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Faisal B. M

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RESEARCH ARTICLE

ANALYTICAL CONSTITUTIVE MODEL FOR STRESS - STRAIN CURVES OF PMMA AT DIFFERENT STRAIN RATES

Faisal B. M

Department of Mechanical Engineering - University of Wasit

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ABSTRACT

A simple equation has been proposed to describe the secant part of the stress-strain curves. This equation based on the stress and strain at the peak point. The unknown parameters of this equation were calculated by applying some conditions at three main points on the stress - strain curve. For the case of uniaxial compression loading of polymethelmethacrylate (PMMA) at different strain rates, it was found that the stress at the peak point and the strain at the beginning of the work hardening are related to the strain rate as exponential functions, whereas the inflection stress is linearly related to the peak stress. The equation proposed here was found to correlate well experimental data.

INTRODUCTION

Most deformational behaviour of thermoplastic under mechanical loads depends to a great extent on time. This influence of time makes the dimensioning of plastic components considerably more complicated than with other materials. Although there has been much development in the theoretical aspects of the behaviour of polymers, experimental investigations have in general been limited to uniaxial tests.

Leterrier *et al.*[1] studied the viscoelastic behaviour of PUR at different strain rates and temperatures and modelled it in terms of a three-parameter constitutive equation whose general form was derived from Kohlrausch relaxation law. Kitagawa *et al.* [2] investigated the stress-strain behaviour of PE at different strain paths using the overstress theory of Krempl. Arruda *et al.*[3] studied the effect of strain rate, temperature on the inelastic response of a glassy PMMA using a fully 3D constitutive model of a large strain inelastic response of glassy polymers. Li and Lambros[4] studied the thermomechanical behavior of (PMMA and PC) under compression dynamic load over strain rates of 0.0001-0.001 s^{-1}. Wu *et al.*[5]used a quasi-static conditions in tensile tests at intermediate strain rate to investigate experimentally the tensile properties of (PMM). Richeton *et al.*[6] used the three dimensional constitutive model based on an elastic-viscoplastic rheolgical approach to describe the mechanical response of (PC and PMMA) over a wide range of temperatures and strain rates. Zaïri *et al.*[7] experimentally studied the stress-strain curve of (RT-PMMA) under uniaxial compression tests at different strain rates and then they used the constitutive model of Boyce-Socrate-Llana to describe this curves. Zhang *et al.*[8]investigated experimentally the mechanical properties of PVB by quasi-static tensile tests at strain rates of 0.008-0.317 s^{-1} and high-speed tensile test at strain rates 8.7-1360 s^{-1}.

A goal in this work is to discuss some experimental stress-strain curves of polymethelmethacrylate (PMMA) at comparatively large strain under uniaxial compression by proposing a new constitutive relationship.

Stress-Strain Curves

All viscoelastic solids are linear for sufficiently small strains, and there are always deviations from linearity. Generally, polymers exhibit a properties of linear viscoelastic behaviour at low stresses where strain below (0.2- 0.5)%. Hook's law can describe the linear range where the strain is linearly related to the applied stress. The experimental stress-strain curves of PMMA at different strain rates given by Arruda *et al.*[3] are shown in Fig. 1. From this figure it can be noted that the modulus changes due to the non-linearity. This modulus which called the secant modulus (E_s) depends on the stress and strain rate and is (0.85) of the initial tangent modulus[9]. The secant modulus decreases with the loading until reaches zero at the peak point. Figure 1 also shows that when the strain is 15% there is an inflection point on the descending part that characterised by the necking phenomenon. Then, the work hardening occurs when the stress tends to be constant.

**Corresponding author: Faisal B. M*
Department of Mechanical Engineering - University of Wasit

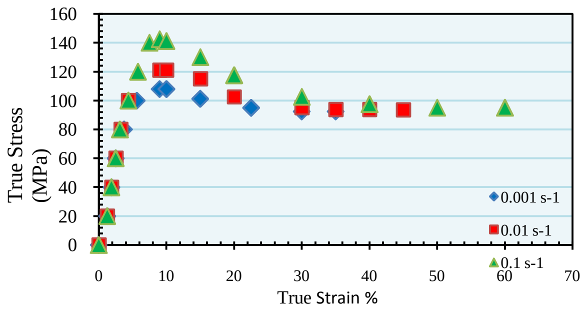

Fig. 1 Experimental uniaxial comprssion stress-strain curves for PMMA at different strain rates. [3]

According to the above notes, it can be concluded that there are five key points for the stress-strain curves as shown in Fig. 1. These points are:

1. The stress and strain at initial point.
2. The stress and strain at the secant point.
3. The stress and strain at the peak point.
4. The stress and strain at the inflection point.
5. The stress and strain at the beginning of the work hardening.

For the ascending part of these curves, the following simple equation can be proposed to describe the relation between the stress and strain:

$$Y = \frac{Ax}{1 + Bx + Cx^2} \qquad \ldots (1)$$

where:

$$Y = \frac{\sigma}{\sigma_p} \; , \; X = \frac{\varepsilon}{\varepsilon_p}$$

σ and ε are the stress and strain at any choosen point on the stress-strain curve.

σ_p and ε_p are the stress and strain at the peak point.

A, B and C are constants.

Based on the experimental data the peak stress σ_p vs. strain rate $\dot{\varepsilon}$ is shown in Fig. 2, which can be fitted to the following relation:

$$\sigma_p = 1 - a_0 e^{b_0 \dot{\varepsilon}} \qquad \ldots (2)$$

where a_0 is constant (MPa).

b_0 is constant (S).

But for the descending part the stress at the inflection point σ_i vs. peak stress σ_p is shown in Fig. 3. This curve can be fitted to the relation of the form:

$$\sigma_i = a_1 + b_1 \sigma_p \qquad \ldots (3)$$

where a_1 is constant (MPa).

b_1 is constant.

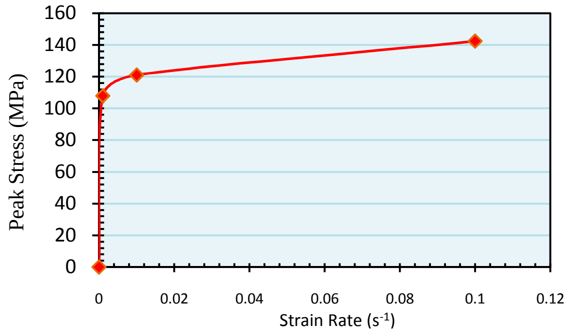

Fig. 2 The relation between the peak stress and the strain rate.

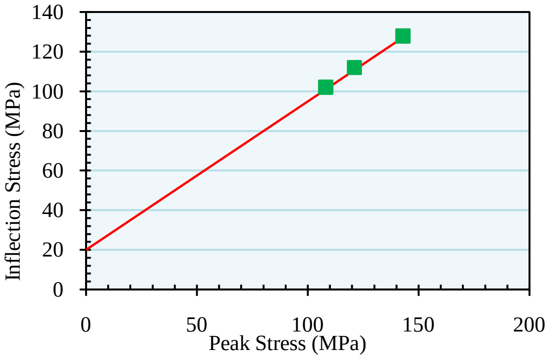

Fig. 3 The relation between the peak stress and inflection stress.

When the true stress tends to be constant, the true strain vs. strain rate is shown in Fig. 4 that can be described by the following relation:

$$\varepsilon_h = 1 - a_2 e^{b_2 \dot{\varepsilon}} \qquad \ldots (4)$$

where a_2 is constant.

b_2 is constant (S).

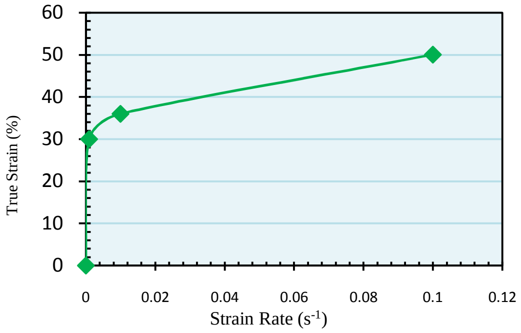

Fig. 4 The relation between the strain rate and the true strain at the beginning of the work hardening.

Evaluation of the Constants

The values of the three constants A, B and C in Eq.(1) can be obtained using the following conditions:

1. $\frac{dY}{dX}=\frac{E_0}{E_p}$ at X= 0 and Y= 0, then:

$$A=\frac{E_0}{E_p} \quad \dots (5)$$

where E_0 is the initial tangent modulus of elasticity.

2. At the peak point, X=1and Y=1, then:

$$1=\frac{A}{1+B+C}$$

And

$$B=A-1-C \quad \dots (6)$$

3. $\frac{dY}{dX}=1$ at X=1 and Y=1, then:

$$1=\frac{(1+B+C)A-A(B+2C)}{(1+B+C)^2} \quad \dots (7)$$

by inserting Eq. (6) in Eq. (7):

$$C=1-A \quad \dots (8)$$

4. At the secant point $E_s=0.85E_0$,where X=0.5, then:

$$\frac{dY}{dX}=\frac{E_s}{E_p}$$

$$\frac{0.85E_0}{E_p}=\frac{(1+0.5B+0.25C)A-0.5A(B+C)}{(1+0.5B+0.25C)^2} \quad \dots (9)$$

substituting Eq. (8) in Eq. (9) yields:

$$B=0.5(A-5)+1.0846522\sqrt{3+A} \quad \dots (10)$$

RESULTS AND DISCUSSION

At the peak point for the case of uniaxial compression loading of PMMA, the peak stress is highly effected by strain rate. The constants in Eq. (2) that describes the relation between them can be obtained by using the least squares method. Then Eq. (2) becomes:

$$\sigma_p=1+111.54622e^{2.4579316\dot{\varepsilon}} \quad \dots (11)$$

Substituting the values of strain rates (0.001, 0.01 and 0.1 s^{-1}) in Eq. (11) to calculate the peak stress. Table (1) shows the values of the theoretical and experimental[3] peak stress.

Table 1 Experimental and theoretical peak stress at different strain rates.

Strain Rate (s^{-1})	Experimental peak stress (MPa)	Theoretical peak stress (MPa)
0.001	108	112.82
0.01	121	115.32
0.1	142.5	143.63

Table 2 The values of the constant of equation (1)

Strain Rate (s^{-1})	A	B	C
0.001	1.75	-3.9889448	-0.75
0.01	1.5619835	-4.0356954	-0.5619835
0.1	1.3263158	-4.0928969	-1.3263158

For the ascending parts of the curves, the constants in Eq. (1) are calculated by substituting the theoretical values of the peak stress in Eqs. (5), (8) and (9). From these equations, it can be noted that these constants are functions of strain rate since the initial modulus (E_0 = 2.1GPa[10]) and the strain at peak points are constants. Table (2) shows the values of these constants at strain rates of 0.001, 0.01 and 0.1 s^{-1}. The values of these constants which given in Table (1) are also shown in Figs. 5, 6 and 7. Figs 5 and 6 show that the values of A and B are decreased as strain rate increased respectively, while Fig. 7 shows that the constant C is increased as strain rate increased.

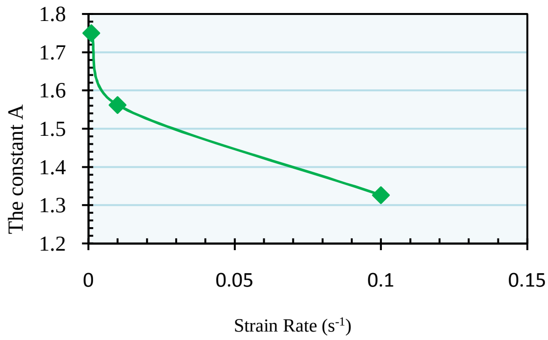

Fig. 5 The varation of the constant A with the strain rate.

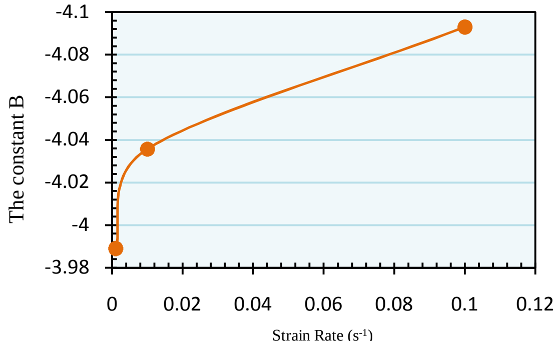

Fig. 6 The varation of the constant B with strain rate.

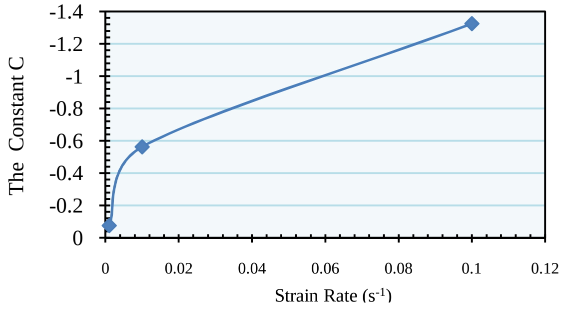

Fig. 7 The varation of the cinstant C with the strain rate.

By substituting the values of A, B and C for each values of strain rate, the ascending part of the stress-strain curves of PMMA can be predict.

Also, the relation between the peak stress and the inflection stress (where the strain tends to be about 15%) are shown in Fig. 3. From this figure it can be noted that the inflection stress is linearly related to the peak stress that it also function of strain rate. Thus equation (3) becomes:

$$\sigma_i = 20 + 0.75\sigma_p \qquad \dots (12)$$

When the work hardening occurs the true stress tends to be constant, while the true strain is highly related to the strain rate as shown in Fig. 4. By using least square method, equation (4) becomes:

$$\varepsilon_h = 1 - 0.6865141 e^{-3.328273\dot{\varepsilon}} \qquad \dots (13)$$

The inflection stress and the strain at the beginning of the work hardening can be obtained by substituting the values of the peak stress and the values of strain rates in Eqs. (12) and (13) respectively. Then, the stress-strain behaviour of PMMA at different strain rates can be obtained by using Eqs. (11), (12) and (13). Figs. 8, 9 and 10 show the comparison between the theoretical and experimental results[3] at strain rates of (0.001, 0.01 and 0.1 s^{-1}) respectively. From these figures it can be noted that Eq. (1) together with Eqs. (11), (12) and (13) gave a good prediction to the true stress - strain behaviour Of PMMA with an average error of (1.05, 1.23 and 0.9) % for the strain rates of (0.001, 0.01 and 0.1 s^{-1}) respectively.

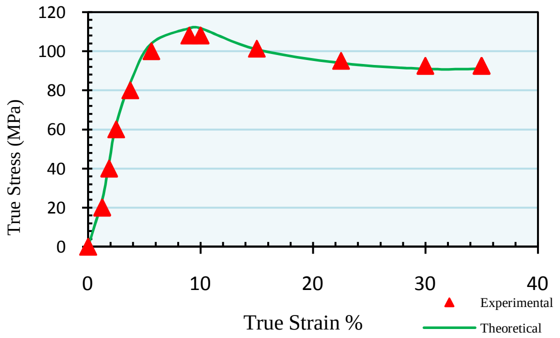

Fig. 8 Uniaxial compression stress-strain curve of PMMA at strain rate of 0.001 s^{-1}.

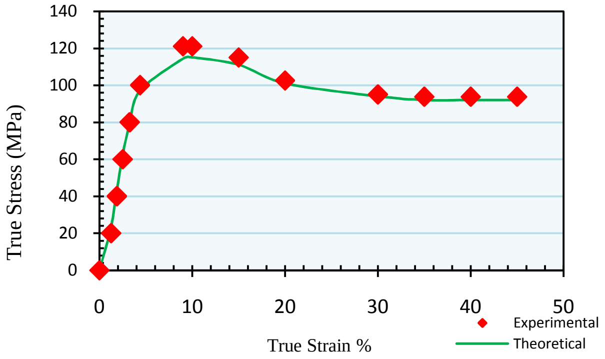

Fig. 9 Uniaxial compression stress-strain curve of PMMA at strain rate of 0.01 s^{-1}.

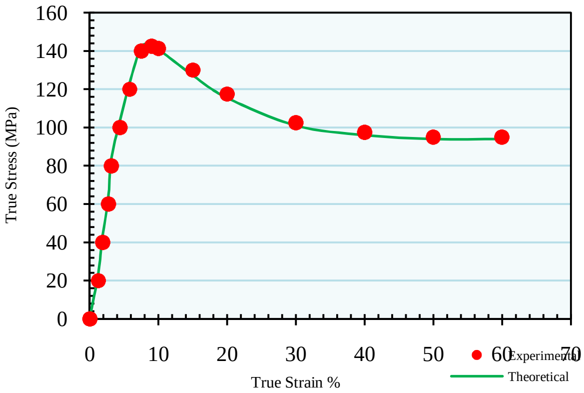

Fig. 10 Uniaxial compression stress-strain curve of PMMA at strain rate of 0.1 s^{-1}.

CONCLUSIONS

From the above discussion, the following conclusions can be drawn:

1. The effect of strain rate on the true stress- strain curve of PMMA under uniaxial compression loading was investigated by proposing a new simple equation. Three main points are required to calculated the constants (A, B and C) of this equation which described the secant part only.
2. All constants of the proposing equation are functions of strain rate.
3. The descant part of stress-strain curves is adequately described by using three main points from which the peak point was a co-point for both the secant and descant parts.
4. At the peak point and the beginning of work hardening, the peak stress and the true strain were related to the strain rate in exponential form, while the inflection stress was linearly related to the peak stress.
5. The proposing equations provide a good description for stress-strain curves with an average total error of (1.06 %).

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