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# THE BEHAVIOR OF THE GENERALIZED POTENTIAL KERNEL OF AXISYMMETRIC CONTACT PROBLEMS AND THE STRUCTURE RESOLVENT KERNEL OF THE FUNDAMENTAL PROBLEMS

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**ABSTRACT**

In this paper, the behavior and the properties of the generalized potential kernel (**GPK**) of the integral equations (**IEs**), in the axisymmetric contact and mixed problems, in the theory of elasticity are considered. Moreover, the behavior of the first and second structure of the **GPK** is discussed. Many special and new cases, from the kernel, are established. In addition, the behavior of the kernel of the first and second fundamental equations of an infinite elastic plate weakened by a curvilinear hole, in two-dimensional problems, in the theory of elasticity, is considered. The curvilinear hole is conformally mapped outside (inside) a unit circle, using a complex rational conformal mapping. Finally, the first and second structure properties of the complex kernel are proved.



## INTRODUCTION

The theory of application of **IEs**, with its different kinds and kernels, is an important subject within applied sciences. The **IEs** are used as mathematical models for many varied physical situations. In addition, the rapid development of computer engineering has aroused the considerable interest of researchers for the development of universal numerical methods for the solution of applied problems. Many different methods can be used to solve the **IEs** analytically, such as Fourier transformation method; orthogonal polynomials method; degenerate kernel method; potential theory method; Cauchy method and Krein's method. More information for the above methods and other analytic methods with its applications can be found in [1-9].

At the same time, the sense of numerical methods takes an important place in solving the **IEs** for different kernels. More information, for the importance of using the numerical methods in mathematical physics and applied sciences, contain in [10-17].

In other view, the first and second fundamental problems of an isotropic homogeneous performed plate, have been discussed. The elastic plate weakened by a curvilinear hole. Some authors used Laurent's theorem to express the solution in a series form, see [18-21]. Others used complex variables method of Cauchy integrals to express the solution in the form of two complexes potential functions, Gaursat functions, by using many rational mappings, see [22-25]. In this case, the first and second fundamental problems tend to integro differential equations, in the complex plane with Cauchy kernel. It is worth mentioning that Exadaktylos *et al*. [19, 20] considered rational mapping functions with complex constants that conformally mapped the holes inside a unit circle, using Laurent's method. In other side, Abdou and Asseri [24, 25] considered more general rational mapping functions with complex constants that conformally maps the holes outside a unit circle, using Cauchy singular method.

**In section two**, the **GPK** in the axisymmetric contact and mixed problems in the theory of elasticity is considered in a general form of Weber-Sonien integral formula. Many special cases when the kernel takes: logarithmic form, Carleman function, elliptic kernel and potential function are considered. In addition, the physical meaning of each kernel, in the contact problems is discussed. Moreover, many new cases are established. Finally, we present the **GPK** in the form of Cauchy problem for the first partial derivatives and nonhomogeneous (homogeneous) wave equation for the second partial derivatives.

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**In section three**, we consider the first and second fundamental problems in the theory of elasticity, in two-dimensional problem. For this, we consider an infinite elastic plate weakened by a curvilinear hole**.** Then, after using a rational mapping function, the curvilinear hole is conformally mapped outside (inside) the unit circle. Such problem leads to an integro differential equation (**IDE**) with singular kernel. Then, the behavior of such kernel with its first and second structure is discussed. Moreover, the corresponding kernel of **FIE**, in the same complex plane, with its structure, is discussed.

**The generalized potential kernel**:

Let us suppose that a concentrated force $\phi(\phi_1(q),\phi_2(q),\phi_3(q))$ is applied at a certain point $q(y_1,y_2,y_3)$ in space. Then, the displacement at an arbitrary point $p(x_1,x_2,x_3)$ can be expressed in a compact form as the product of a certain matrix $\Gamma(p,q)$, called the Kelvin **–** Somigliana matrix, and the vector $\phi(q)$ is

$$u(p) = \Gamma(p,q)\ \varphi(q) \tag{2.1}$$

The expressions of the matrix elements of $\Gamma(p,q)$ are (see [3]),

$$\Gamma_{ij} = \frac{1}{4\pi(\lambda+2\mu)r(p,q)}\left[(\lambda+\mu)\frac{\partial r}{\partial x_i}\frac{\partial r}{\partial x_j}+(\lambda+3\mu)\ \delta_{ij}\right], \tag{2.2}$$

where λ and μ are called Lame's constants.

Now, let us suppose for a certain closed Liapunov surface $S^l$ in space that the forces $\varphi(p)$ are given, and then the integral $V(p)=\int_S \Gamma(p,q)\ \varphi(q)\ ds_q$ called a vector function satisfying Lame's equations in the entire space except the surface *S*. In analogy with the harmonic potential, this function named "the generalized elastic potential function". Further, we assume that the density function $\varphi(p)$ and the vector function $V(p)$ may be determined directly at points on the surface and their limiting values are identical and equal to the proper values. Consequently, the elastic potential function of a single layer is a vector function that is continuous everywhere in space. The potential *V* (p) tends to zero as $r^{-1}$, or term $\{V(p)\ O(r^{-1})\}$ tends to infinity and $r=\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}.$
Now, we mention certain properties of the potential *V (p)*:

1- The potential $V(p)\to 0$ as $p\to\infty$;

2- If we define a certain point $p$ in space and a small area element whose normal direction is denoted by $\nu$, the stress vector acting on this area element is

$$T_\nu\ V(p) = \int_S \Gamma_1(p,q)\ \varphi(q)\ dS_q \tag{2.3}$$

The latter formula is an **IE** for which $\Gamma_1(p,q)$ is called its kernel.

In the case of an elastic material, the three dimensional kernel can take the form:

$$\Gamma_{i,j} = -[\alpha\delta_{ij}+\delta\frac{(x_i-y)(x_j-y_j)}{r^2}]\frac{\sum_{\ell=1}^{3}(x_\ell-y_\ell)V_{l(p)}}{r^3}+\varepsilon[V_{i(p)}\frac{(x_j-y_j)}{r^3}-V_{j(p)}\frac{(x_i-y_i)}{r^3}] \tag{2.4}$$

$$\left\{\alpha=\frac{1}{2\pi}\left(\frac{\mu}{\lambda+2\mu}\right),\quad \delta=\frac{3}{2\pi}\left(\frac{\lambda+\mu}{\lambda+2\mu}\right)\right\}.$$

Referring to the regularity of the **IE** (2.3), one can interchange the order of differentiation and integration at all points in the space, with the exception of *S*.

3- By concerning the points lying in close proximity of the surface *S*, so that the normal direction *V* can be uniquely determined at each of these points, a stress vector is generated by the single layer potential and the above-mentioned directions of the normal. Hence, we have:

$$T_{\nu(q_1)}\ V(p) = \int_S \Gamma_1(p,q)\ \varphi(q)\ dS_q \tag{2.5}$$

*q* is a point on the surface *S* such that the normal passes through the point *p*.

$$\lim_{p\to q(p\in D^-)} T_{\nu(q_1)}V(p) = \lim_{p\to q_1}\int_S \Gamma_1(p,q)\ \varphi(q)\ dS_q = T_\nu^-\ V(q_1) \quad \text{(External)}$$

$$\lim_{p\to q(p\in D^+)} T_{\nu(q_1)}V(p) = \lim_{p\to q_1}\int_S \Gamma_1(p,q)\ \varphi(q)\ dS_q = T_\nu^+\ V(q_1) \quad \text{(Internal)}$$

By supposing the general **IE**

$$\mu\ \varphi(p) \pm \lambda \int_c \Gamma(p,q)\ \varphi(q_1)\ dS_{q_1} = \mp F(q), \tag{2.6}$$

in which the upper sign corresponds to the external problem $(T_\nu^-)$ and internal problem $(T_\nu^+)$, respectively. The parameter $\mu$ defines the kind of the **IE.** The constant λ, may be complex and possesses different physical meanings.

***The structure kernel of the general axisymmetric contact problem***

When the modules of elasticity in the contact problems changes in the layer surface according to the power law $\sigma_i = K_0\varepsilon_i^\nu, i=1,2,3,\ (0\le\nu<1).$ Here, $\sigma_i$ and $\varepsilon_i$ are the stress and strain rate intensities, respectively. While $K_0$ and ν are the constants depending on the physical properties of the elastic material, where $K_0$ ν is called the modulus of the elasticity material and ν is called Poisson ratio. In this case, we have, see Abdou [26]

$$\Gamma(x-\zeta, y-\eta) = \left[(x-\zeta)^2+(y-\eta)^2\right]^{-\nu}\ (0\le\nu<1). \tag{3.1}$$

Using the following notations $\phi(x,y)=\phi(r\cos\theta, r\sin\theta)=\Phi(r,\theta)=\phi_m(r)\cos m\theta$

$f(x,y)=f(r\cos\theta, r\sin\theta)=\bar{f}(r,\theta)=f_m(r)\cos m\theta;\ (m\ge 0)$, in (8), (9) to obtain:

$$\mu\phi_m(r)-\lambda\int_{-\pi}^{\pi}\int_0^a \frac{\rho\phi_m(\rho)\ d\rho d\vartheta}{\left[r^2+\rho^2-2r\rho\cos\vartheta\right]^\nu}=f_m(r). \tag{3.2}$$

The above formula represents **FIE** of the second kind with generalized in polar coordinate. To represent the **GPK** in the form of Weber- Sonien integral formula, we use the following three formulas, see Bateman and Ergelyi [27],

$$\int_{-\pi}^{\pi}\frac{\cos m\vartheta d\vartheta}{\left[1-2z\rho\cos\vartheta+\rho^2\right]^\nu}=\frac{2\pi(\nu)_m z^m}{m}\ {}_2F_1(\nu,\nu+\nu,1+m,z^2),\left((\nu)_m=\frac{\Gamma(\nu+m)}{\Gamma(\nu)}\right),$$

$${}_2F_1\left(\alpha,\alpha+\tfrac{1}{2}-\beta;\beta+\tfrac{1}{2};z^2\right)=(1+z)^{-2\alpha}\ {}_2F_1\left(\alpha,\beta;2\beta;\frac{4z}{(1+z^2)^2}\right),$$

$$\int_0^\infty J_n(ax)J_n(bx)x^{-\ell}dx=\frac{(ab)^n 2^{-\ell}\Gamma(n+\frac{1-\ell}{2})}{(a+b)^{2n+\ell+1}\Gamma(1+n)\Gamma(\frac{1+\ell}{2})}\ {}_2F_1(n+\tfrac{1-\ell}{2},n+\tfrac{1}{2};2n+1;\frac{4ab}{(a+b)^2}),$$

Hence, **FIE** of Eq. (3.2) yields

$$\mu X_m(r)-\lambda\int K_m^\nu(r,\rho)X_m(\rho)d\rho=g_m(r);\left(X_m(\rho)=\sqrt{\rho}\,\phi_m(\rho),\ \sqrt{r}f_m(r)=g_m(r)\right), \tag{3.3}$$

With general kernel

$$K_m^\nu(r,\rho)=c\sqrt{r\rho}\int_0^\infty u^{2\nu-1}J_m(ru)J_m(\rho u)du,\quad c=\frac{\pi\Gamma(1-\nu)\,2^{(1-2\nu)}}{\Gamma(1+\nu)}. \tag{3.4}$$

Here, $\Gamma(n)$ is the gamma function, $(\nu)_m$ is called the Pochammer symbol, $J_m(ru)$ is the Bessel function and ${}_2F_1(a,b;c;z)$ is the Gauss hypergeometric function. The **GPK** of Eq. (3.4) called Weber- Sonien integral formula.

3.1- **Special cases and discussion:** Many special cases can derive from the **GPK** of (3.4)

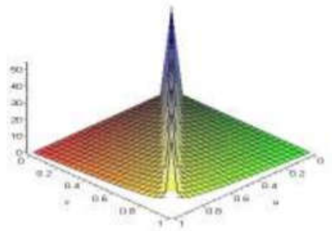
1. Logarithmic kernel: Let in (3.4), $\nu = 0.5;\ m = \pm 0.5$ , to have

$$K^{0}_{\pm\frac{1}{2}}(r,\rho) = 2\pi\sqrt{r\rho}\int_0^\infty J_{\pm\frac{1}{2}}(ru)J_{\pm\frac{1}{2}}(\rho u)du = \ln|r-\rho|.$$

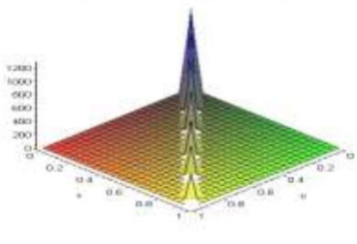
Many problems in the continuum mechanics, and in the axisymmetric contact problems in the theory of elasticity, lead to integral equation of the second kind with logarithmic kernel, see Aleksandrov, Covalence [28]. In addition, many problems in semi symmetric contact problem and mathematical physics lead to **FIE** of the first kind with logarithmic kernel. The solution of such problems, using orthogonal polynomial method and potential theory method in [29] and [30] respectively, obtained in the form of spectral relationships, i.e in the linear combination form of eigenvalues and eigenfunctions of the problem. Carleman kernel, see Figs. (1-2): Let in Eq. (3.4) the harmonic order $m = \pm 0.5$ to have

$$K^{\nu}_{\pm\frac{1}{2}}(r,\rho) = c\sqrt{r\rho}\int_0^\infty u^{2\nu-1}J_{\pm\frac{1}{2}}(ru)J_{\pm\frac{1}{2}}(\rho u)du = |r-\rho|^{-\nu};(0<\nu<1).$$

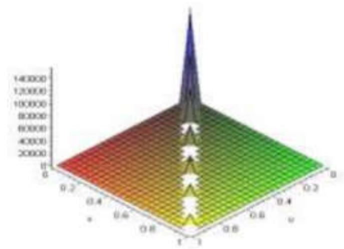
The importance of Carleman kernel comes from the work of Arutiunion [31], which has shown that the first approximation of the nonlinear theory of plasticity leads to **FIE** with Carleman kernel.



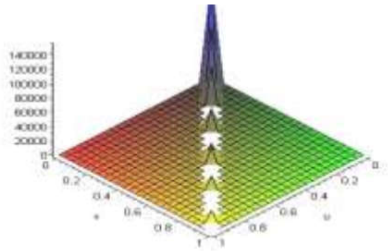
$(1a:\ m = 0.5, \nu = 0.33)$



$(1b: m = 0.5, \nu = 0.55)$



$(1c: m = 0.5, \nu = 0.87)$



$(1d: m = 0.5, \nu = 0.93)$

(**Fig.1** Contains the shape of Carleman kernel for $m$ =0.5 and different values of $V$.) From the previous figures of Carleman function, we deduced that as $V$ increases the cracks in the elastic material increase.

(3) Elliptic kernel: Let, in (3.4), $m = 0,\ \nu = 0.5$, we have the elliptic kernel

$$K^{\frac{1}{2}}_{0}(r,\rho) = \pi\sqrt{r\rho}\int_0^\infty J_0(ur)J_0(u\rho)du = E\left(\frac{\sqrt{r\rho}}{r+\rho}\right)$$

The importance of the elliptic kernel comes from the work of Covalence [2], who developed the **FIE** of the first kind for the mechanics mixed problem of continuous media and obtained an approximate solution of it.

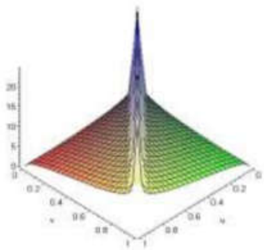


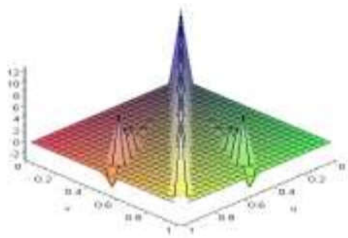
Fig. (2)

(**Fig.2** Contains the shape of elliptic kernel.)

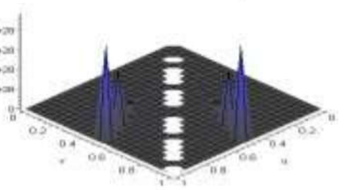
(4) Potential kernel: Let, in Eq. (3.4), $\nu$=0.5, we have the potential kernel

$$K^{\frac{1}{2}}_{m}(r,\rho) = 2\pi\sqrt{r\rho}\int_0^\infty J_m(ur)J_m(u\rho)du.$$

The potential kernel of the integral equation is investigated from the semi- symmetric Hertz problem of two different materials in three dimensions, see Abdou [32,33].



$(3.1: m = 50, \nu = 0.5)$

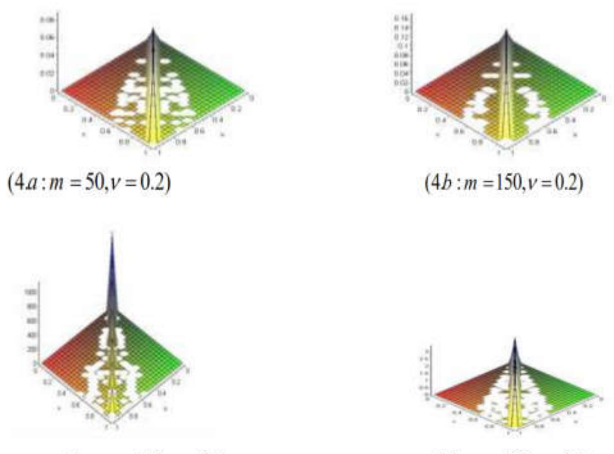


$(3b: m = 150, \nu = 0.5)$

(**Fig.3** Contains the shape of potential kernel for different values of m.)

The above Figures (3) contain two ships of the potential kernel for different harmonic m, we see that as the harmonic m increases the cracks increases.

(5) **General cases:** From the **GPK** of Eq. (3.4) we consider the following:



$(4a: m=50, v=0.2)$ $(4b: m=150, v=0.2)$

$(4c: m=150, v=0.7)$ $(4d: m=120, v=0.4)$

(**Fig.4** contains the shape of potential kernel for different values of m,ν.)

***The structure of the kernel***

***Theorem1:*** The structure of the kernel (3.4) represents Cauchy problem for the first derivatives order and wave equation for the derivatives of second order.

***Proof:*** Differentiate the kernel of Eq. (3.4) with respect to $r$ and $\rho$ respectively, then after using the properties of Bessel function, we get

$$\{\frac{\partial}{\partial r}+\frac{\partial}{\partial \rho}-(a_m(r)+a_m(\rho))\}K^v_{m,m}(r,\rho)=(K^v_{m-1,m}(r,\rho)+K^v_{m,m-1}(r,\rho))$$

$$a_m(x)=(\frac{1}{2x}-\frac{m}{x}),\quad K^v_{m,m}(r,\rho)=K^v_m(r,\rho);\quad K^v_{m-1,m}(r,\rho)=c\sqrt{r\rho}\int_0^\infty u^{2v-1}J_{m-1}(ru)J_m(\rho u)du \tag{3.5}$$

The formula (3.5) represents Cauchy problem in the nonhomogeneous case.

The second derivatives lead us to the following

$$\left(\frac{\partial^2}{\partial r^2}-\frac{\partial^2}{\partial \rho^2}\right)K^v_m(r,\rho)=[h_m(r)-h_m(\rho)]K^v_m(r,\rho),\quad [h_m(x)=(m^2-0.25)x^{-2}; m\neq \pm 0.5]., \tag{3.6}$$

The above formula represents a nonhomogeneous wave equation. So we can decide that the second derivative of the **GPK** of Eq. (3.4) represents a nonhomogeneous wave equation except at m=±0.5 we have a homogeneous wave equation.

The resolvent kernel of Eq. (3.4, takes the following form:

$$\mu R^v_{m,m}(u,v;\alpha)=K^v_{m,m}(u,v)+\lambda\int_0^1 K^v_{m,m}(u,s)\,R^v_{m,m}(s,v;\alpha)\,ds;((I-\alpha K)(I+R)=I). \tag{3.7}$$

$$\mu R^v_{m,m}(u,v;\alpha)=K^v_{m,m}(u,v)+\lambda\int_0^1 R^v_{m,m}(u,s;\alpha)\,K^v_{m,m}(s,v)\,ds;((I+R)(I-\alpha K)=I). \tag{3.8}$$

***The structure kernel of an infinite elastic plate weakened by a curvilinear hole***

It is known that, see Muskhelishvili [4], the first and second fundamental problems, in the plane theory of elasticity, are equivalent to finding two analytic functions $\phi_1(z)$ and $\psi 1(z)$ of one complex argument $z=x+iy, i=\sqrt{-1}$. These analytic potential functions, Gaursat functions, must satisfy the boundary conditions

$$k\,\varphi_1(t)-t\,\overline{\varphi_1'(t)}-\overline{\psi_1(t)}=f(t);\quad k=\chi=\frac{\lambda+3\mu}{\lambda+\mu}>1. \tag{4.1}$$

Where, $k=-1$ and $f(t)$ is a given function of stress for the first fundamental problems. While $f(t)=2\mu g(t)$ is a given function of the displacement for the second fundamental problems; $\lambda$, $\mu$ are called the Lame's constants and $t$ denotes the affix of a point on the boundary. The two complex potential functions $\phi_1(z), \psi_1(z)$ in this case, take the forms

$$\varphi_1(z)=-\frac{X+iY}{2\pi(1+\chi)}\ln\zeta+c\Gamma\zeta+\varphi(\zeta),\quad \psi_1(z)=\frac{\chi(X-iY)}{2\pi(1+\chi)}\ln\zeta+c\Gamma^*\zeta+\psi(\zeta). \tag{4.2}$$

Where $X$, $Y$ are the components of the resultant vector of all external forces acting on the boundary and $\Gamma, \Gamma^*$ are complex constants. The two complex functions $\phi(\zeta)$ and $\psi(\zeta)$ are single valued analytic functions within the region outside the unit circle and $\phi(\infty)=\psi(\infty)=0$. For the **FFP** we have $X=Y=0$ and $\Gamma=\Gamma$.

In the absence of body forces, Muskhelishvili [4] considered the stress components in the plane theory of elasticity in the form $\sigma_{xx}+\sigma_{yy}=4\,\mathrm{Re}\{\varphi'(z)\}, \quad \sigma_{yy}-\sigma_{xx}+2i\,\sigma_{xy}=2[\bar{z}\,\varphi''(z)+\psi'(z)]$

Using the conformal mapping function $z=c\omega(\zeta),\ c>0$, such that $\omega'(\zeta)$ does not vanish or become infinite, we deduce that the formula (4.2) leads us to an **IDE** in the complex plane, as the following:

$$\varphi(\zeta)+\frac{1}{2\pi i}\int_\gamma L(\sigma,\zeta)\,\overline{\varphi'(\sigma)}\,d\sigma=A(\zeta), \tag{4.3}$$

$$A(\zeta)=F(\zeta)-ck\Gamma\zeta+\frac{c\overline{\Gamma^*}}{\zeta}+N(\zeta)\,\omega(\zeta),\ N(\zeta)=c\,\overline{\Gamma}-\frac{X-iY}{2\pi(1+\chi)}\zeta;\quad F(\zeta)=f(t).$$

Here, the kernel of the **IDE** (4.3) takes the form

$$L(\sigma,\zeta)=\frac{1}{\overline{\omega'(\sigma)}}\left[\frac{\omega(\sigma)-\omega(\zeta)}{\sigma-\zeta}\right]. \tag{4.4}$$

We write the formula (4.3) in the form of **FIE, in** the complex plane as:

$$X(\zeta)+\frac{1}{2\pi i}\int_\gamma K(\sigma,\zeta)\,\overline{X(\sigma)}\,d\sigma=H(\zeta),$$
$$X(\zeta)=\phi'(\zeta);\quad H(\zeta)=A'(\zeta)-k\,\omega'(\zeta). \tag{4.5}$$

The kernel of the **FIE,** in this case, is given in the form

$$K(\sigma,\zeta)=\frac{1}{\overline{\omega'(\sigma)}}\frac{\partial}{\partial\zeta}\left[\frac{\omega(\sigma)-\omega(\zeta)}{\sigma-\zeta}\right]=\frac{\partial L(\sigma,\zeta)}{\partial\zeta} \tag{4.6}$$

From the above, we deduce that Eq. (4.3) which represents the first and second fundamental problems takes the form of an **IDE** with singular kernel L ($\sigma$, $\zeta$). In addition, its equivalent formula (4.5) represents **FIE** with singular kernel K($\sigma$, $\zeta$) given by (4.6). The structure resolvent of the first and second partial derivatives of the kernel (4.4), respectively is given in the following forms:

$$(\frac{\partial}{\partial\zeta}+\frac{\partial}{\partial\sigma})L(\sigma,\zeta)=\frac{1}{\omega'(\sigma)}\left[\frac{\omega'(\sigma)-\omega'(\zeta)}{\sigma-\zeta}\right], \tag{4.7}$$

$$(\frac{\partial^2}{\partial\zeta^2}-\frac{\partial^2}{\partial\sigma^2})L(\sigma,\zeta)=\frac{1}{\omega'(\sigma)}\left[2\frac{\omega'(\sigma)-\omega'(\zeta)}{(\sigma-\zeta)^2}-\frac{\omega''(\sigma)+\omega''(\zeta)}{\sigma-\zeta}\right]. \tag{4.8}$$

In addition, the first and second structure resolvent of Fredholm kernel of Eq. (4.6), are

$$(\frac{\partial}{\partial\zeta}+\frac{\partial}{\partial\sigma})K(\sigma,\zeta)=\frac{1}{\omega'(\sigma)}\frac{\partial}{\partial\zeta}\left[\frac{\omega'(\sigma)-\omega'(\zeta)}{\sigma-\zeta}\right], \tag{4.9}$$

$$(\frac{\partial^2}{\partial\zeta^2}-\frac{\partial^2}{\partial\sigma^2})K(\sigma,\zeta)=\frac{1}{\omega'(\sigma)}\frac{\partial}{\partial\zeta}\left[2\frac{\omega'(\sigma)-\omega'(\zeta)}{(\sigma-\zeta)^2}-\frac{\omega''(\sigma)+\omega''(\zeta)}{\sigma-\zeta}\right]. \tag{4.10}$$

***Some Applications: The first and second structure of (4.4) and (4.6)***

In this section, the complex potential functions for an infinite elastic plate weakened by a curvilinear hole, using some different conformal mapping functions, are determined. The first and second structure the kernels of **IDE** and **FIE**, in each case, are computed.

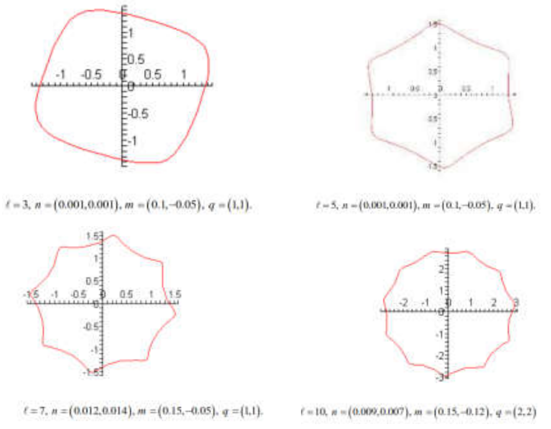
***Application1***: Consider the conformal mapping with complex coefficients

$$z=c\omega(\zeta)=\frac{q\zeta+m\zeta^{-\ell}}{1-n\zeta^{-\ell}};\quad \ell=1,2,...,N;(q,m,n\ \text{are comple cons tants}). \tag{4.11}$$

From the rational mapping, we can discuss the following

1. The rational mapping has (l+1) corners.
2. The shapes of the holes depending on the values of q's, *n*'s and *m*'s.
3. Entering none zero values of the complex constants *m* and *d* never gives symmetric graphs. While, entering zero values for all imaginary parts of both *m* and *d*, we get symmetric shapes around the *x*-axis. On the other hand, entering zero values for all real parts of both *m* and *d,* we get symmetric shape around the *y*-axis.
4. The complex constant $m = m_1 + im_2$ works on circling the shape from the symmetry situation and the circling angleθ. Positive values of θ means that the circling will be in the positive direction i.e. in the anti-clockwise direction and for negative values the circling will be in the negative direction i.e. in clockwise direction.

The physical interest of the mapping (4.11) comes from its special cases and its different shapes of holes, see Figs. (5).




(**Fig.5**: Contains different shapes of the conformal mapping of Eq.(4.11))

Write the conformal mapping in the form:

$$\frac{\omega(\zeta)}{\omega'(\zeta^{-1})}=\alpha(\zeta)+\overline{\beta(\zeta)};\quad \alpha(\zeta)=\frac{h}{\zeta^{\ell}-n};\quad h=\frac{(d\,n^{\nu}+m)(1-n\bar n)^{\ell}}{\bar d\left[1-n\bar n(1+\ell)-l\bar m\,n^{\nu}\right]},\left(\nu=1+\frac{1}{\ell}\right). \tag{4.12}$$

The two complex potential functions, Gaursat functions, take the form

$$-k\phi(\zeta)=A(\zeta)-\frac{d\,\Gamma^*}{\zeta}-\frac{h}{\zeta^{\ell}-n}\left(b+N\left(n^{\nu-1}\right)\right),$$

$$\psi(\zeta)=\frac{k\bar d\,\bar\Gamma}{\zeta}-\frac{\omega(\zeta^{-1})}{\omega'(\zeta)}\varphi_*(\zeta)+\frac{\bar h\,\zeta^{\ell}}{1-\bar n\,\zeta^{\ell}}\varphi_*\left(n^{1-\nu}\right)+B(\zeta)-B_*, \tag{4.13}$$

In (4.13), we assumed

$$\psi_*(\zeta)=\psi(\zeta)+\beta(\zeta)\phi'_*;\quad N(\zeta)=\bar d\,\bar\Gamma-\frac{X-iY}{2\pi(1+\chi)}\zeta;\quad A(\zeta)=\frac{1}{2\pi i}\sum_{j=0}^{\infty}\zeta^{-(j+1)}\int_{\gamma}\sigma^j F(\sigma)\,d\sigma,$$

$$B(\zeta)=\frac{1}{2\pi i}\int_{\gamma}\frac{\overline{F(\sigma)}}{\sigma-\zeta}d\sigma,\quad \frac{1}{2\pi i}\int_{\gamma}\frac{\overline{F(\sigma)}}{\sigma}d\sigma;\quad \phi_*(\zeta)=\phi'(\zeta)+\bar N \tag{4.14}$$

The free function F(ζ)with its derivatives must satisfy the Holder condition.

Here, the behavior kernel of **IDE** of Eq. (4.4) and the behavior kernel of **FIE** of Eq. (4.6) using the conformal mapping (4.11) are shown in Figs. (6) and (7), respectively. In addition, we consider in Fig. (8), the first structure of the partial derivatives of (4.7) of **IDE.** In Fig. (9), we compute the first structure of the first partial derivatives of (4.9) of **FIE.** Moreover, the second structure of the second partial derivatives of (4.8) and (4.10) are computed, respectively in Figs. (10) and (11). The above results is computed for the values

($\ell=2$, $q=1+i$, $n=0.1+0.1i$ and $m=0.2-0.2i$ ).

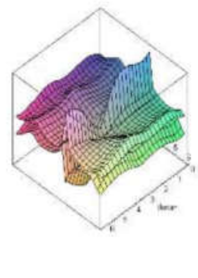


Fig. (6) kernel of **IDE** of (4.4)

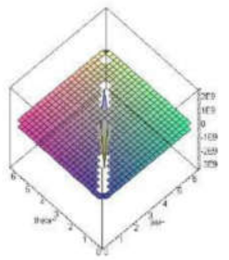


Fig.(7) kernel of **FIE** of (4.6)

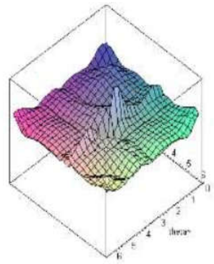


Fig. (8) **StFPDE** of **IDE**

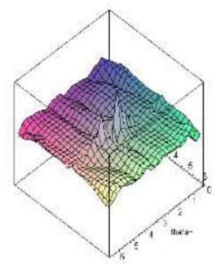


Fig.(9) **StFPDE** of **FIE**

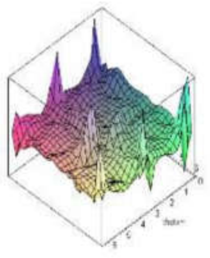


Fig. (10) **StSPDE** of **IDE**

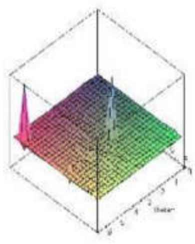


Fig. (11) **StSPDE** of **FIE**

**Application2:** *For the conformal mapping*

$$z = c\,\frac{\zeta + m\zeta^{-\ell}}{1 - n\zeta^{-q}},\ (c = 2,\ \ell = 3,\ q = 2,\ n = 0.1 \text{ and } m = -0.01) \quad (4.15)$$

The kernels of Eq. (4.4) and of Eq. (4.6) with the aid of conformal mapping (4.15) are computed, respectively in Figs. (12) - (13). Also, the first structure of Eqs. (4.7), (4.9) is considered, respectively in Figs. (14) - (15). Moreover, the second structures of (4.8), (4.10) are computed, respectively in Figs. (16) - (17).

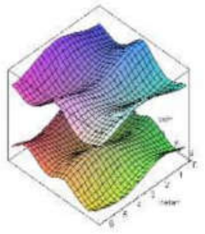


Fig. (12) kernel of **IDE**

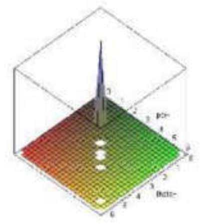


Fig.(13) kernel of **FIE**

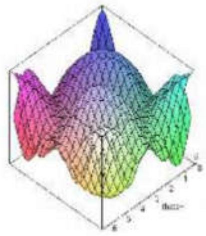


Fig. (14) **StFPDE** of **IDE**

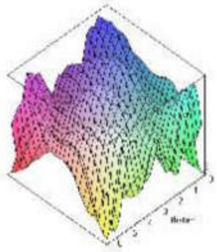


Fig.(15) **StFPDE** of **FIE**

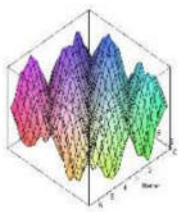


Fig. (16) **StSPDE** of **IDE**

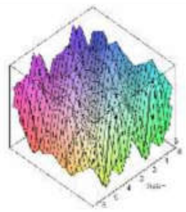


Fig. (17) **StSPDE** of **FIE**

## CONCLUSION

From the previous work, we can establish the following:

In the semi symmetric or in the axisymmetric contact and mixed problems, in the theory of elasticity and mathematical physics problems, when the modules of elasticity changes according to the power law $\sigma_i = K_0\,\varepsilon_i^{\nu},\ i = 1,2,3,\ (0 \le \nu < 1)$, we have an integral equation with a kernel in the form of generalized potential function. This kernel takes the form of Weber- Sonien integral formula

$$K_m^{\nu}(r,\rho) = c\sqrt{r\rho}\int_0^{\infty} u^{2\nu-1} J_m(ru) J_m(\rho u)\,du,\quad c = \frac{\pi\,\Gamma(1-\nu)\,2^{(1-2\nu)}}{\Gamma(1+\nu)}.$$

In general, we can write the above kernel in the Legendre polynomial form to have.

$$K_m^{\nu}(r,\rho) = c\,2^{-2w^-}(r\rho)^{m+\frac{1}{2}}\sum_{n=0}^{\infty}\frac{\Gamma^2(n+m+1-w^-)P_n^m(r)P_n^m(\rho)}{\Gamma^2(n+m+1).(2n+m+1-w^-)^{-1}};\quad w^{\pm} = \tfrac{1}{2}(1\pm\nu).$$

Many special cases can be derived from the generalized kernel: a-Logarithmic kernel $(\nu = 0.5,\ m = \pm 0.5)$ b- Carleman kernel $(m = \pm 0.5)$ c- Elliptic kernel: $(m = 0,\ \nu = 0.5)$, d- Potential kernel ($\nu = 1/2$). In addition, different degree of harmonic oscillator for positive and negative values of $m$ can be derived and discussed.

The structure of the generalized kernel represents Cauchy problem for the first order of derivatives. Also, it represents a nonhomogeneous wave equation for the second order of derivatives under the condition $m \neq \pm 0.5$ and homogeneous wave equation if $m = \pm 0.5$.

In the plane theory of elasticity, the first and second fundamental problems or the first and second boundary value problems, for an infinite plate weakened by a curvilinear hole, after using suitable conformal mapping, leads to an **IDE** of the second kind with Cauchy kernel. In addition, the same problem leads to **FIE** with discontinuous kernel. The structure resolvent of the first and second order of partial derivatives, for the **IDE** and **FIE** is hold and computed.

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