(gx_n,gp)$$
$$i.e\ d(fp,fp) \geq\lambda d(q,gp)\ as\ n\to\infty$$

Since $\lambda>1$, $d(fp,gp)=0$. So we get $fp=gp=q$

Now we prove the uniqueness of the point of coincidence. For this, assume that there exists another point of coincidence $q_1$ in $X$ such that $fp_1=gp_1=q_1$. From (6) we get

$$d(fp_1,fp) \geq\lambda d(gp_1,gp)$$
$$d(q_1,q) \geq\lambda d(q_1,q).$$

Since $\lambda>1$ $d(q_1,q)=0$ which shows that $q_1=q$. Hence theorem is proved.

By proposition (7) $f$ and $g$ have a unique common fixed point.

Corollary 1 (14 Theorem 3.1) *Let (X,d) be a cone metric space. Suppose that two mappings $f,g:X\to X$ satisfy the following condition*

$$d(fx,fy)\geq\lambda d(gx,gy), \quad \forall x,y\in X$$

where $\lambda>1$. If the range of f contains the range of g and one of the subsets f(X) and g(X) is complete, then f and g have a unique point of coincidence in X. More over, if f and g are weakly compatible, f and g have unique fixed point.

***Theorem***

Let $(X,d)$ be a cone b-metric space. suppose that the mappings $f,g: X\to X$ satisfies the condition

$$d(fx,fy)\geq\lambda[d(fx,gx)+d(fy,gy)] \quad \forall x,y\in X,\ x\neq y \tag{7}$$

where $\lambda\in(\frac{1}{2},1)$. If the range of $f$ contains the range of $g$, and one of the subsets $f(X)$ and $g(X)$ is complete subspace of $X$ then $f$ and $g$ have a point of coincidence in $X$.

***Proof*** Let $x_0$ be arbitrary. Since $f(X)\supset g(X)$ we can choose $x_1\in X$ such that $y_0=g(x_0)=f(x_1)$. Let $x_2\in X$ be such that $y_1=g(x_1)=f(x_2)$. Continuing this process, having chosen $x_n\in X$, we choose $x_{n+1}$ in $X$ such that

$$y_n=gx_n = fx_{n+1}$$
$$Now,\ d(y_n,y_{n-1}) = d(fx_{n+1},fx_n)$$
$$\geq \lambda[d(fx_{n+1},gx_{n+1})+d(gx_n,fx_n)]$$
$$\geq \lambda[d(y_n,y_{n+1})+d(y_n,y_{n-1})]$$
$$i.e,\ d(y_n,y_{n+1}) \leq \frac{1-\lambda}{\lambda}d(y_n,y_{n-1})$$
$$= hd(y_n,y_{n-1})\ \ where\ h=\frac{1-\lambda}{\lambda}<1$$
$$d(y_n,y_{n+1}) \leq hd(y_{n-1},y_n)$$
$$\leq h^2d(y_{n-2},y_{n-1})$$
$$\ldots\ldots\ldots$$
$$\leq h^nd(y_0,y_1).$$

Now we show that $\{y_n\}$ is Cauchy sequence.

By the triangle inequality for $p\geq1$ we have

$$d(y_n,y_{n+p}) \leq ad(y_n,y_{n+1})+a^2d(y_{n+1},y_{n+2})+\cdots\cdots a^pd(y_{n+p-1},y_{n+p})$$
$$\leq [ah^n+a^2h^{n+1}+\cdots+a^ph^{n+p-1}]\ d(y_1,y_0)$$
$$\leq \frac{ah^n}{1-ah}d(y_1,y_0)\longrightarrow0\ \ as\ n\to\infty$$

Let $0\ll c$. By using the lemma 8 we get

$d(y_n,y_{n+p})\leq \frac{ah^n}{1-ah}d(y_1,y_0)\ll c$ by setting $a_n=\frac{ah^n}{1-ah}\ d(y_1,y_0)$

Hence $\{y_n\}$ is Cauchy sequence. Since $f(X)$ is complete, there exist a $q$ in $f(X)$ such that $y_n\to q$ as $n\to\infty$. Consequently we can find $p$ in $X$ such that $f(p)=q$.

From (7) we get

$$d(fx_n,fp) \geq \lambda[d(fx_n,gx_n)+d(fp,gp)$$

$$d(fp,fp) \geq \lambda[d(fp,gp)] \quad as n\to\infty$$

$$d(fp,gp) = 0$$

$fp=gp$.

***Corollary 2 (13, Theorem 3.7)***

*Let (X,d) be a cone metric space. suppose that the mappings f,g: X→X satisfies the condition* $d(fx,fy)\geq\lambda[d(fx,gx)+d(fy,gy)]\quad \forall x,y\in X\ x\neq y$

*where* $\lambda\in(\frac{1}{2},1)$. *If the range f contains the range of g, and one of the subsets f(X) and g(X) is complete subspace of X then f and g have a point of coincidence in X.*

***Theorem 4***

Let $S$ and $I$ be commuting mappings and $T$ and $J$ be commuting mappings of a complete cone b-metric space $(X,d)$ into itself satisfying.

$$d(Sx,Ty)\geq\lambda d(Ix,Jy),\ forall x,y\in X \tag{8}$$

where $\lambda>1$. If $S(X)\supset J(X)$ and $T(X)\supset I(X)$ and if $S$ and $T$ are continuous, then all $S,T,I$ and $J$ have a unique common fixed point.

***Proof.*** Let $x_0$ in $X$ be arbitrary. Since $S(X)\supset J(X)$ we can choose $x_1\in X$ such that $Jx_0=Sx_1$. Let $x_2\in X$ be such that $Ix_1=Tx_2$ as $T(X)\supset I(X)$ . In general, $x_{2n+1}\in X$ is chosen such that $Jx_{2n}=Sx_{2n+1}$ and $x_{2n+2}\in X$ such that $Ix_{2n+1}=Tx_{2n+2}$.

*Take,* $y_{2n}=Jx_{2n}=Sx_{2n+1}\quad n\geq 0$

$y_{2n+1}=Ix_{2n+1}=Tx_{2n+2}\quad n\geq 0$

Now, we shall show that $y_n$ is a Cauchy sequence.

For this we have

$$
\begin{aligned}
d(y_{2n},y_{2n-1}) &= d(Sx_{2n+1},Tx_{2n})\\
&\geq \lambda d(Ix_{2n+1},Jx_{2n})\ from\ (5)\\
&\geq \lambda[d(y_{n+1},y_n)]\ for n\geq 1\\
d(y_n,y_{n-1}) &\geq \lambda d(y_{n+1},y_n)\\
d(y_{n+1},y_n) &\leq \frac{1}{\lambda}d(y_n,y_{n-1})\\
&\cdots\cdots\cdots\\
&\cdots\cdots\cdots\\
&\leq (\frac{1}{\lambda})^n d(y_1,y_0)\\
&\leq h^n d(y_1,y_0)\ where h=\frac{1}{\lambda}<1.
\end{aligned}
$$

By the triangle inequality, for $p\geq 1$ and using definition of cone b- metric space, we have

$$
\begin{aligned}
d(y_n,y_{n+p}) &\geq ad(y_n,y_{n+1})+a^2d(y_{n+1},y_{n+2})+\cdots\cdots a^pd(y_{n+p-1},y_{n+p})\\
&\leq ah^n+a^2h^{n+1}+\cdots+a^ph^{n+p-1}\ d(y_1,y_0)\\
&\leq ah^n+a^2h^{n+1}+\cdots+a^ph^{n+p-1}\ d(y_1,y_0)\\
&\leq \frac{ah^n}{1-ah}d(y,y_0)\to 0\quad as n\to\infty
\end{aligned}
$$

Let $0\ll c$. By using the lemma 8 we get $d(y_n,y_{n+p})\ll c$. It follows that $\{y_n\}$ is Cauchy sequence.

Let $y\in X$ be such that

$$\lim_{n\to\infty}Jx_{2n}=\lim_{n\to\infty}Sx_{2n+1}=\lim_{n\to\infty}Ix_{2n+1}=\lim_{n\to\infty}Tx_{2n+2}=y. \tag{9}$$

Since $S$ is continuous , $S$ and $I$ commute, and from (6) it follows that

$$\lim_{n\to\infty}S^2x_{2n+1}=Sy,\lim_{n\to\infty}SIx_{2n+1}=\lim_{n\to\infty}ISx_{2n+1}=Sy. \tag{10}$$

From (8)

$$d(S(S(x_{2n+1}),Tx_{2n+2}) \geq\lambda d(ISx_{2n+1},Jx_{2n+2})$$

$$\lambda d(ISx_{2n+1},Jx_{2n+2}) \leq\frac{1}{\lambda}d(S^2x_{2n+1},Tx_{2n+2}).$$

Taking the limit as $n\to\infty$, we get

$d(Sy,y)\leq\frac{1}{\lambda}d(Sy,y)$.

Since $0<\lambda<1$, we get $d(Sy,y)=0$ which implies $Sy=y$.

Similarly, since $T$ is continuous, $J$ and $T$ commute, and from (9) it follows that

$$\lim_{n\to\infty}T^2x_{2n+2}=Ty,\ \lim_{n\to\infty}TJx_{2n}=\lim_{n\to\infty}JTx_{2n}=Ty \tag{11}$$

From(8).

$$d(Sx_{2n+1},T(Tx_{2n+2})) \geq\lambda d(Ix_{2n+1},JTx_{2n+2})$$

$$d(Ix_{2n+1},JTx_{2n+2}) \leq\frac{1}{\lambda}d(Sx_{2n+1},T^2x_{2n+2}).$$

Taking the limit as $n\to\infty$, and using the inequalities (9) and(10) we get

$d(y,Ty)\leq\frac{1}{\lambda}(y,Ty)\Rightarrow Ty=y$

Next, We prove $Iy=y$. From (8)

$$d(Sy,Tx_{2n+2}) \geq\lambda d(Iy,Jx_{2n+2})$$

$$d(Iy,Jx_{2n+2})\leq\frac{1}{\lambda}d(Sy,Tx_{2n+2}).$$

Taking the limit as $n\to\infty$, using inequality(9) we get

$d(Iy,y)\leq\frac{1}{\lambda}d(y,y)\Rightarrow Iy=y$

Again from (8) we get,

$$d(Sy,Ty) \geq \lambda d(Iy,Jy)$$

$$d(Iy,Jy) \leq \frac{1}{\lambda}(Sy,Ty)$$

$$\leq \frac{1}{\lambda}(y,y) \Rightarrow Iy=Jy$$

Thus we $Sy=Ty=Iy=Jy=y$

Next, to prove uniqueness of the common fixed point,assume that there exist

another common fixed point $x$ in $X$ of all $S,T,I$ and $J$, then

$d(x,y)=d(Sx,Ty)\geq\lambda d(Ix,Jy)\geq\lambda d(x,y)$

Since $\lambda>1$ $d(x,y)=0$ i.e $y$ is the unique common fixed point of all $S,T,I$ and $J$. Hence we proved the theorem

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