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International Journal of Recent Scientific Research Vol. 7, Issue, 8, pp. 13096-13104, August, 2016 International Journal of Recent Scientific Rezearch

Research Article

A STUDY OF RADIATION AND MASS TRANSFER EFFECTS ON UNSTEADY MHD FREE CONVECTIVE FLUID FLOW EMBEDDED IN A POROUS MEDIUM WITH HEAT GENERATION/ABSORPTION

Nandakishore S1* and Ravikumar D2

¹Department of Mathematics, N.B.K.R. Institute of Science & Technology, Vidyanagar, A.P. India ²Department of Mathematics, Rayalaseema University, Kurnool, A.P. India

ARTICLE INFO

ABSTRACT

Article History: Received 17th May, 2016 Received in revised form 12th June, 2016 Accepted 04th July, 2016 Published online 28th August, 2016

Key Words:

Radiation, MHDflow, Buoyancy effects, Heat and mass transfer, Galerkin finite element method, Free convective fluid flow. An analysis has been done for unsteady, two dimensional, hydro magnetic, laminar boundary layer flow of a viscous in compressible electrically conducting fluid along a vertical plate in the presence of thermal and buoyancy effects. The dimensionless governing equations of the flow are solved analytically by an efficient Galerkin finite element method. Nusset number and Sherwood number are present and discussed. It has been found that thermal conductivity and concentration buoyancy effects shows linear variation where as the fluid velocity varies with temperature. The effect with variation of the parameters $P_r S_{c}M_{,Q}K$ has been investigated with the help of graphs.

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INTRODUCTION

MHD is concerned with the study of the interaction of magnetic fields and electrically conducting fluids in motion. There are numerous applications of MHD principles including MHD generators, MHD pumps and flow meters etc. Convection problems of electrically conducting fluid in presence of transfers' magnetic field have got much importance because of its wide applications in many engineering and environmental processes. Several investigations were performed using both analytical and numerical methods under different sets of thermal conditions which are continuous and well defined at the boundary wall.

In the last five years, many investigations dealing with heat flow and mass transfer over a verticatal porous plate with variable suction, heat absorption/generation or Hall current have been reported. Singh studied MHD free convection and mass transfer flows with hall current, viscous dissipation, Joule heating and thermal diffusion. Azzam presented radiation effects on the MHD mixed free-fixed convective flow past a semi-infinite moving vertical plate for high temperature differences. Cookey *et al* investigated the influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Hayat and Abbas presented the radiation effects on MHD flow in a porous space. Ogulu *et al.* Studied unsteady MHD free convective flow of a compressible fluid past a moving plate in the presence of radiative heat transfer. Recently, Prakash *et al* have studied MHD free convection and mass transfer flow of a micro-polar thermally radiating and reacting fluid with time dependent suction. Raptis discussed the flow of a micro polar fluid past a continuously moving plate in the presence of radiation. Sunitha *et al.* presented radiation and mass transfer effects on MHD free convection flow past an impulsively started isothermal vertical plate with dissipation.

In this paper, effect of thermal radiation, time-dependent suction and chemical reaction on the two-dimensional flow of an incompressible Boussinesq fluid in the presence of uniform magnetic field applied perpendicular to the flow has been studied... The problem is governed by the system of coupled non-linear partial differential equations whose exact solutions are not obtained in usual manner. Here we can apply Galerkin finite element method to obtain its solution, which is more convenient from computational point of view.

Mathematical formulation of the problem

Consider the problem of unsteady two-dimensional, laminar, boundary layer flow of a viscous, incompressible, electrically conducting fluid along a semi-infinite vertical plate in the presence of thermal and concentration buoyancy effects. Time dependent suction is considered normal to the flow. The *x*-axis is taken along the plate in the direction of the flow and y'-axis is perpendicular to it. Further, due to the semi-infinite plane surface assumption the flow variables are the functions of normal distances y and t. A uniform magnetic field is applied perpendicular to the direction of the flow. Under the usual Boussinesq's approximation, the equations governing of the flow problem are:

Equation of continuity:

$$\frac{\partial \mathbf{v}'}{\partial \mathbf{y}'} = \mathbf{0} \tag{1}$$

Equation of Momentum:

$$\frac{\partial u'}{\partial t'} + v^1 \frac{\partial u'}{\partial y'} = v \frac{\partial^2 u'}{\partial y'^2} + g \alpha (T - T_{\infty}) + g \alpha (C - C_{\infty}) - \sigma \frac{B_0^2}{\rho} u' - \frac{v}{\kappa'} u'$$
(2)

Equation of Energy:

$$\frac{\partial T}{\partial t'} + \mathbf{V}\frac{\partial T}{\partial y} = \mathbf{v}\frac{\partial^2 T}{\partial y'} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'} + Q_0(\mathbf{T} - T_\infty)$$
(3)

Equation of Concentration:

$$\frac{\partial C}{\partial t'} + V'\frac{\partial C}{\partial y} = v \frac{\partial^2 C}{\partial y'} - k_r'^2 C$$
(4)

By using the Rosseland approximation, the radiative flux vector q_r can be written as:

$$q_r = \frac{4\sigma}{3k} \frac{\partial T^4}{\partial y'} \tag{5}$$

All the variables are defined in the nomenclature. It is assumed that the temperature differences within the flow are sufficiently small so that T^4 can be expanded in a Taylor series about the free stream temperature T_{∞} so that the higher order terms are neglected.

$$T^4 \approx 4T_{\infty}^3 \text{ T} - 3T_{\infty}^4 \tag{6}$$

Using radiative flux and the Taylor series, the energy equation can be written as

$$\frac{\partial T}{\partial t'} + V'\frac{\partial T}{\partial y'} = V \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial {y'}^2} + \frac{16\sigma T_{\infty}^3}{3\rho c_p k} \frac{\partial^2 T}{\partial {y'}^2} + Q_0 (T - T_{\infty})$$

$$\tag{7}$$

From Eq (1) one can see that the suction is a function of time only, so it is assumed in the form⁵:

$$\mathbf{v} = -\mathbf{V}_0 \left(1 + \varepsilon \mathbf{A} e^{s' t'}\right) \tag{8}$$

Where A is the suction parameter and $\varepsilon A \ll 1$. Here V₀ is mean suction velocity, which are a non-zero positive constant and the minus sign indicates that the suction is moving towards the plate. Now we can introduce the following dimensionless parameters for our convenience.

$$u = \frac{u'}{u_0}, \ y = \frac{U_0 y'}{v}, \ t = \frac{U_0^2 t'}{v}, \ y = \frac{\rho c_p v}{k}, \ Sc = \frac{v}{D},$$

$$\theta = \frac{T - T_{\infty}}{T - T_{w}}, \quad = \frac{C - C_{\infty}}{C - C_{w}}, \ G_r = \frac{g\beta v(T - T_{\infty})}{U_0^3}, \ G_m = \frac{g\beta v(C - C_{\infty})}{U_0^3}, \ S = \frac{vs'}{U_0^2}, \ K = \frac{U_0^2 k'}{v^2}, \ Q = \frac{Q_0 v}{U_0^2}$$

$$M = \frac{\sigma \beta_0^2}{\rho U_0^2}, \ k_r^2 = \frac{k_r^2 v}{U_0^2}, \ Nr = \frac{16\sigma \ T_{\infty}^3}{3k \ k}$$
(9)

Now convenient to introduce the following dimensionless parameters:

On substitution of Eq. (9) into Eqs (2), (4) and (7), the following governing equations are obtained in non-dimensional form:

$$\frac{\partial u}{\partial t} \quad (1 + \varepsilon A e^{st}) \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G_r \theta + G_m \varphi \quad \left(M + \frac{1}{\kappa}\right) u \tag{10}$$

$$\frac{\partial\theta}{\partial t} \quad (1 + \varepsilon A e^{st}) \frac{\partial\theta}{\partial y} = \left(\frac{1 + Nr}{P_r}\right) \frac{\partial^2 \theta}{\partial y^2} + Q\theta \tag{11}$$

$$\frac{\partial}{\partial t} \quad (1 + \varepsilon A e^{st}) \frac{\partial}{\partial y} = \frac{1}{sc} \frac{\partial^2}{\partial y^2} \quad k_r^2 \tag{12}$$

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The corresponding initial and boundary conditions are:

$$u=1, \theta = (1+\varepsilon e^{st}), \quad = (1+\varepsilon e^{st}) \text{ on } y=0 \quad u \to 0, \theta \to 0 \text{ as } y \to \infty$$
(13)

The mathematical formulation of the problem is now completed. Eqs (10)-(12) are coupled nonlinear systems of partial differential equations, and are to be solved by using the initial and boundary conditions given in Eq. (13). However, exact solutions are difficult if possible. Hence these equations are solved by Gale kin finite element method.

Method of solution

By applying the Gale kin finite element method for Eq. (10) over the element (e), $(y_i \ll y \ll y_k)$ is:

$$\int_{y_j}^{y_k} N^{(e)t} \left[\frac{\partial^2 u^{(e)}}{\partial y^2} + P_1 \frac{\partial u^{(e)}}{\partial y} - \frac{\partial u^{(e)}}{\partial t} - M_1 u^{(e)} + (G_r \theta + G_m \varphi) \right] dy = 0$$
(14)

Where $P_1 = (1 + \varepsilon A e^{st})$, and $M_1 = \left(M + \frac{1}{\kappa}\right)$. Integrating the first term in Eq. (14) by parts

on obtains
$$N^{(e)^T} \left\{ \frac{\partial u^{(e)}}{\partial y} \right\}_{y_j}^{y_k} = \int_{y_j}^{y_k} \left\{ \frac{\partial N^{(e)^T}}{\partial y} \frac{\partial u^{(e)}}{\partial y} - N^{(e)^T} \left(\left[P_1 \frac{\partial u^{(e)}}{\partial y} + \frac{\partial u^{(e)}}{\partial t} + M_1 u^{(e)} - (G_r \theta + G_m \varphi) \right] \right\} dy = 0$$
 (15)

Neglecting the first term in Eq. (15), one gets:

$$\int_{y_j}^{y_k} \left\{ \frac{\partial N^{(e)^T}}{\partial y} \frac{\partial u^{(e)}}{\partial y} - N^{(e)^T} \left(\left[P_1 \frac{\partial u^{(e)}}{\partial y} - \frac{\partial u^{(e)}}{\partial t} - M_1 u^{(e)} + (G_r \theta + G_m \varphi) \right] \right\} dy = 0$$

Let $u^{(e)} = N^{(e)} \phi^{(e)}$ be the linear piecewise approximation solution over the element (e) $(y_j \le y \le y_k)$, Where $N^{(e)} = [N_j N_k]$, $\phi^{(e)} = [u_j u_k]^T$ and $N_j = \frac{y_k - y_j}{y_k - y_j}$, $N_k = \frac{y - y_j}{y_k - y_j}$ are the basis functions. One obtains:

$$\frac{1}{l^{(e)}} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} - \frac{P_1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{l^{(e)}}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{M_1 l^{(e)}}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} u_j \\ u_j \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} u_j \\ u_j \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} u_j \\ u_j \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} u_j \\ u_j \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} u_j \\ u_j \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} u_j \\ u_j \end{bmatrix} = (G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} u_j \\ u_$$

Where dot denotes the differentiation w.r.t t. assembling the element equations for two consecutive elements $y_{i-1} \le y \le y_i$ and $y_i \le y \le y_{i+1}$, following is obtained:

$$\frac{1}{l^{(e)}} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} - \frac{P_1}{2} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{l^{(e)}}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{M_1 l^{(e)}}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} = G_r \theta + G_m \varphi) \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$
(16)

Now put row corresponding to the node I to zero, from Eq. (16) the difference schemes with $l^{(e)}$ =h is:

$$u_{i-1} + 4u_i + u_{i+1} = \frac{1}{h^2} \begin{pmatrix} 6 & 3P_1 h & M_1 \end{pmatrix} u_{i-1} + \frac{4}{h^2} \begin{pmatrix} 3 + M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_{i+1} + 6 \begin{pmatrix} G_r \theta + G_m \varphi \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix} u_i + \frac{1}{h^2} \begin{pmatrix} 6 + 3P_1 h & M_1 \end{pmatrix}$$

Applying the trapezoidal rule, following system of equations in Crank –Nicholson method are obtained:

$$A_1 u_{i-1}^{j+1} + A_2 u_i^{j+1} + A_3 u_{i+1}^{j+1} = A_4 u_{i-1}^j + A_5 u_i^j + A_6 u_{i+1}^j + 6k \left(G_r \theta_i^j + G_m \phi_i^j \right)$$
(17)

Now from Eqs (11) and (12), following equations are obtained:

$$B_1 \theta_{i-1}^{j+1} + B_2 \theta_i^{j+1} + B_3 \theta_{i+1}^{j+1} = B_4 u_{i-1}^j + B_5 \theta_i^j + B_6 \theta_{i+1}^j$$
(18)

$$C_1 \phi_{i-1}^{j+1} + C_2 \phi_i^{j+1} + C_3 \phi_{i+1}^{j+1} = C_4 \phi_{i-1}^j + C_5 \phi_i^j + C_6 \phi_{i+1}^j$$
(19)

Where

$$\begin{array}{rrrr} A_{1} = 1 - 3r + \frac{3}{2} rP_{1}h + \frac{1}{2} rM_{1} & ^{2} \\ A_{2} = 4 + 6r + 2rM_{1} & ^{2} \\ A_{3} = 1 - 3r & \frac{3}{2} rP_{1}h + \frac{1}{2} rM_{1} & ^{2} \\ A_{4} = 1 + 3r & \frac{3}{2} rP_{1}h & \frac{1}{2} rM_{1} & ^{2} \\ A_{5} = 4 & 6r & 2rM_{1} & ^{2} \\ A_{6} = 1 + 3r + \frac{3}{2} rP_{1}h & \frac{1}{2} rM_{1} & ^{2} \\ B_{1} = 1 - 3rP_{2} + \frac{3}{2} rP_{1}h & \frac{1}{2} rQ & ^{2} \\ B_{2} = 4 + 6rP_{2} & 2rQ & ^{2} \\ B_{3} = 1 - 3rP_{2} & \frac{3}{2} rP_{1}h & \frac{1}{2} rQ & ^{2} \\ B_{4} = 1 + 3rP_{2} & \frac{3}{2} rP_{1}h + \frac{1}{2} rQ & ^{2} \\ B_{5} = 4 & 6rP_{2} + 2rQ & ^{2} \end{array}$$

 $B_{6} = 1 + 3rP_{2} + \frac{3}{2}rP_{1}h + \frac{1}{2}rQ^{2}$ $C_{1} = Sc - 3r + \frac{3}{2}rP_{1}Sch + \frac{1}{2}rSc^{2}k_{r}^{2}$ $C_{2} = 4Sc + 6r + 2rSc^{2}k_{r}^{2}$ $C_{3} = Sc - 3r + \frac{3}{2}rP_{1}Sch + \frac{1}{2}rSc^{2}k_{r}^{2}$ $C_{4} = Sc + 3r + \frac{3}{2}rP_{1}Sch + \frac{1}{2}rSc^{2}k_{r}^{2}$ $C_{5} = 4Sc + 6r + \frac{3}{2}rP_{1}Sch + \frac{1}{2}rSc^{2}k_{r}^{2}$ $C_{6} = Sc + 6r + \frac{3}{2}rP_{1}Sch + \frac{1}{2}rSc^{2}k_{r}^{2}$ Here $P_{2} = \frac{1+Nr}{P_{r}}$, $r = \frac{k}{h^{2}}$

And h, k is the mesh sizes along y-direction and time t-direction respectively. Index I refers to the space and j refers to the time. In Eqs (17) - (19), taking I =1(1) n and using initial and boundary conditions (13), the following system 0f equations are obtained:

$$A_i X_i = B_i$$
 $i = 1(1)3$ (20)

Where A_i 's is matrices of order n and X_i , B_i 's column matrices having n-components. The solutions of above system of equations are obtained by using Thomas algorithm for velocity, temperature and concentration. Also, numerical solutions for these equations are obtained by C- program. In order to prove the convergence and stability of Galerkin finite element method, the same C- program was run with slightly changed values of h and k and no significant change was observed in the values of u, θ and ϕ . Hence, the Galerkin finite element method is stable and convergent.

Skin – friction, rate of heat and mass transfer

Skin- friction coefficient in terms of Nusselt number (τ) at the plate is

Heat transfer coefficient in terms of Nusselt number (Nu) at the plate is:

Mass transfer coefficient in terms of Sherwood number (Sh) at the plate is:

RESULTS AND DISCUSSION

In order to get clear insight of the physical problem, Some numerical calculations are carried out for the non-dimensional velocity u, temperature θ , concentration ϕ , skin- friction coefficient (τ) and heat and mass transfer coefficient in terms of Nusselt number (Nu) and Sherwood number (Sh) respectively. To be realistic, We recall that the values of Prandtl number (P_r =0.71), corresponds physically to air.

Electrolytic solution (P_r =1.0), water (P_r =7.0), and water at 4^oC (P_r =11.4). The values of Schmidt number are chosen for hydrogen (Sc =0.22), water-vapour (Sc = 0.60), ammonia (Sc =0.78), methanol (Sc =1.0), and propyl benzene at 20^o C (Sc =2.62).

Figure 1 depicts the temperature distribution, highlighting the effect of radiation. An increasing of the Prandlt number, observed decrease in the temperature and temperature



boundary layer while an increase in the thermal radiation Nr leads to increase in the temperature and temperature boundary layer. The temperature is observed to decrease steeply and exponentially away from the plate. Figure 2 depicts the effect of heat source parameter Q on the temperature field. It is observed that an increase in Q increase the temperature field. Figure 3 depicts the effect of the Schmidt number Sc and chemical reaction rate constant k_r on the species concentration. It is observed that an increase in the Schmidt number or chemical reaction rate decrease in the concentration and concentration boundary layer. From Figs 1 and 3, it is observed that the suction has little or no effect on the temperature and concentration boundary layers.

The temperature and the species concentration are coupled to the velocity via free-convection parameters G_r and G_m as seen in Eq. (10). Figures 4-11 display the effects of material parameters such as P_r , Sc, Q, and Nr k_r , A M, K, G_r and G_m , when the plate is

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$
$$\operatorname{Nu} = \left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$
$$\operatorname{Sh} = \left(\frac{\partial \phi}{\partial y}\right)_{y=0}$$

cooled by free –convection currents ($G_r > 0$). It is observed that an increase in P_r , Sc, k_r and M leads to decrease in the velocity field while an increase in Q, Nr, K, G_r and G_m leads to increase in the velocity field. From Figs 7 and 8, it is also observed that an increase in the suction parameter A leads to the decrease in the velocity field. A comparison of velocity field curves due to cooling of the plate shows that velocity increase rapidly near the plate and after attaining a maximum value, it decreases as y 'increases.



rig.o-Effect of K1 and Aon velocity field u for cooling of the plate when G_r =5, G_m =5, P_r =0.71, Sc=0.22, Q=1.0, Nr=0.5, M=0.5, k=1.0, ε =0.02, A=0.3, ε =0.02, s=0.5, t=1.0



Fig.3- Concentration profiles When Pr=0.71, Nr=0.5, A=0.3



Fig: 5- Effect of Sc on velocity field u for cooling of the plate when Gr=5, Gm=5, Pr=0.71, Q=1.0, Nr=0.5, Kr=0.5 M=0.5, K=1.0, A=0.3, ε=0.02, s=0.5t=1.0



Fig.7-Effect of Nr and A velocity field u for cooling of the plate when G_r =5, Gm= 5, P_r =0.71, Sc=0.22, Q=1.0, k_r =0.5, M=0.5, K=1.0, ε =0.02, s=0.5, t=1.0



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Fig.10-Effect of Gr on velocity field u for cooling of the plate when G_m =5, P_r =0.71, Sc=0.22, Q=1.0, Nr=0.5, k_r =0.5, M=0.5, k=1.0, A=0.3, ε =0.02, s=0.5, t=1.0



Fig.11-Effect of Gm on velocity field u for cooling of the plate G_r =5, P_r =0.71, Sc=0.22, t=1.0 Q=1.0, Nr=0.5, k_r =0.5, M=0.5, k=1.0, A=0.3, ε =0.02, s=0.5,

Figures 12 to 19 show the effects material parameters P_r , Sc, Q, Nr, $K_r A$, M, K, Gr and G_m , when the plate is heated by freeconvention currents ($G_r < 0$). It is observed that an increase in P_r , M, Gr and G_m leads to increase in the velocity field while an increase in Sc, Q, Nr, k_r and K leads to decrease in the velocity field. From Figs 15 and 16, it can be seen that an increase in the suction parameter A leads to increases in the velocity field.



Fig.12-Effect of Pr on velocity field u for Cooling of the plate when G_r = -5.0, G_m =5, Sc=0.22, Q=1.0, Nr=0.5, k_r =0.5, M=0.5, k=1.0, A=0.3, ε =0.02, s=0.5, t=1.0



1.5 0.5 0 12 10 2 -0.5 u -1 Sc =0.22 Sc =0.60 -1.5 Sc =0.78 -2 Sc =1.00 Sc =2.62 -2.5 -3

Fig.13-Effect of Sc on velocity field u for cooling of the plate when G_r = -5.0, Gm=5 P_r=0.71, Q=1.0, Nr=0.5, k_r =0.5, M=0.5, k=1.0, A=0.3, ε =0.02, s=0.5, t=1.0



Fig.14-Effect of Q on velocity field u for heating of the plate when $G_r = -5.0$, $G_m = 5$, $P_r = 0.71$, Sc=0.22, Nr=0.5, $k_r = 0.5$, M=0.5, k=1.0, A=0.3, $\varepsilon = 0.02$, s=0.5, t = 1.0 **Fig.15**-Effect of Nr and A on velocity field u for heating of the plate when $G_r = -5.0$, f = 0.5, $G_m = 5$, $P_r = 0.71$, Sc=0.22, Q=1.0, $k_r = 0.5$, M=0.5, k=1.0, $\varepsilon = 0.02$, s=0.5, t=1.0



Fig.16-Effect of k_r and A on velocity field u for heating of the plate when $G_r = -\frac{Fig.17}{Effect}$ of M and K on velocity field u for heating of the plate when $G_r = -\frac{5.0, G_m = 5, P_r = 0.71, Sc = 0.22, Q = 1.0, Nr = 0.5, M = 0.5, k = 1.0, \varepsilon = 0.02, s = 0.5, t = 1.0}{5.0, G_m = 5, P_r = 0.71, Sc = 0.22, Q = 1.0, Nr = 0.5, k = 0.3, \varepsilon = 0.02, s = 0.5, t = 1.0}$





Fig.18-Effect Gr on velocity field u for heating of the plate when $G_m = 5$,

Pr=0.71, Sc=0.22, M=0.5, k=1.0, Q=1.0, Nr=0.5,

kr=0.5, A=0.3, ε =0.02, s=0.5, t=1.0



Table 1 Presents numerical values of the skin-friction coefficient (τ)for variations in P_r , Sc, Q, Nr, k_r , A, M, K, G_r , and G_m , for cooling of the plate ($G_r < 0$). It is observed that, an increase in P_r , Sc, k_r and M leads to decrease in the value of skin-friction coefficient while an increase in Q, Nr, K, G_r and G_m leads to increase in the value of skin-friction coefficient.

Table 2 presents numerical values of the skin-friction coefficient (τ) for variations in P_r, Sc, Q, Nr, k_r, A, M, K, G_r and G_m for heating of the plate (G_r<0). It is observed that an increase in P_r, M, G_r and G_m leads to increase in the value of skin-friction Coefficient while an increase in Sc, Q, Nr k_r, and K leads to decrease in the value of skin-friction coefficient.

Table 3 presents numerical values of heat transfer coefficient in terms of Nusselt number (Nu) for different values of the prandtl number P_r , heat source parameter Q and thermal radiation Nr, respectively. It is observed that, an increase in the prandtl number leads to decrease in the value of heat transfer coefficient while an increase in the heat source parameter or thermal radiation leads to increase in the value of heat transfer coefficient.

Table 4 presents numerical values of mass transfer coefficient in terms of Sherwood number (Sh) for different values of Schmidt number Sc and chemical Reaction rate constant k_r , respectively. It is observed that, an increase in the Schmidt number or chemical reaction rate constant leads to decrease in the value of mass transfer coefficient.

Pr	Sc	Q	Nr	k,	М	к	Gr	Gm	τ
0.71	0.22	1.0	0.5	0.5	0.5	1.0	5.0	5.0	5.404040
7.00	0.22	1.0	0.5	0.5	0.5	1.0	5.0	5.0	1.312256
0.71	0.60	1.0	0.5	0.5	0.5	1.0	5.0	5.0	4.525942
0.71	0.22	1.5	0.5	0.5	0.5	1.0	5.0	5.0	8.510313
0.71	0.22	1.0	1.0	0.5	0.5	1.0	5.0	5.0	5.738986
0.71	0.22	1.0	0.5	1.0	0.5	1.0	5.0	5.0	4.956634
0.71	0.22	1.0	0.5	0.5	1.0	1.0	5.0	5.0	4.369522
0.71	0.22	1.0	0.5	0.5	0.5	2.0	5.0	5.0	6.881610
0.71	0.22	1.0	0.5	0.5	0.5	1.0	10.0	5.0	9.514360
0.71	0.22	1.0	0.5	0.5	0.5	1.0	5.0	10.0	7.935074

Table 1--- Skin – friction coefficient (τ) for cooling of the plate (G_r>0)

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Pr	Sc	Q	Nr	k,	М	K	Gr	Gm	τ
0.71	0.22	1.0	0.5	0.5	0.5	1.0	- 5.0	5.0	-2.816585
7.00	0.22	1.0	0.5	0.5	0.5	1.0	- 5.0	5.0	1.275202
0.71	0.60	1.0	0.5	0.5	0.5	1.0	- 5.0	5.0	-3.694690
0.71	0.22	1.5	0.5	0.5	0.5	1.0	- 5.0	5.0	-5.922676
0.71	0.22	1.0	1.0	0.5	0.5	1.0	- 5.0	5.0	-3.151528
0.71	0.22	1.0	0.5	1.0	0.5	1.0	- 5.0	5.0	-3.263996
0.71	0.22	1.0	0.5	0.5	1.0	1.0	- 5.0	5.0	-2.678056
0.71	0.22	1.0	0.5	0.5	0.5	2.0	- 5.0	5.0	-2.998256
0.71	0.22	1.0	0.5	0.5	0.5	1.0	- 10.0	5.0	-6.926902
0.71	0.22	1.0	0.5	0.5	0.5	1.0	-5.0	10.0	0.287558

Table 2--- Skin – friction coefficient (τ) for heating of the plate (G_r<0)

Table 3 --- Heat transfer coefficient in terms of Nusselt number (Nu)

P,	Q	Nr	Nu
0.71	1.0	0.5	-0.375516
3.00	1.0	0.5	-1.237810
0.71	2.0	0.5	0.016614
0.71	1.0	1.0	-0.288008

Table 4	Mass transfe	r coefficient in	terms of Sherwood	l number (Sh)
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Sc	k,	Sh
0.22	0.5	-0.443266
0.60	0.5	-0.800992
0.22	1.0	- 0.558000

CONCLUSIONS

The problem of two-dimensional fluid flow in the presence of thermal and concentration buoyancy effects under the influence of uniform magnetic field applied normal to the flow is formulated and solved numerically. Governing the flow of equations are solved by adopting a Galerkin finite element method. The results obtained are compared with previous works and found to be in good agreement. It is found that increasing the Prandlt number results in a decrease in the temperature and temperature boundary layer while increasing the thermal radiation leads to a rise in the temperature and temperature boundary layer. An increase in the Schmidt number or chemical reaction rate constant decrease in the concentration and concentration boundary layer. Also, it has been observed that the suction has a negligible effect on the temperature and concentration. The value of the skin-friction coefficient (τ) decrease as increase in the Prandtl number Pr and Schmidt number Sc. The value of the skin-friction coefficient is maximum for cooling of the plate than in case of heating of the plate. Also, it is observed that, the increase in the Prandtl number Pr and Schmidt number Sc decreases Nusselt number and Sherwood numbers. The results of this study might find potential applications in fluid flows.

Nomenclature

u, v velocity components

T dimensional temperature

- q_r radiative flux vector
- x,y Cartesian coordinates
- t time
- g acceleration due to gravity
- α Coefficient of volume expansion due to temperature
- α^* Coefficient of volume expansion due to concentration
- P Density
- C_p specific heat at constant pressure
- V kinematic viscosity
- M magnetic parameter
- K thermal conductively
- U₀ mean velocity
- Sc Schmidt number
- P_r Brandt number
- \dot{K}_{r}^{2} chemical reaction rate constant
- ε Small reference parameter <<1
- G_r free-convection parameter due to temperature
- G_m free-convection parameter due to concentration
- A suction parameter
- s constant exponential index
- D molar diffusivity
- Nr thermal radiation parameter
- K permeability of the porous medium
- Q heat source parameter

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How to cite this article:

Nandakishore S and Ravikumar D.2016, A Study of Radiation And Mass Transfer Effects on Unsteady MHD Free Convective Fluid Flow Embedded In A Porous Medium With Heat Generation/Absorption. *Int J Recent Sci Res.* 7(8), pp. 13096-13104.