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Research Article

REFINED BEAM THEORY FOR FLEXURAL ANALYSIS OF COMPOSITE BEAM

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ABSTRACT

Present paper deals with static analysis of composite deep beam by using refined beam theory and considering transverse shear deformation effect. In this paper hyperbolic shear deformation theory is used for analysis of composite beam by using general solution scheme, whereas the governing equation boundary conditions are obtained by using principal of virtual work theorem. The final results are obtained in non-dimensional form and are compared with the present theories.

INTRODUCTION

Composite material are widely used in space craft, aircrafts, automotive, shipbuilding and other industries and hence it became necessary to predict accurately the static behavior of composite beam which subjected to deformation due shear. This created new theory which is used for analysis deep composite beam and gave correct structural behavior of deep beam by considering correct warping and bending behavior this theory is known as shear deformation theory.

Sayyad A.S. [1] carried out a comparative study of refined beam theories for the free vibration analysis of thick beams, taking into account transverse shear deformation effect. Meghare T.K. *et al*. [2] carried out astudy of a new shear deformation theory for the bending analysis of thick isotropic beams made up of steel. The transverse shear stress can be obtained directly from the constitutive relations satisfying the shear stress free surface conditions on the top and bottom surfaces of the beam, hence the theory does not require shear correction factor. Salve S. *et al*. [3] studied a trigonometric shear deformation theory for flexure of thick beams, taking into account transverse shear deformation effects. In his paper he has examined isotropic beam fixed in position in position and subjected to parabolic and cosine loading. Dahake A.G. *et al*. [4] studied A trigonometric shear deformation theory for flexure of thick beams, taking into account transverse shear deformation effects. The thick simply supported isotropic beams are considered for the numerical studies to demonstrate the efficiency of the theory. Bhimaraddi A. *et al*. [5] studied a parabolic shear deformation beam theory assuming higher order variations for axial displacement. In that theory, the axial displacement variation can be selected so that it results in a suitable admissible transverse shear-strain variation across the depth of the beam. All these variations for transverse shear-strain satisfy the requirement that the shear-strain be zero at the extreme fibre and non-zero elsewhere along the depth of the beam. Ghugal Y.M. *et al*. [6] studied a Hyperbolic Shear Deformation Theory (HPSDT) taking into account transverse shear deformation effects, which was used for the static flexure analysis of thick isotropic beams. Sawant M.K. *et al*. [7] carried out a study of a new hyperbolic shear deformation theory for flexure of deep beams, in which number of variables is same as that in the hyperbolic shear deformation theory. The noteworthy feature of theory that the transverse shear stresses can be obtained directly from the use of constitutive relations with efficacy, satisfying the shear stress free condition on the top and bottom surfaces of the beam.

Aim and scope

An enormous amount of literature on the structural analysis of structures including the flexure of thick beams are available, still this area is open for research both at fundamental and applied level. The specific aim and objective of the present research work is to develop the theory in a simple way and having the features such as utilization simplicity, accuracy, free of shear correction factors.

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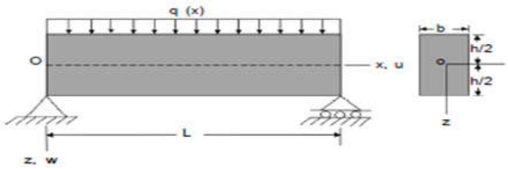
Beam Under Consideration

The beam under consideration occupies the region



Figurer 1 Composite beam subjected to uniformly distributed load

Where *x*, *y*, *z* are Cartesian coordinates, *L* is the length of beam, *b* is the width and *h* is the total depth of beam. The beam is subjected to transverse load of intensity q(x) per unit length of the beam.

Assumptions Made in Theoretical Formulation

1. The in-plane displacement u in x direction consists of two parts:
   A. A displacement component analogous to displacement in elementary beam theory of bending;
   B. Displacement component due to shear deformation which is assumed to be parabolic in nature with respect to thickness coordinate.
2. The transverse displacement w in z direction is assumed to be a function of x coordinate.
3. One dimensional constitutive law is used.
4. The beam is subjected to lateral load only.
5. Body forces are ignored.

Displacement Field

Based on the before mentioned assumptions, the displacement field of the present unified refined beam theory is given as below:

$$u\left(x,z\right) = u_0 - z\frac{dw_0}{dx} + f\left(z\right)\phi_x \qquad (1)$$

$$w = w_0\left(x\right) \qquad (2)$$

Here u and w are the axial and transverse displacements of the beam centre line. The functions f(z) assigned according to the shearing stress distribution through the thickness of the beam are as follows

Present theory:

$$f(z) = \left[\frac{h}{\pi}sin\frac{\pi z}{h}\right]$$

The normal and transverse shear strains are obtained from linear theory of elasticity.

$$\varepsilon_x = \frac{du}{dx} = \frac{du_0}{dx} - z\frac{d^2 w_0}{dx^2} + f\left(z\right)\phi \qquad (3)$$

$$\gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \frac{d}{dz}f\left(z\right)\phi \qquad (4)$$

One dimensional law is used to obtained normal bending and transverse shear stresses.

$$\sigma_x^k = E^k\left[\frac{du_0}{dx} - z\frac{d^2 w_0}{dx^2} + f\left(z\right)\left(\frac{d\phi}{dx}\right)\right] \qquad (5)$$

$$\tau_{zx}^k = G^k \frac{d}{dz}f\left(z\right)\phi \qquad (6)$$

Governing Equations

Using the Eqns. (2) through (6) for strains, stresses and principle of virtual work, variationally consistent differential equations for the beam under consideration are obtained. The principle of virtual work when applied to the beam leads to:

$$b\int_{x=0}^{x=L}\int_{z=-h/2}^{z=+h/2}\left(\sigma_x\delta\varepsilon_x + \tau_{zx}\delta\gamma_{xz}\right)dxdz - \int_{x=0}^{x=L}q\left(\delta w_b + \delta w_s\right)dx = 0 \qquad (7)$$

Where the symbol δ denotes the variational operator. Employing the Green's theorem in Eqn. (7) successively and collecting the coefficients of $\partial w_b\, and\, \partial w_s$ the governing equations obtained are as follows:

$$D\frac{d^4 w_0}{dx^4} - E\frac{d^3\phi}{dx^3} = q \qquad (8)$$

$$E\frac{d^3 w_0}{dx^3} - F\frac{d^2\phi}{dx^2} + H\phi = 0 \qquad (9)$$

Thus, the variationally consistent governing differential equations and boundary conditions are obtained. The constants appear in the governing equations are the stiffness and inertia given as follows.

$$D = E_1\int_{-h/2}^{h/2} z^2 dz; C = E_1\int_{-h/2}^{h/2} zf\left(z\right)dz; F = E_1\int_{-h/2}^{h/2}\left[f\left(z\right)\right]^2 dz; G = G_{31}\int_{-h/2}^{h/2}\left[1 - \frac{d}{dz}f\left(z\right)\right]^2 dz$$

General Solution Scheme for analysis of composite beam

The general solution for transverse displacement w_0 and warping function ϕ is obtained using equations 8 and 9by solution of linear differential equations.

$$\frac{d^3 w_0}{dx^3} = \frac{E}{D}\frac{d^2\phi}{dx^2} + \frac{Q_{(x)}}{D} \qquad (10)$$

Where $Q_{(x)}$ is the generalized shear force for beam and it is given by $Q_{(x)} = \int_0^x q dx + C_1$.

Now the equation number 9 is rearranged in the following form

$$\frac{d^3 w_0}{dx^3} = \frac{F}{E}\frac{d^2\phi}{dx^2} - \beta\phi \qquad (11)$$

A single equation in terms of ϕ is obtained by using equation 10 and 11 as:

$$\frac{d^2\phi}{dx^2} - \lambda^2\phi = \frac{Q_{(x)}}{\alpha D_0} \qquad (12)$$

Where $\alpha = \frac{F_0}{E_0} - \frac{E_0}{D_0}, \beta = \frac{H_0}{E_0}\, and\, \lambda^2 = \frac{\beta}{\alpha}$

The general solution of equation 12 is given by

$$\phi\left(x\right) = C_2\cosh\lambda x + C_3\sin\lambda x - \frac{Q_{(x)}}{\beta D_0} \qquad (13)$$

Transverse displacement w(x) can be obtained by substituting the value of $\phi(x)$ in equation 11

$$w(x)=\iiint\int qdxdxdxdx+\frac{C_1x^3}{6}+\frac{A_0}{\lambda D_0}\left[C_2\cosh\lambda x+C_3\sin\lambda x\right]+\frac{C_4x^2}{2}+C_5x+C_6 \quad (14)$$

Where C_1–C_6 are the arbitrary constants of integration and can be obtained by imposing natural (forced) and kinematic (geometric) boundary conditions of beam.

Illustrative Example

The general solution for transverse displacement w_0 and warping function ϕ is obtained using equations 8 and 9by solution of linear differential equations.

$$\frac{d^3w_0}{dx^3}=\frac{E}{D}\frac{d^2\phi}{dx^2}+\frac{Q_{(x)}}{D} \quad (10)$$

Where $Q_{(x)}$ is the generalized shear force for beam and it is given by $Q_{(x)}=\int_0^x qdx+C_1$.

Now the equation number 9 is rearranged in the following form

$$\frac{d^3w_0}{dx^3}=\frac{F}{E}\frac{d^2\phi}{dx^2}-\beta\phi \quad (11)$$

A single equation in terms of ϕ is obtained by using equation 10 and 11 as:

$$\frac{d^2\phi}{dx^2}-\lambda^2\phi=\frac{Q_{(x)}}{\alpha D_0} \quad (12)$$

Where $\alpha=\frac{F_0}{E_0}-\frac{E_0}{D_0},\beta=\frac{H_0}{E_0}\,and\,\lambda^2=\frac{\beta}{\alpha}$

The general solution of equation 12 is given by

$$\phi(x)=C_2\cosh\lambda x+C_3\sin\lambda x-\frac{Q_{(x)}}{\beta D_0} \quad (13)$$

Transverse displacement w(x) can be obtained by substituting the value of $\phi(x)$ in equation 11

$$w(x)=\iiint\int qdxdxdxdx+\frac{C_1x^3}{6}+\frac{A_0}{\lambda D_0}\left[C_2\cosh\lambda x+C_3\sin\lambda x\right]+\frac{C_4x^2}{2}+C_5x+C_6 \quad (14)$$

Where C_1–C_6 are the arbitrary constants of integration and can be obtained by imposing natural (forced) and kinematic (geometric) boundary conditions of beam.

As shown in figure a simply supported beam uniform beam of rectangular cross-section occupying the region given by figure1 is considered for detailed numerical study.

Example

A simply supported beam with rectangular cross-section (b × h) is subjected to uniformly distributed load (UDL) q over the span L at surface z = h/2 acting in the downward z direction. The origin of beam is taken at left end support, i.e. at x = 0. The boundary conditions associated with simply supported beam are as follow.

$$\frac{d^3w}{dx^3}=\frac{d^2\phi}{dx^2}=\frac{dw}{dx}=\phi=0 \qquad at\ \ x=L/2 \qquad (15)$$

$$\frac{d^2w}{dx^2}=\frac{d\phi}{dx}=w=0 \qquad at\ \ x=0 \qquad (16)$$

The boundary condition, $\phi=0$ at x = L/2 is used from the condition of symmetry of deformation, in which the middle cross-section of the beam must remain plane without warping [Gere and Timoshenko (1986)]. From the general solution of beam, expressions for ϕ and w are obtained as follows:

$$\phi=\frac{qL}{2\beta D_0}\left[\left(1-\frac{2x}{L}\right)-\frac{\sinh\lambda(L/2-x)}{(\lambda L/2)\cosh(\lambda L/2)}\right] \quad (17)$$

$$w=\frac{qL^4}{24D_0}\left[\left(\frac{x}{L}\right)^4-2\left(\frac{x}{L}\right)^3+\left(\frac{x}{L}\right)\right]+\frac{qL^2E^2}{2D_0^2H}\left[\frac{x}{L}\left(1-\frac{x}{L}\right)-\frac{2}{(\lambda L)^2}\left(1-\frac{\cosh\lambda(L/2-x)}{\cosh(\lambda L/2)}\right)\right] \quad (18)$$

RESULT AND DISCUSSION

The result for of transverse displacement ($\bar{w}$), axial stress σ_x and transverse shear stress τ_{zx} for composite beam subjected uniform distributed load are presented in non-dimentional form for purpose of presenting the results in this paper.

$$\bar{w}=\frac{w10bh^3E[L/2,0]}{ql^4},\quad \bar{\sigma}_x=\frac{\sigma_{xx}b}{q},\ \tau_{zx}=\frac{b\tau_{zx}}{q}$$

CONCLUSION

From the static flexural analysis of simply composite beams following conclusions are drawn.

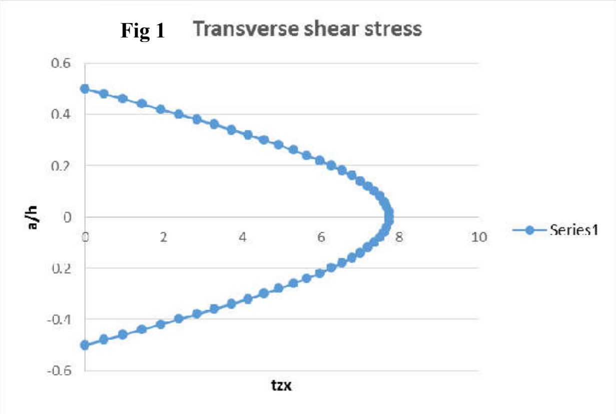
1. The Present refined theory are effectively assessed the combined effect of shear and bending deformation.
2. The results of maximum transverse deflection obtained by present refined beam theory are in excellent agreement with the exact solution.
3. The transverse shear stress obtained from constitutive relation using present theory gives near to exact values.
4. The present theories are variationally consistent and obviate need of shear correction factor.

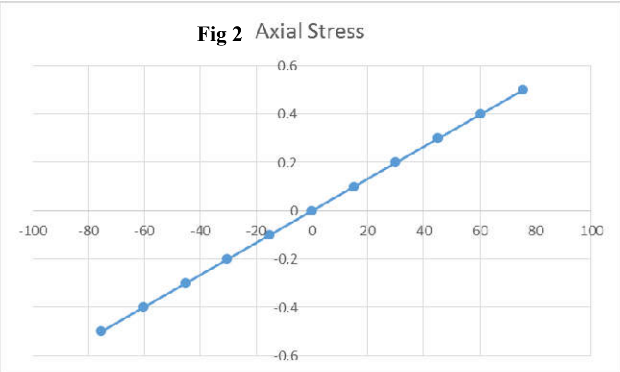
Comparison of transverse displacement ($\bar{w}$), axial stress (σ_x) and transverse shear stress (τ_{zx}) for beam subjected uniformly distributed load.

a/h	Theory	Model	$\bar{w}$	$\bar{\sigma}_x$	τ_{xz}^{CR}
10	Present	HSBT	1.5790	75.2800	7.7400
	Ghughal and Sharma	HPSDT	1.6020	75.2580	7.5600
	Timoshenko	FSDT	1.5950	75.0000	4.999
	Euler and Bernoulli	EBT	1.5630	75.0000	----
4	Present	HSBT	1.7984	12.3201	3.1023
	Ghughal and Sharma	HPSDT	1.8060	12.2580	3.0740
	Timoshenko	FSDT	1.7660	12.0000	2.0000
	Euler and Bernoulli	EBT	1.5630	12.0000	----

Variation of transverse shear stress through the thickness of composite beam subjected to uniformly distributed load for aspect ratio 10 (Fig 1).

Variation of bending stress through the thickness of composite beam subjected to uniformly distributed load for aspect ratio 10 (Fig 2).





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