Available Online at http://www.recentscientific.com

ISSN: 0976-3031

International Journal of Recent Scientific Research
Vol. 6, Issue, 6, pp.4418-4420, June, 2015

International Journal of Recent Scientific Research

RESEARCH ARTICLE

AN ALTERNATIVE METHOD FOR SOLVING LINEAR FRACTIONAL PROGRAMMING PROBLEMS

Kirtiwant P. Ghadle and Tanaji S. Pawar

Department of Mathematics, Dr. Babasaheb Ambedkar Marathwada University, Aurangabad, Maharashtra, India

ARTICLE INFO

Article History:

Received 14th, May, 2015
Received in revised form 23rd, May, 2015
Accepted 13th, June, 2015
Published online 28th, June, 2015

Key words:

Linear fractional programming problem, Alternative method, Simplex method, Optimal solution

ABSTRACT

Linear fractional programming problem (LFPP) is a problem in which the objective function is a linear fractional function, while the constraint functions are in the form of linear inequalities. LFPP are useful tools in production planning, financial and corporate planning, health care and hospital planning. In this paper, an alternative method for solving LFPP is introduced. This can be illustrated with the help of some numerical examples. It is powerful method to reduce number of iterations and save valuable time.

INTRODUCTION

Linear programming problem (LPP) is a mathematical technique to find optimal solution (maximum or minimum) of a problem subject to certain constraints (Bajalinov, 2013), while a linear fractional programming (LFPP) problem is one whose objective function is linear fractional function.
In general LFPP be:

$$\text{Maximize } Z = \frac{Cx+\alpha}{C'x+\beta}$$

Subject to the constraints: $Ax = b, x \geq 0$. In this denominator is positive for all the assumptions.

The linear fractional programming problems (LFPP) have great importance in non-linear programming. LFPP are very useful in production planning, financial and corporate planning, health care and hospital planning. Several methods to solve this problem have been proposed (Bitran *et al.*, 1973). (Isbell *et al.*, 1956) first discussed an example of fractional programming problem. In military, programming games have been form when troops are in the field and decision is taken to be how to distribute the fire among several possible types of targets. However for a single objective linear fractional programming can be used to transform the problem into a linear programming problem (Charnes *et al.*, 1962). (Odior, 2012) solved the LFP problem by algebraic approach which depends on the duality concept and the partial fractions. (Jayalakshmi, 2012) have proposed a method namely, bound and decomposition method to a fully fuzzy linear programming (FFLP) problem to obtain an optimal fuzzy solution. (Nachammai *et al.*, 2012) considered FFLFP problem by using ranking method based on metric distance. Resently (Tantawy, 2007, 2008) has suggested a feasible direction approach and a duality approach to solve a linear fractional programming problem.

In this paper, an attempt has been made to solve linear fractional programming problem (LFPP) by an alternative method. This method is different from Isbell, Tantawy, Charnes and Cooper, Pandian *et al*. method.

An Alternative Algorithm For Linear Fractional Programming Problem

To find optimal solution of any LFPP by an alternative method, algorithm is given as follows:

Corresponding author:* **Kirtiwant P. Ghadle
Department of Obstetrics & Gynaecology, Kasturba Medical College, Mangalore, Manipal University, Karnataka, India

Step 1 Check objective function of LPP is of maximization type or minimization. If it is to be minimization type then convert it into a maximization type by using the result:

$$\text{Min. } Z = -\text{ Max.}(-Z).$$

Step 2 Check whether all b_i (RHS) are non-negative. If any b_i is negative then multiply the corresponding equation of the constraints by -1.

Step 3 Express the given LPP in standard form then obtain an initial basic feasible solution.

Step 4 Choose greatest ratio of coefficients in objective function.

1. If greatest ratio of coefficients is unique, then variable corresponding to this column is incoming variable.
2. If greatest ratio of coefficients is not unique, then use tie breaking technique.

Step 5 Compute the ratios with X_B. Choose minimum ratio, then variable corresponding to this row is outgoing variable. The element corresponding to incoming variable and outgoing variable becomes pivotal (leading) element.

Step 6 Use usual simplex method for this table and go to next step.

Step 7 Find net evaluations Δ_j for each variables x_j by the formula:

$$\Delta_j = \frac{z^1}{z^2} \text{ where } z^1 = (C_B x_B + \alpha) - C_j \text{ and } z^2 = (C'_B x_B + \beta) - C'_j.$$

Step 8 If all $\Delta_j \geq 0$, current solution is an optimal solution, otherwise go to step 6. Thus optimum solution is obtained and which is optimum solution of given LFPP.

Solved Problems

Problem 3.1 Solve the following linear fractional programming problem

Maximize $Z = \frac{5x_1+3x_2}{5x_1+2x_2+1}$

Subject to: $3x_1 + 5x_2 \leq 15$ $5x_1 + 2x_2 \leq 10$ $x_1, x_2 \geq 0.$

Solution: LPP is in standard form

Maximize $Z = \frac{5x_1+3x_2}{5x_1+2x_2+1}$

Subject to: $3x_1 + 5x_2 + s_1 = 15$
$5x_1 + 2x_2 + s_2 = 10$
$x_1, x_2, s_1, s_2 \geq 0$

where s_1, s_2 are slack variables.

By solving above problem, we obtain the following final simplex table:

Simplex table

				5	**3**	**0**	**0**	
				5	2	0	0	
[illegible]	[illegible]	BVS	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3	2	[illegible]	3	3/5	1	1/5	0	
0	0	[illegible]	4	19/5	0	-2/5	1	
			[illegible]	-16/5	0	3/5	0	
			[illegible]	-14/5	1	7/5	1	
		Z=9/7	[illegible]	1.14	0	0.42	0	

Since all $\Delta_j \geq 0$, current solution is an optimum solution. Therefore optimum solution is:

$x_1 = 0,\ x_2 = 3.$ Max. $Z = \frac{9}{7}$.

Problem 3.2 Solve the following LFPP:

Maximize $Z = \frac{6x_1+5x_2}{2x_1+7}$

Subject to: $x_1 + 2x_2 \leq 3$
$3x_1 + 2x_2 \leq 6$
$x_1, x_2 \geq 0.$

Solution: LPP is in standard form:

Maximize $Z = \frac{6x_1+5x_2}{2x_1+7}$

Subject to: $x_1 + 2x_2 + s_1 = 3$
$3x_1 + x_2 + s_2 = 6$
$x_1, x_2, s_1, s_2 \geq 0$

where s_1, s_2 are slack variables.

By solving above problem, we obtain the following final simplex table

Simplex table

				6	**5**	**0**	**0**	
				2	0	0	0	
[illegible]	[illegible]	BVS	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5	0	[illegible]	3/4	0	1	3/4	-1/4	
6	2	[illegible]	3/2	1	0	-1/2	1/2	
		51/4	[illegible]	0	0	3/4	7/4	
		10	[illegible]	7	7	6	8	
		Z=51/40	[illegible]	0	0	0.125	0.218	

Since all $\Delta_j \geq 0$, current solution is an optimum solution. Therefore optimum solution is:

$x_1 = \frac{3}{2},\ x_2 = \frac{3}{4}.$ Max. $Z = \frac{51}{40}$.

Problem 3.3 Solve the following linear fractional programming problem

Maximize $Z = \frac{x_1+3x_2+2x_3}{2x_1+x_2+4x_3+1}$

Subject to: $x_1 + 3x_2 + 6x_3 \leq 8$
$2x_1 + x_2 + 4x_3 \leq 5$
$x_1, x_2, x_3 \geq 0.$

Solution: LPP is in standard form:

Maximize $Z = \frac{x_1+3x_2+2x_3}{2x_1+x_2+4x_3+1}$

Subject to: $x_1 + 3x_2 + 6x_3 + s_1 = 8$

$$2x_1 + x_2 + 4x_3 + s_2 = 5$$

$$x_1,\ x_2, x_3, s_1, \quad s_2 \geq 0$$

where $s_1,\ s_2$ are slack variables.

By solving above problem, we obtain the following final simplex table

Simplex table

				1	**3**	**2**	**0**	**0**		
				2	1	4	0	0		
[illegible]	[illegible]	BVS	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3	1	[illegible]	8/3	1/3	1	2/3	1/3	0		
0	0	[illegible]	7/3	5/3	0	10/3	-1/3	1		
		8	[illegible]	0	0	0	1	0		
		11/3	[illegible]	-2/3	1	-7/3	4/3	1		
		Z=24/11	[illegible]	0	0	0	0.75	0		

Since all $\Delta_j \geq 0$, current solution is an optimum solution. Therefore optimum solution is: $x_1 = 0,\ x_2 = \frac{8}{3},\ x_3 = 0$. Max. $Z = \frac{24}{11}$.

CONCLUSIONS

An alternative method to obtain the solution of linear fractional programming problems has been derived. A number of algorithms have been developed, each applicable to specific type of LFPP only. Our approach is general purpose method for solving LFPP to reduce number of iterations by selecting pivot element and gives more efficiency in result.

The numbers of application of LFPP are very large and it is not possible to give a comprehensive survey of all of them.

However, an efficient method for the solution of general LFPP is still. This technique is useful to apply on numerical problems, reduces the labour work and save valuable time.

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How to cite this article:

Kirtiwant P. Ghadle and Tanaji S. Pawar., An Alternative Method For Solving Linear Fractional Programming Problems. *International Journal of Recent Scientific Research Vol. 6, Issue, 6, pp.4418-4420, June, 2015*

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