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RESEARCH ARTICLE

ON SEMI TOPOLOGICAL GROUPS WITH RESPECT TO IRRESOLUTENESS

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ABSTRACT

In this study, we investigate some properties of semi topological groups with respect to irresoluteness defined [6].We show that if $(G,*,\tau)$ is a semi topological groups with respect to irresoluteness then $(G,*,\tau^1)$ is also semi topological groups with respect to irresoluteness. Later we prove that every semi-open subgroup of semi topological groups with respect to irresoluteness is semi-closed.

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INTRODUCTION

In [13], Levini defined a semi-open set in a topological space as a set A such that there exists an open set such that $O \subset A \subset Cl(O)$ and showed that a set A is semi-open if and only if $A \subset Cl(Int(A))$. He also showed that arbitrary union of semi-open sets is semi-open and intersection of two semi-open sets may not be semi-open. But that it was proved by Croosley and Hildebrand ([16]) the intersection of a semi-open set and an open set is semi open.

In [16], Crossley and Hildebrand defined semi-closed sets, semi-closure and semi-interior in a manner analogous to corresponding concepts of closed sets, closure and interior and showed that A is semi-open if and only if sInt(A)=A and A is semi-closed if and only if sCl(A)=A.

A mapping $f: X \rightarrow Y$ between topological spaces X and Y is called:

- Semi-continuous (resp. irresolute [9]) if for each open (resp, semi-open) set $V \subset Y$, the set $f^{-1}(V)$ is semi-open in X ([13]). Equivalently, the mapping f is ([14]) semi-continuous (resp. irresolute ([15])) if for each $x \in X$ and for each open (semi-open) neighborhood V of f(x) there exists a semi-open neighborhood U of x such that $f(U) \subset V$;
- pre-semi-open if for every semi-open set A of X, f(A) is semi-open in Y ([9]);
- semi-homeomorphism if f is bijective, irresolute and pre - semi-open ([9]);

In the literature, there are different generalization of topological group and semi topological groups by using semi continuity and irresoluteness (see [1,2,3,4,5,6,7,18]).By replacing the continuity in the definition of semi topological groups ([12]) with irresoluteness, authors introduced the notions of semi topological groups with respect to irresoluteness as follows:

A semi-topological group with respect to irresoluteness $(G,*,\tau)$ is a group $(G.*)$ endowed with a topology τ such that the left translations , the right translations and the symmetry map are irresolute. Later properites of these spaces invastigated in [5]

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and [7]. In this study, we investigate some properties of semi topological groups with respect to irresoluteness defined in [6].We show that if $(G,*,\tau)$ is a semi topological groups with respect to irresoluteness then $(G,*,\tau^{-1})$ is a also semi topological groups with respect to irresoluteness. Later we prove that every semi-open subgroup of semi topological groups with respect to irresoluteness is semi-closed.

RESULTS

Lemma 1 ([9])

If $f: X\rightarrow Y$ is a semi-homeomorphism, then
(1) $sCl(f(A))=f(sCl(A))$ for all $A\subset X$;
(2) $sInt(f(A))=f(sInt(A))$ for all $A\subset X$.

Lemma 2

Let $(G,*,\tau)$ be semi topological groups with respect to irresoluteness and $A\subset G$. Then $(sCl(A))^{-1}=sCl(A^{-1})$ and $(sInt(A))^{-1}=sInt(A^{-1})$

Proof

Since i is semi-homeomorphism for a subset A of G, we have $(sCl(A))^{-1}=sCl(A^{-1})$ and $(sInt(A))^{-1}=sInt(A^{-1})$ by Lemma 1.

Theorem 3

([5]) Let (G,τ) be a topological spaces. If U is semi-open (G,τ), then U^{-1} is semi-open (G,τ^{-1}).

Theorem 4

Let $(G,*,\tau)$ be a semi topological groups with respect to irresoluteness. Then $(G, *, \tau^{-1})$ is also semi topological groups with respect to irresoluteness where $\tau^{-1}=\{U^{-1}:U\in\tau\}$ is the conjugate topology of G.

Proof

By Theorem 3, (G,τ^{-1}) is a topological spaces. We need to show that $I:(G,\tau^{-1})\rightarrow(G,\tau^{-1})$, $I(x)=x^{-1}$, ${}_gL:(G,\tau^{-1})\rightarrow(G,\tau^{-1})$, ${}_gL(x)=g*x$ and $R_g:(G,\tau^{-1})\rightarrow(G,\tau^{-1})$, $R_g(x)=x*g$ are irresolute.
For ${}_gL$: Let V be a semi-open in (G,τ^{-1}). Then V^{-1} is semi-open in (G,τ). Since $r_{g^{-1}}$ is irresolute, $r_{g^{-1}}^{-1}(V^{-1})=V^{-1}*g$ is a semi-open in (G,τ) that is $(V^{-1}*g)^{-1}=g^{-1}*V$ is semi-open in (G,τ^{-1}). This implies ${}_gL^{-1}(V)=g^{-1}*V$ is semi-open in (G,τ^{-1}). Hence ${}_gL$ is irresolute. Similarly we can prove that R_g is irresolute.
For I (x): Let V be a semi-open in (G, τ^{-1}). Then V^{-1} is semi-open in (G,τ). Since i is irresolute, $i^{-1}(V^{-1})=V$ is semi-open in (G,τ) that is V^{-1} is semi-open in (G,τ^{-1}). This implies $I^{-1}(V)=V^{-1}$ is semi-open in (G,τ^{-1}). Hence I is irresolute.

Theorem 5

Let $(G,*,\tau)$ be a semi topological groups with respect to irresoluteness, then every semi-open subgroup H of G is also semi-closed.

Proof

Since all ${}_gl$ are semi-homeomorphism and H is semi-open, for all $g\in G$, $g*H$ is semi-open. Hence $Y=\bigcup_{g\in G-H} g*H$ is semi-open and H=G-Y is semi-closed.

Definition 6

A topological spaces is said to be semi-homogeneous if for all $x,y\in G$, there exists a semi- homeomorphism f such that f(x)=y.

Corollary 7

Every semi topological groups with respect to irresoluteness $(G,*,\tau)$ is semi-homogeneous.

Proof

For $x,y\in G$, choose $z=x^{-1}*y$. Since r_z is semi-homeomorphism, we have $r_z(x) = x*z=x*x^{-1}*y=y$. Therefore $(G,*,\tau)$ is semi-homogeneous.

Theorem 8

Every subgroup H of a semi topological groups with respect to irresoluteness $(G,*,\tau)$ is also a semi topological groups with respect to irresoluteness.

Proof

Let ${}_al,r_a,i:(G,\tau)\rightarrow(G,\tau)$ semi-homeomorphisms of $(G,*,\tau)$. Since restrictions ${}_al|_H$, $r_a|_H$, $i|_H:(H,\tau_H)\rightarrow(H, \tau_H)$ are also semi-homeomorphism, $(H,*, \tau_H)$ is a semi topological groups with respect to irresoluteness.

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