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**RESEARCH ARTICLE**

**ON BIANCHI TYPE –III STRANGE QUARK MASSIVE STRING UNIVERSE IN GENERAL THEORY OF RELATIVITY**

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**ABSTRACT**

Bianchi type –III strange quark universe is investigated in general theory of relativity. We have obtained exact solutions by assuming that the expansion  $\mu$  in the model is proportional to the shear  $\dagger$  which leads to  $C = A^n$  where A and C are functions of time t only. The physical and geometrical behaviours of universe are discussed.

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**INTRODUCTION**

Space-times admitting a three parameter group of automorphisms are important in discussing the cosmological models. The case where the group is simply transitive over the 3-dimensional and useful for two reasons. First, Bianchi has shown that there are only nine distinct sets of structure constants for groups of this type. Therefore, we can use algebra to classify the homogeneous Space -times. The second reason for the importance of Bianchi type Space -times is the simplicity of the field equations.

When we study the Bianchi type models, we observe that the models contain isotropic special cases and they permit arbitrarily small anisotropic levels at some instant of cosmic times.

Bianchi type cosmological models are important in the sense that these models are homogeneous and anisotropic, from

which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view, anisotropic universe has a greater generality than isotropic models. The simplicity of the field equations made Bianchi space time useful in constructing models of spatially homogeneous and anisotropic cosmologies.

It is still a challenging problem before us to know the exact physical situation at very early stages of the formation of our universe. The string theory is a useful concept before the creation of the particle in the universe. The strings are nothing but an important topological stable defect due to the phase transition that occurs as the temperature below some critical temperature at the very early stages of the universe.

The present day configurations of the universe are not contradicted by the large scale network of strings in the early universe. Moreover, the galaxy formation can be explained by the density fluctuations of the vacuum strings.

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Lorenz [1] has presented Tilted electromagnetic Bianchi type III cosmological solution. Tikekar and Patel [2] obtained some exact solutions of massive string of Bianchi type –III space time presence and absence of magnetic field. Bali and Jain [3] have studied Bianchi type –III non-static magnetized cosmological model for perfect fluid distribution in general relativity. Amirhaschi H. Zainuddin [4-5] have obtained Bianchi type III string cosmological models for perfect fluid distribution and also studied magnetized massive string with bianchi type III in general relativity. Pradhan [6] have presented Massive string cosmology in Bianchi type III space-time with electromagnetic field. Pradhan and Amirhaschi H. Zainuddin [7] have studied Dark energy model in anisotropic Bianchi type –III space time with variable EoS parameter. Adhav [8] have obtained Bianchi type –III magnetized wet dark fluid cosmological model in general relativity. Upadhaya and Dave [9] have investigated some magnetized Bianchi type –III Massive string Cosmological models in general relativity. S. P. Kandalkar, S. W. Samdurkar, S. P. Gawande [10] have obtained Bianchi type –III string cosmological models in the presence of magnetic field in general relativity, G. S. Rathore, K. Mandawat, D. S. Chauhan [11] have investigated Bianchi type –III string cosmological models with bulk viscosity and electromagnetic field. S. D. Deo, G. S. Punwatkar, U. M. Patil [13] also studied Bianchi type III cosmological model electromagnetic field with cosmic string in general theory of relativity. S. D. Deo, G. S. Punwatkar, U. M. Patil [14] have studied LRS Bianchi type III Universe with magnetized wet dark fluid in general theory of relativity.

In this paper, we have investigated Bianchi type –III Strange quark string cosmological Universe in General theory of relativity. To obtained exact solutions of field equations, we have considered massive strings. The physical and geometrical behaviour of the model are also discussed.

**The Metric and Field Equations**

Here, we consider the spatially homogeneous and anisotropic Bianchi type –III metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2ax} dy^2 + C^2 dz^2 \tag{1}$$

Where  $a$  is non -zero constant and A, B and C are function of t only.

$T_i^j$  is energy momentum tensor for the matter cosmic cloud string [Letelier,1979,[12]] is given by

$$T_i^j = \dots u_i u^j - \} x_i x^j \tag{2}$$

Where, ... is the rest energy density for cloud of string with particle attached along the extension.

$$\dots = \dots_p + \} \tag{3}$$

Where,  $\dots_p$  is the particle energy,  $\}$  is the tension density of

the string,  $u^i$  is the flow vector of matter and  $x^i$  represents the direction of anisotropy is z-axis.

$$u_4 u^4 = -1, x_3 x^3 = 1 \text{ and } u^i x_i = 0$$

As we know that string is free to vibrate, the different vibrating modes of the string represent different types of particles because these different modes are seen as different masses. Therefore here we will take quark instead of particles in the string cloud. Here we consider strange quark matter energy density instead particle energy density in the string cloud.

$$\dots = \dots_q + \} + B_c \tag{4}$$

We have energy momentum tensor of strange quark matter attach to the string cloud as

$$T_i^j = (\dots_q + \} + B_c) u_i u^j - \} x_i x^j \tag{5}$$

The Einstein field equation in the general relativity is given by

$$R_i^j - \frac{1}{2} R g_i^j = -8f k T_i^j \tag{6}$$

Where,  $R_i^j$  is known as Ricci tensor and  $R = g^{ij} R_{ij}$  is the Ricci scalar and  $T_i^j$  is energy momentum tensor for matter.

The field equations (6) together with the line element (1) with equations (5) we get

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = 0 \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = 0 \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{a^2}{A^2} = 8f G \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{a^2}{A^2} = 8f G \dots \tag{10}$$

$$a \left( \frac{\dot{B}}{B} - \frac{\dot{A}}{A} \right) = 0 \tag{11}$$

From equation (11) we get

$$\frac{\dot{B}}{B} = \frac{\dot{A}}{A} \tag{12}$$

Integrating,(12) have

$$B = kA \tag{13}$$

Where,  $k$  is a constant of integration

As we consider  $B = A$ , by taking  $k = 1$   
The field equations (07) to (10) reduce to

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = 0 \tag{14}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{a^2}{A^2} = 8fG \tag{15}$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{C}}{AC} - \frac{a^2}{A^2} = 8fG \dots \tag{16}$$

**Solution of the Field Equations**

We assume that the expansion is proportional to the shear which is physical conditions. This condition leads to

$$C = A^n \tag{1}$$

Where  $n$  is a constant

**Case-I for Massive strings**

We have  $\dots + \} = 0$

From equation (15), (16) and (2), we have

$$\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{\dot{A}\dot{C}}{AC} - \frac{a^2}{A^2} = 0 \tag{3}$$

Using equation (1), the above equation reduce to

$$\frac{\ddot{A}}{A} + (n+1)\frac{\dot{A}^2}{A^2} = \frac{a^2}{A^2} \tag{4}$$

Now put  $\dot{A} = g(A)$  in equation (4), we get

$$\frac{d}{dA}(g^2) + 2\frac{(n+1)}{A}g^2 = 2\frac{a^2}{A} \tag{5}$$

Integrating equation (5), we get

$$g^2 = \left(\frac{dA}{dt}\right)^2 = \frac{a^2}{n+1} + k_1 A^{-2n-2} \tag{6}$$

Where  $k_1$  is integration constant

The Bianchi type -III model in this case reduces to the form

$$ds^2 = -\left(\frac{dt}{dA}\right)^2 dA^2 + A^2 [dx^2 + e^{-2ax} dy^2] + A^{2n} dz^2 \tag{7}$$

Without loss of generality, if we choose  $A=T, x=X, y=Y, z=Z$  then equation (7) becomes

$$ds^2 = -\left[\frac{dT^2}{\frac{a^2}{n+1} + k_1 T^{-2n-2}}\right] + T^2 [dX^2 + e^{-2aX} dY^2] + T^{2n} dZ^2 \tag{8}$$

**Physical and Geometrical Behaviour of the Model**

The rest energy density  $(\dots)$  and string tension density  $(\})$  for the model (8), we have

$$\dots = \frac{1}{8fG} \left[ \frac{na^2}{(n+1)T^2} + \frac{(2n+1)k_1}{T^{2(n+2)}} \right] \tag{9}$$

$$\} = -\frac{1}{8fG} \left[ \frac{na^2}{(n+1)T^2} + \frac{(2n+1)k_1}{T^{2(n+2)}} \right] \tag{10}$$

Using (4) we get

$$\dots_q = \frac{1}{8fG} \left[ \frac{2na^2}{(n+1)T^2} + \frac{2(2n+1)k_1}{T^{2(n+2)}} \right] - B_c \tag{11}$$

And expansion scalar  $\mu$  and shear scalar  $\dagger$  is defined as

$$\mu = (n+2) \left[ \frac{a^2}{(n+1)T^2} + \frac{k_1}{T^{2(n+2)}} \right]^{\frac{1}{2}} \tag{12}$$

$$\dagger^2 = \frac{(n-1)^2}{3} \left[ \frac{a^2}{(n+1)T^2} + \frac{k_1}{T^{2(n+2)}} \right] \tag{13}$$

Where

$$\frac{\dagger^2}{\mu^2} = \frac{(n-1)^2}{3(n+2)^2} = \text{constant} \tag{14}$$

The particle energy  $\dots_p$  and tension density  $\}$  of the cloud string vanish asymptotically in general if  $(n+2)>0$ . The expansion of the model stops when  $n=-2$ . The model starts expanding with a big bang at  $T=0$  and the expansion in the model decreases as time increases if  $(n+2)>0$ . Since  $\lim_{t \rightarrow \infty} \frac{\dagger}{\mu} \neq 0$ , the model does not approach isotropy for large values of  $t$ .

**Case II for n=1**

When n=1 in equation (1), is given by

$$C = A \tag{15}$$

From equation (14) and (15), we get

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = 0 \tag{16}$$

Now put  $\dot{A} = f(A)$  in equation (16), we get

$$\frac{d}{dA}(g^2) + \frac{2g^2}{A} = 0 \tag{17}$$

Integrating equation (17), we get

$$g^2 = \left(\frac{dA}{dt}\right)^2 = \frac{k_2}{A^2} \tag{18}$$

Where  $k_2$  is integration constant

The Bianchi type –III model in this case reduces to the form

$$ds^2 = -\left(\frac{dt}{dA}\right)^2 dA^2 + A^2 [dx^2 + e^{-2ax} dy^2 + dz^2] \tag{19}$$

By choice of coordinate transformation A=T, x=X, y=Y, z=Z we get

$$ds^2 = -\left(\frac{T^2}{k_2}\right) dT^2 + T^2 [dX^2 + e^{-2aX} dY^2 + dZ^2] \tag{20}$$

**Physical and Geometrical Behaviour of the Model**

The rest energy density ( ... ) and string tension density ( } ) for the model (20), we have

$$\dots = \frac{1}{8fG} \left[ \frac{3k_2 - T^2 a^2}{T^4} \right] \tag{21}$$

$$\} = -\frac{1}{8fG} \left[ \frac{3k_2 - T^2 a^2}{T^4} \right] \tag{22}$$

Using (4) we get

$$\dots_q = \frac{1}{8fG} \left[ \frac{6k_2 - 2T^2 a^2}{T^4} \right] - B_c \tag{23}$$

Scalar expansion  $\mu$  and shear  $\dagger$  is obtained as

$$\mu = 3 \left[ \frac{k_2}{T^4} \right]^{1/2} \tag{24}$$

$$\dagger^2 = 0 \tag{25}$$

Where

$$\frac{\dagger^2}{\mu^2} = 0 \tag{26}$$

**CONCLUSION**

In the present study, we have investigated the effect of strange quark matter attach to the string cloud in Bianchi type III Universe. Einstein’s field equations have been solved exactly suitable physical and kinematical parameter. In order to solve field equations we have used more general equations of state for the proper rest energy density and string tension density.

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