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RESEARCH ARTICLE

SOME MORE RESULTS ON SOFT PREOPEN SETS IN SOFT TOPOLOGY

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ABSTRACT

The aim of this paper is to define soft pre-neighbourhood, soft pre-frontier and soft pre-exterior and study their basic properties. Several important results relating soft pre-interior, soft pre-frontier and soft pre-exterior are established and characterized some results on soft preopen setsin soft topology. An attempt is made to arrive at further results on soft preopen sets.

INTRODUCTION

Soft set is parametrized general mathematical tools which deal with a collection of approximate descriptions of objects. In 1999, Russian researcher Molodtsov [14] introduced the concept of a soft set as a new approach for modeling uncertainties. In 2011, Shabir and Naz [15] initiated the study of soft topology. Cagmen *et al*[16] defined basic notions and concepts of soft topological spaces such as soft open and soft closd sets, soft interior, soft closure, soft basis, soft neighbourhood of a point, soft limit point of a soft set , soft difference and soft compliment. Also they established several properties of these notions. In 1982, A.S. Mashhour *et al* [11] have defined the notion of preopen sets in general topology. The concepts of preclosure and preinterior of a set are also due to A.S.Mashhour *et al*[12]. Navalagi[23] , in 2002, has defined preneighbourhoods, pre-interior point , pre-limit point, pre derived set and prefrontier of a set. G.Navalgi proved some results on preopen and preclosed sets. Also she defined pre-exterior of a set and studied some of its properties. Mrudula Ravindran[24] introduced soft preopen sets and proved some of its properties. In this paper further results on soft preopen sets and soft preclosed sets are characterized.

Preliminaries

Definition1 ([14])

Let U be an initial universe set and *E* be the set of parameters .Let P (U) denotes the power set of U and AN $\subset$ E. A pair (F, A) is called a *soft set over U, where F* is a mapping given by **F: A**$\rightarrow$*P (U).*
In other *words,* a soft set over U is a parametrized family *of subsets of the* universe U.

Definition 2([15])

Let τ be a collection of soft sets over a universe U and A be a non-empty subset
Of the set of parameters E, then τ is said to be a soft topology on U if

1. The null soft set (¤ ,A) and absolute soft set (U,A) belong to τ
2. The union of any number of soft sets in τ belongs to τ
3. The intersection of any two soft sets in τ belongs to τ.
4. The triplet (U,τ ,A) is called soft topological space over U. The members of τ are called soft open sets in U and complements of them are called soft closed sets in U.

Definition 3 (Cartesian product of two soft sets) [26]

Let (*F*, *A*) and (*G*,*B*) be two soft sets over a common universe *U* , then the cartesian product of these two soft sets is denoted by (*F*, *A*)×(*G*, *B*) and is defined by (*F*, *A*)×(*G*,*B*) = (*H*, *A*×*B*) where H(a,b) = *F*(a)×*G*(b).

Definition 4([22])

Let (U, A,²) be a soft topological space and let (G,A) be a soft set . Then

I. The soft closure of (G,A) is the soft set
II. $\tilde{s}cl(G,A) = \tilde{\bigcap}$ {(S,A): (S,A) is soft closed and (G,A) $\tilde{\subseteq}$ (S,A)
III. The soft interior of (G,A) is the soft set $\tilde{s}\mathrm{int}(G,A) = \tilde{\bigcup}$ {(S,A):(S,A) is soft open and (S,A) $\tilde{\subseteq}$ (G,A)}

Definition 5([24]) Soft preopen sets

In a soft topological space (U, A, τ), a soft set

I. (G, A) is said to be soft preopen set if (G, A) $\tilde{\subseteq} \tilde{s}\mathrm{int}(\tilde{s}cl(G,A))$
II. (F, A) is said to be soft preclosed set if (F, A) $\tilde{\supseteq} \tilde{s}cl(\tilde{s}\mathrm{int}(F,A))$

A soft preclosed set is nothing but the complement of a soft preopen set

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Definition 6(soft preinterior) ([24])

Let (U, A,^{2}) be a soft space over U. Then the soft preinterior of the soft set (F, A) over U is denoted by $\tilde{s}p\,\text{int}(F,A)$ and defind as the union of all the soft preopen sets contained in (F, A).

Definition 7(soft preclosure) ([24])

Let (U, A,^{2}) be a soft space over U. Then the soft preclosure of the soft set (F, A) over U is denoted by $\tilde{s}pcl(F,A)$ and defined as the intersection of all soft preclosed sets contained in (F, A).

Theorem 1 ([24])

Let (U, A,^{2}) be a soft topological space and (G,A) and (K,A) be two soft sets over U. Then,

(i) $(G,A)\tilde{\subseteq}(K,A)\Rightarrow \tilde{s}p\,\text{int}(G,A)\tilde{\subseteq}\tilde{s}p\,\text{int}(K,A)$

(ii) $(G,A)\tilde{\subseteq}(K,A)\Rightarrow \tilde{s}pcl(G,A)\tilde{\subseteq}\tilde{s}pcl(K,A)$

(iii) $\tilde{s}pcl((G,A)\tilde{\bigcup}(K,A))=\tilde{s}pcl(G,A)\tilde{\bigcup}\tilde{s}pcl(K,A)$

(iv) $\tilde{s}p\,\text{int}((G,A)\tilde{\bigcap}(K,A))=\tilde{s}p\,\text{int}(G,A)\tilde{\bigcap}\tilde{s}p\,\text{int}(K,A)$

(v) $\tilde{s}pcl((G,A)\tilde{\bigcap}(K,A))\tilde{\subset}\tilde{s}pcl(G,A)\tilde{\bigcap}\tilde{s}pcl(K,A)$

(vi) $\tilde{s}p\,\text{int}((G,A)\tilde{\bigcup}(K,A))\tilde{\supset}\tilde{s}p\,\text{int}(G,A)\tilde{\bigcup}\tilde{s}p\,\text{int}(K,A)$

Definition 8 (soft pre-neighbourhood)

In a soft topological space (U, A,^{2}), a soft set (F, A) is called a soft pre-neighbourhood of the soft point e_F $\tilde{\in}$ (U, A) if there exists a soft preopen set (H, A) such that e_F $\tilde{\in}$ (H,A) $\tilde{\subseteq}$ (F,A)

Theorem 2

For soft sets (F, A) and (G,A) over a common universe U we have

$\tilde{s}p\,\text{int}((F,A)-(G,A))\tilde{\subset}$
$\tilde{s}p\,\text{int}(F,A)-\tilde{s}p\,\text{int}(G,A)$

Proof

Let $e_F\tilde{\in}\tilde{s}p\,\text{int}((F,A)-(G,A))$ which implies that there exists a soft pre-neighbourhood (H,A) of e_F such that (H,A) $\tilde{\subseteq}$ (F,A) –(G,A) $\tilde{\subseteq}$ (F,A). From this we get (H, A)$\tilde{\bigcap}$(G,A) = (¤ ,A).

Hence $e_F\tilde{\notin}\tilde{s}p\,\text{int}(G,A)$.

Result 1

$\tilde{s}p\,\text{int}((F,A)-(G,A))\neq\tilde{s}p\,\text{int}(F,A)-\tilde{s}p\,\text{int}(G,A)$

Example 1 [24]

Let U = {a, b}, A = {e_1,e_2 }. Define

(F_1,A) = {(e_1,ϕ),(e_2 ,ϕ)}, (F_2,A) = {(e_1,ϕ),(e_2 ,{a})}, (F_3,A) = {(e_1,ϕ),(e_2 ,{b})}, (F_4,A) = {(e_1 ,ϕ),(e_2 ,{a, b})},

(F_5,A) = {(e_1,{a}),(e_2 ,ϕ)}, (F_6,A) = {(e_1,{a}),(e_2 ,{a})},

(F_7,A) = {(e_1,{a}),(e_2 ,{b})}, (F_8,A) = {(e_1,{a}),(e_2 ,{a, b})},

(F_9,A) = {(e_1,{b}),(e_2 , ϕ)}, (F_{10},A) = {(e_1,{b}),(e_2 ,{a})},

(F_{11},A) = {(e_1,{b}),(e_2 ,{b})}, (F_{12},A) = {(e_1,{b}),(e_2 ,{a, b})},

(F_{13},A) = {(e_1,{a,b}),(e_2 , ϕ)}, (F_{14},A) = {(e_1,{a,b}),(e_2 ,{a})},

(F_{15},A) = {(e_1,{a,b}),(e_2 , {b})}, (F_{16},A) = {(e_1 ,{a,b}),(e_2 ,{a,b})},

Are all soft sets on universal set U under the parameter set A.

τ = { (F_1,A), (F_5,A), (F_7,A), (F_8,A), (F_{16},A) } is a soft topology over U.

Soft preopen sets are (F_1,A), (F_5,A), (F_6,A), (F_7,A), (F_8,A), (F_{13},A), (F_{14},A), (F_{15},A), (F_{16},A)

Let (F,A) = (F_8,A) = $\{(e_1,\{a\}),(e_2,\{a,b\})\}$ and (G,A) = $(F_7,A)=\{(e_1,\{a\}),(e_2,\{b\})\}$.

(F,A) and (G,A) are soft preopen sets.

(F,A) – (G,A) = $\{(e_1,¤),(e_2,\{a\})\}$

$\tilde{s}p\,\text{int}((F,A)-(G,A))=\{(e_1,¤),(e_2,¤)\}$

$\tilde{s}p\,\text{int}(F,A)-\tilde{s}p\,\text{int}(G,A)=\{(e_1,\{a\}),(e_2,\{a,b\})\}-\{(e_1,\{a\}),(e_2\{b\})\}$

= $\{(e_1,¤),(e_2,\{a\})\}$

$\tilde{s}p\,\text{int}((F,A)-(G,A))\neq\tilde{s}p\,\text{int}(F,A)-\tilde{s}p\,\text{int}(G,A)$

Theorem 3

For soft sets (F,A) and (G,B) we have

$\tilde{s}cl((F,A)\times(G,B))=\tilde{s}cl(F,A)\times\tilde{s}cl(G,B)$

$\tilde{s}\text{in}\,t((F,A)\times(G,B))=\tilde{s}\text{in}\,t(F,A)\times\tilde{s}\text{in}\,t(G,B)$

Proof

Let H(a,b)$\tilde{\in}\tilde{S}$ cl((F,A)×(G,B)) .We will show that F(a) $\tilde{\in}\tilde{S}$ cl((F,A) and G(b)$\tilde{\in}\tilde{S}$ cl(G,B)

Let F(a) $\tilde{\in}$ (J,A) be soft closed in (U,A,^{2}). Since H(a,b) $\tilde{\in}$ (J,A)x(U,B) which is soft closed

(¤ ,A×B)≠ ((J,A)×(U,B))$\tilde{\bigcap}$ (F,A)×(G,B))

= ((J,A)$\tilde{\bigcap}$ (F,A))×((U,B)$\tilde{\bigcap}$ (G,B))

=((J,A)$\tilde{\bigcap}$ (F,A))×(G,B)

This implies that (J,A) $\tilde{\bigcap}$ (F,A)≠(¤ ,A) and F(a) $\tilde{\in}\tilde{S}$ cl((F,A) . Similarly we can show that

G(b)$\tilde{\in}\tilde{S}$ cl(G,B).

Hence $\tilde{s}cl((F,A)\times(G,B))\tilde{\subset}\tilde{s}cl(F,A)\times\tilde{s}cl(G,B)$

Let H(a,b) $\tilde{\in} \tilde{S}$ cl((F,A)×(G,B)) . That is F(a) $\tilde{\in} \tilde{S}$ cl((F,A) and G(b) $\tilde{\in} \tilde{S}$ cl(G,B)

If possible assume that H(a,b) $\tilde{\notin} \tilde{S}$ cl((F,A)×(G,B)) . Then there exists a soft open set

(U, $K_1 \times K_2$) containing H(a,b) and (U, $K_1 \times K_2$) $\tilde{\cap}$ (H,A×B) = (¤ ,($K_1 \times K_2$) $\tilde{\cap}$ (A×B))

ie (U $\tilde{\cap}$ H,($K_1 \times K_2$) $\tilde{\cap}$ (A×B)) = (¤ ,($K_1 \times K_2$) $\tilde{\cap}$ (A×B))

ie (U,($K_1 \tilde{\cap}$ A)×($K_2 \tilde{\cap}$ B)) = (¤ ,($K_1 \tilde{\cap}$ A)×($K_2 \tilde{\cap}$ B))

$\Rightarrow$ either (U,($K_1 \tilde{\cap}$ A)) =(¤ ,($K_1 \tilde{\cap}$ A) or (U, ($K_2 \tilde{\cap}$ B)) = (¤ ,($K_2 \tilde{\cap}$ B))

By the above theorem (U, K_1) is soft open and (U, K_2) is soft open. So either F(a) $\tilde{\notin} \tilde{S}$ cl(F,A) or G(b) $\tilde{\notin} \tilde{S}$ cl(G,B) which is a contradiction to the hypothesis. Hence H (a,b) $\tilde{\notin} \tilde{S}$ cl((F,A)×(G,B)).Therefore

$$\tilde{s}cl(F,A)\times\tilde{s}cl(G,B)\tilde{\subseteq}\tilde{s}cl((F,A)\times(G,B))$$

By similar arguments we can show

$$\tilde{s}\mathrm{int}((F,A)\times(G,B))=\tilde{s}\mathrm{int}(F,A)\times\tilde{s}\mathrm{int}(G,B)$$

Theorem 4

(F,A) and (G,A) are soft preopen subsets of soft topological space $(U,A,^{2})$ and $(U,B,^{2})$ respectively iff (F,A) × (G,A) = (H,A×B) where H(a,b) = F(a) × G(b) is soft preopen in $(U,A\times B,^{2})$.

Proof

(F,A) is soft preopen in $(U,A,^{2})$ then (F,A) $\tilde{\subseteq} \tilde{s}\mathrm{int}(\tilde{s}cl(F,A))$ and (G,B) is soft preopen in $(U,B,^{2})$ then (G,B) $\tilde{\subseteq} \tilde{s}\mathrm{int}(\tilde{s}cl(G,B))$

$\tilde{s}\mathrm{int}(\tilde{s}cl(F,A)\times(G,B)) = \tilde{s}\mathrm{int}(\tilde{s}cl(F,A)\times\tilde{s}cl(G,B))$

$= \tilde{s}\mathrm{int}(\tilde{s}cl(F,A))\times\tilde{s}\mathrm{int}(\tilde{s}cl(G,B))$

$\tilde{\supseteq}(F,A)\times(G,B)$ Hence (F,A)×(G,B) is soft preopen in $(U,A\times B,^{2})$

Conversely, if (F,A)×(G,B) is soft preopen in $(U,A\times B,^{2})$ then (F,A)×(G,B) $\tilde{\subseteq} \tilde{s}\mathrm{int}(\tilde{s}cl(F,A)\times(G,B))$

$= \tilde{s}\mathrm{int}(\tilde{s}cl(F,A)\times\tilde{s}cl(G,B))$

$= \tilde{s}\mathrm{int}(\tilde{s}cl(F,A))\times\tilde{s}\mathrm{int}(\tilde{s}cl(G,B))$

Then (F,A) $\tilde{\subseteq} \tilde{s}\mathrm{int}(\tilde{s}cl(F,A))$ and (G,B) $\tilde{\subseteq} \tilde{s}\mathrm{int}(\tilde{s}cl(G,B))$

(F,A) and (G,B) are soft preopen

Theorem 5

For soft sets (F,A) and (G,B) we have

$\tilde{s}pcl((F,A)\times(G,B)) = \tilde{s}pcl(F,A)\times\tilde{s}pcl(G,B)$ and

$\tilde{s}p\,\mathrm{int}((F,A)\times(G,B)) = \tilde{s}p\,\mathrm{int}(F,A)\times\tilde{s}p\,\mathrm{int}(G,B)$

Proof

Let H(a,b) $\tilde{\in} \tilde{S}$ pcl((F,A)×(G,B)) .We will show that F(a) $\tilde{\in} \tilde{S}$ pcl((F,A) and G(b) $\tilde{\in} \tilde{S}$ pcl(G,B)

Let F(a) $\tilde{\in}$ (J,A) be soft preopen in $(U,A,^{2})$. Since H(a,b) $\tilde{\in}$ (J,A)x(U,B) which is soft preopen

(¤ ,A×B) ≠ ((J,A)×(U,B)) $\tilde{\cap}$ ((F,A)×(G,B))

= ((J,A) $\tilde{\cap}$ (F,A))×((U,B) $\tilde{\cap}$ (G,B))

=((J,A) $\tilde{\cap}$ (F,A))×(G,B)

This implies that (J,A) $\tilde{\cap}$ (F,A) ≠ (¤ ,A) and F(a) $\tilde{\in} \tilde{S}$ pcl((F,A) . Similarly we can show that G(b) $\tilde{\in} \tilde{S}$ pcl(G,B). Hence

$$\tilde{s}pcl((F,A)\times(G,B))\tilde{\subseteq}\tilde{s}pcl(F,A)\times\tilde{s}pcl(G,B)$$

Let H(a,b) $\tilde{\in} \tilde{S}$ pcl((F,A)×(G,B)) . That is F(a) $\tilde{\in} \tilde{S}$ pcl((F,A) and G(b) $\tilde{\in} \tilde{S}$ pcl(G,B)

If possible assume that H(a,b) $\tilde{\notin} \tilde{S}$ pcl((F,A)×(G,B)) . Then there exists a soft preopen set

(U, $K_1 \times K_2$) containing H(a,b) and (U, $K_1 \times K_2$) $\tilde{\cap}$ (H,A×B) = (¤ ,($K_1 \times K_2$) $\tilde{\cap}$ (A×B))

ie (U $\tilde{\cap}$ H,($K_1 \times K_2$) $\tilde{\cap}$ (A×B)) = (¤ ,($K_1 \times K_2$) $\tilde{\cap}$ (A×B))

ie (U,($K_1 \tilde{\cap}$ A)×($K_2 \tilde{\cap}$ B)) = (¤ ,($K_1 \tilde{\cap}$ A)×($K_2 \tilde{\cap}$ B))

$\Rightarrow$ either (U,($K_1 \tilde{\cap}$ A)) =(¤ ,($K_1 \tilde{\cap}$ A) or (U, ($K_2 \tilde{\cap}$ B)) = (¤ ,($K_2 \tilde{\cap}$ B))

By the above theorem (U, K_1) is soft preopen and (U, K_2) is soft preopen. So either

F(a) $\tilde{\notin} \tilde{S}$ pcl(F,A) or G(b) $\tilde{\notin} \tilde{S}$ pcl(G,B) which is a contradiction to the hypothesis. Hence H(a,b) $\tilde{\notin} \tilde{S}$ pcl((F,A)×(G,B)) .Therefore

$$\tilde{s}pcl(F,A)\times\tilde{s}pcl(G,B)\tilde{\subseteq}\tilde{s}pcl((F,A)\times(G,B))$$

By similar arguments we can show

$$\tilde{s}p\,\mathrm{int}((F,A)\times(G,B))=\tilde{s}p\,\mathrm{int}(F,A)\times\tilde{s}p\,\mathrm{int}(G,B)$$

Soft pre-limitpoint

Definition 9

Let $(U,A,^{2})$ be a soft topological space. A soft element $F^{x}_{e_1} \in £$ is said to be a soft pre-limitpoint of a soft set (F,A) over U if every soft preopen set containing $F^{x}_{e_1}$ contains atleast one soft element of (F,A) other than $F^{x}_{e_1}$.

Remark 1

The point $F^{x}_{e_1}$ is said to be a soft prelimit point of (F,A) iff for each soft preopen set (H,A) containing $F^{x}_{e_1}$,

(H, A) $\tilde{\cap}$ ((F, A) - $F^{x}_{e_1}$) ≠ (¤ , A).

Definition 10

The set of all soft pre-limit points of (F,A) is said to be the soft prederived set of (F,A) and is denoted by $\tilde{s}pd$ (F,A). Also, (F,A) $\tilde{\bigcup}\tilde{s}$ pd(F,A) is soft preclosed iff $\tilde{s}pcl(F,A) = (F,A)\tilde{\bigcup}\tilde{s}pd(F,A)$

Theorem 6

Let {(F,A)$_{\sim}$: ~ ∈I} be any family of soft subsets of (U,A,²). If $\underset{\sim\in I}{\tilde{\bigcup}}\tilde{s}pcl$ (F,A)$_{\sim}$ is soft preclosed then $\underset{\sim\in I}{\tilde{\bigcup}}\tilde{s}pcl$ (F,A)$_{\sim}$ = $\tilde{s}pcl(\underset{\sim\in I}{\tilde{\bigcup}}(F,A)_{\sim})$

Proof

As (F,A)$_{\sim}$ $\tilde{\subseteq}\underset{\sim\in I}{\tilde{\bigcup}}$ (F,A)$_{\sim}$

Therefore $\tilde{s}pcl$ (F,A)$_{\sim}$ $\tilde{\subseteq}\tilde{s}pcl\underset{\sim\in I}{\tilde{\bigcup}}$ (F,A)$_{\sim}$

We will show that $\tilde{s}pcl(\underset{\sim\in I}{\tilde{\bigcup}}(F,A)_{\sim})\tilde{\subseteq}\underset{\sim\in I}{\tilde{\bigcup}}\tilde{s}pcl(F,A)_{\sim}$

Let $e_F\tilde{\in}\tilde{s}pcl\underset{\sim\in i}{\tilde{\bigcup}}(F,A)_{\sim}$

Now if possible let $e_F\tilde{\notin}\underset{\sim\in I}{\tilde{\bigcup}}\tilde{s}pcl(F,A)_{\sim}$.We have $\underset{\sim\in I}{\tilde{\bigcup}}\tilde{s}pcl(F,A)_{\sim}$ is soft preclosed. Therefore it contains all its soft pre-limit points and e_F is not a soft pre-limit point of $\underset{\sim\in I}{\tilde{\bigcup}}\tilde{s}pcl(F,A)_{\sim}$ and therefore there exists a soft pre-neighbourhood (H, A) of e_F such that (H,A) $\tilde{\bigcap}$ $\underset{\sim\in I}{\tilde{\bigcup}}\tilde{s}pcl(F,A)_{\sim}$ = (¤ ,A).

This implies that (H,A) $\tilde{\bigcap}\underset{\sim\in I}{\tilde{\bigcup}}\tilde{s}pcl(F,A)_{\sim}$ = (¤ ,A) for every α∈I.

Therefore (H, A) $\tilde{\bigcap}$(F, A)$_{\sim}$ = (¤ ,A) for every α∈I, a contradiction to $e_F\tilde{\in}\tilde{s}pcl\underset{\sim\in i}{\tilde{\bigcup}}(F,A)_{\sim}$.

Therefore $\tilde{s}pcl(\underset{\sim\in I}{\tilde{\bigcup}}(F,A)_{\sim})\tilde{\subseteq}\underset{\sim\in I}{\tilde{\bigcup}}\tilde{s}pcl(F,A)_{\sim}$ and hence the result.

Soft Pre-Frontier

Definition 11

The set $\tilde{s}pcl(F,A)-\tilde{s}p\operatorname{int}(F,A)$ is said to be the soft prefrontier of (F,A) and is denoted by $\tilde{s}pfr(F,A)$

Theorem 7

$\tilde{s}pfr(F,A) = \tilde{s}pcl(F,A)\tilde{\bigcap}\tilde{s}pcl((U,A)-(F,A))$

Proof

$\tilde{s}pfr(F,A) = \tilde{s}pcl(F,A)-\tilde{s}p\operatorname{int}(F,A)$

Ie $e_F\tilde{\in}\tilde{s}pfr(F,A) \Rightarrow e_F\tilde{\in}\tilde{s}pcl(F,A)$ and $e_F\tilde{\notin}\tilde{s}p\operatorname{int}$ (F,A)

ie $e_F\tilde{\in}\tilde{s}pcl(F,A)$ and $e_F\tilde{\in}\tilde{s}pcl((U,A)-(F,A))$

Therefore $\tilde{s}pfr(F,A)$ = $\tilde{s}pcl(F,A)\tilde{\bigcap}\tilde{s}pcl((U,A)-(F,A))$

Theorem 8

In general for any soft set (F,A) of (U,A,τ) we have $\tilde{s}pfr(\tilde{s}pfr(F,A))\tilde{\subset}\tilde{s}pfr(F.A)$

Proof

$\tilde{s}pfr(\tilde{s}pfr(F,A)) = \tilde{s}pcl(\tilde{s}pfr(F,A))\tilde{\bigcap}\tilde{s}pcl((U,A)-\tilde{s}pfr(F,A))\tilde{\subset}\tilde{s}pcl(\tilde{s}pfr(F,A)) = \tilde{s}pfr(F,A)$

As $\tilde{s}pfr(F,A)$ is soft preclosed.

Theorem 9

For a soft set (F,A) of (U,A,²) we have $\tilde{s}pfr(\tilde{s}p\operatorname{int}(F,A))\tilde{\subseteq}\tilde{s}pfr(F,A)$ and $\tilde{s}pfr(\tilde{s}pcl(F,A))\tilde{\subset}\tilde{s}pfr(F,A)$

Proof

$\tilde{s}pfr(\tilde{s}p\operatorname{int}(F,A)) = \tilde{s}pcl(\tilde{s}p\operatorname{int}(F,A))-\tilde{s}p\operatorname{int}(\tilde{s}p\operatorname{int}(F,A))$

$= \tilde{s}pcl(\tilde{s}p\operatorname{int}(F,A))-(\tilde{s}p\operatorname{int}(F,A))$

= $\tilde{s}pfr(F,A)$

$\tilde{s}pfr(\tilde{s}pcl(F,A)) = \tilde{s}pcl(\tilde{s}pcl(F,A))-\tilde{s}p\operatorname{int}(\tilde{s}pcl(F,A))$

$= \tilde{s}pcl((F,A))-\tilde{s}p\operatorname{int}(\tilde{s}pcl(F,A))$

$\tilde{\subset}\tilde{s}pcl(F,A)-\tilde{s}p\operatorname{int}(F,A) =$ $\tilde{s}pfr(F,A)$

Theorem 10

A soft set (F,A) of (U,A,²) is soft preopen iff $\tilde{s}pfr(F,A)$ = $\tilde{s}pd$ (F,A).

Proof

Let (F,A) be soft preopen. Then $\tilde{s}p\operatorname{int}(F,A)$ =(F,A)

Now

$\tilde{s}pfr(F,A) = \tilde{s}pcl(F,A)-\tilde{s}p\operatorname{int}(F,A) = \tilde{s}pcl(F,A)-(F,A)$

As $\tilde{s}pcl(F,A) = (F,A)\tilde{\bigcup}\tilde{s}pd(F,A)$

So $\tilde{s}pfr(F,A) = \tilde{s}pcl(F,A)-(F,A)$

$= ((F,A)\tilde{\bigcup}\tilde{s}pd(F,A))-(F,A)$

= $\tilde{s}pd$ (F,A).

Conversely, let $\tilde{s}pfr(F,A)$ = $\tilde{s}pd$ (F,A).

That is $\tilde{s}pd(F,A) = \tilde{s}pcl(F,A)-\tilde{s}p\operatorname{int}(F,A)$

$= ((F,A)\tilde{\bigcup}\tilde{s}pd(F,A))-\tilde{s}p\operatorname{int}(F,A)$

Hence (F,A)- $\tilde{s}p\operatorname{int}(F,A)$ = (¤ ,A)

Therefore (F,A) $\tilde{\subset}\tilde{s}p\operatorname{int}(F,A)$ but $\tilde{s}p\operatorname{int}(F,A)\tilde{\subset}(F,A)$ and hence (F,A) = $\tilde{s}p\operatorname{int}(F,A)$ which shows that (F,A) is preopen.

Soft Pre-Exterior of A Soft Set

Definition 12

Let (U, A, ²) be a softspace over U and (F,A) be a soft set on U.An e_x∈(U,A) is said to be a soft pre-exterior point of (F,A) if e_x is a preinterior point of $(F,A)^c$. That is there exists a soft open set (G,A) such that An $e_x \in (G,A) \tilde{\subseteq} (F,A)^c$.The soft pre-exteior of (F,A) is denoted by $\tilde{s}pExt(F,A)$. Thus $\tilde{s}pExt(F,A) = \tilde{s}p\,\mathrm{int}((U,A)-(F,A)) = \tilde{s}p\,\mathrm{int}\,(F,A)^c$.

Theorem 11

If (U, A,²) be a softspace over U and (F, A) and (G, A) be two softsets then the following properties hold for the soft pre-exterior (spExt) operator but the converses are not true in general.

(i) (F,A) $\tilde{\subset}$ (G,A) then $\tilde{s}pExt(G,A) \tilde{\subset} \tilde{s}pExt(F,A)$

(ii) $\tilde{s}pExt((F,A)\tilde{\bigcup}(G,A)) \tilde{\subset} \tilde{s}pExt(F,A)\tilde{\bigcup}\tilde{s}pExt(G,A)$

(iii) $\tilde{s}pExt((F,A)\tilde{\bigcap}(G,A)) \tilde{\subset} \tilde{s}pExt(F,A)\tilde{\bigcap}\tilde{s}pExt(G,A)$

Proof

(i)If (F,A)$\tilde{\subset}$(G,A) then $(G,A)^c \tilde{\subset} (F,A)^c$ and hence $\tilde{s}p\,\mathrm{int}(G,A)^c \tilde{\subset} \tilde{s}p\,\mathrm{int}(F,A)^c$

This implies that $\tilde{s}pExt(G,A) \tilde{\subset} \tilde{s}pExt(F,A)$

(ii)Since (F,A) $\tilde{\subset}$ (F,A)$\tilde{\bigcup}$ (G,A) and (G,A) $\tilde{\subset}$ (F,A)$\tilde{\bigcup}$ (G,A)

So by (i) $\tilde{s}pExt((F,A)\tilde{\bigcup}(G,A)) \tilde{\subset} \tilde{s}pExt(F,A)$ and $\tilde{s}pExt((F,A)\tilde{\bigcup}(G,A)) \tilde{\subset} \tilde{s}pExt(G,A)$

Therefore

$\tilde{s}pExt((F,A)\tilde{\bigcup}(G,A)) \tilde{\subset} \tilde{s}pExt(F,A)\tilde{\bigcup}\tilde{s}pExt(G,A)$

(iii)Since (F,A)$\tilde{\bigcap}$ (G,A) $\tilde{\subset}$ (F,A) and (F,A)$\tilde{\bigcap}$ (G,A) $\tilde{\subset}$ (G,A)

By (i) $\tilde{s}pExt(F,A) \tilde{\subset} \tilde{s}pExt((F,A)\tilde{\bigcap}(G,A))$ and $\tilde{s}pExt(G,A) \tilde{\subset} \tilde{s}pExt((F,A)\tilde{\bigcap}(G,A))$

Hence

$\tilde{s}pExt((F,A)\tilde{\bigcap}(G,A)) \tilde{\subset} \tilde{s}pExt(F,A)\tilde{\bigcap}\tilde{s}pExt(G,A)$

The following example illustrates the above theorem

Example 2

From example 1, let U ={a,b} and A = {e_1, e_2 }

(F,A) = (F_7, A)={(e_1,{a}),(e_2,{b})} and (G,A) = (F_{15}, A) ={(e_1,{a,b}),(e_2,{b})} are two soft preopen sets.

Clearly, (F,A)$\tilde{\subset}$(G,A)

$\tilde{s}pExt(F,A) =$ {(e_1,{b}),(e_2,{a})}

$\tilde{s}pExt(G,A) =$ {(e_1,¤),(e_2,{a})}

So $\tilde{s}pExt(G,A) \tilde{\subset} \tilde{s}pExt(F,A)$

Converse need not be true.

$\tilde{s}pExt((F,A)\tilde{\bigcup}(G,A)) = \tilde{s}pExt$ {(e_1,{a,b}),(e_2,{b})}

= {(e_1, ¤),(e_2,{a})}

$\tilde{s}pExt((F,A)\tilde{\bigcup}(G,A)) =$ {(e_1,{b}),(e_2,{a})}

$\tilde{s}pExt((F,A)\tilde{\bigcup}(G,A)) \tilde{\subset} \tilde{s}pExt(F,A)\tilde{\bigcup}\tilde{s}pExt(G,A)$

$\tilde{s}pExt((F,A)\tilde{\bigcap}(G,A)) = \tilde{s}pExt$ {(e_1,¤),(e_2,{a})}

= {(e_1,{b}),(e_2,{a})}

$\tilde{s}pExt(F,A)\tilde{\bigcap}\tilde{s}pExt(G,A) =$ {(e_1,¤),(e_2,{a})}

$\tilde{s}pExt((F,A)\tilde{\bigcap}(G,A)) \tilde{\subset} \tilde{s}pExt(F,A)\tilde{\bigcap}\tilde{s}pExt(G,A)$

Theorem 12

For soft sets (F, A) and (G,A) of a soft topological space (U,A,²) the following properties hold for the soft pre-exterior operator

(i) $\tilde{s}Ext(F,A) \tilde{\subset} \tilde{s}pExt(F,A)$

(ii) $\tilde{s}pExt(U,A) = (¤, A)$ and $\tilde{s}pExt(¤, A) = (U,A)$

(iii) $\tilde{s}pExt(F,A)$ is soft preopen

(iv) $\tilde{s}pExt(F,A) = (U,A) - \tilde{s}pcl(F,A)$

(v) $\tilde{s}pExt(\tilde{s}pExt(F,A)) = \tilde{s}p\,\mathrm{int}(\tilde{s}pcl(F,A))$

(vi) $\tilde{s}pExt((F,A)\tilde{\bigcup}(G,A)) = \tilde{s}pExt(F,A)\tilde{\bigcap}\tilde{s}pExt(G,A)$

(vii) $\tilde{s}pExt(F,A) = \tilde{s}pExt((U,A) - \tilde{s}pExt(F,A))$

(viii) $\tilde{s}p\,\mathrm{int}(F,A) \tilde{\subset} \tilde{s}pExt(\tilde{s}pExt(F,A))$

(ix) (F,A)$\tilde{\bigcap}\tilde{s}pExt(F,A) = (¤, A)$

(x) $\tilde{s}p\,\mathrm{int}(F,A), \tilde{s}pExt(F,A)$ and $\tilde{s}pfr(F,A)$ are mutually disjoint

(xi) (U,A)= $\tilde{s}p\,\mathrm{int}(F,A)\tilde{\bigcup}\tilde{s}pExt(F,A)\tilde{\bigcup}\tilde{s}pfr(F,A)$

Proof

$(i) e_x \tilde{\in} \tilde{s}Ext(F,A) \Rightarrow e_x \tilde{\in} \tilde{s}\,\mathrm{int}((U,A)-(F,A)) \tilde{\subset} \tilde{s}p\,\mathrm{int}((U,A)-(F,A)) = \tilde{s}pExt(F,A)$

$(ii) \tilde{s}pExt(U,A) = \tilde{s}p\,\mathrm{int}((U,A)-(U,A))$

$= \tilde{s}p\,\mathrm{int}(¤, A)$

= (¤ ,A)

$\tilde{s}pExt(¤, A) = \tilde{s}p\,\mathrm{int}((U,A)-(¤, A))$

$= \tilde{s}p\,\mathrm{int}(U,A)$

= (U,A)

$(iii) \tilde{s}pExt(F,A) = \tilde{s}p\,\mathrm{int}((U,A)-(F,A))$

Therefore $\tilde{s}pExt(F,A)$ is soft preopen

$(iv) \tilde{s}pExt(F,A) = \tilde{s}p\,\mathrm{int}((U,A)-(F,A)) = (U,A) - \tilde{s}pcl(F,A)$

$(v) \tilde{s}pExt(\tilde{s}pExt(F,A)) = \tilde{s}p\,\mathrm{int}((U,A) - \tilde{s}pcl(F,A))$

$= \tilde{s}p\,\mathrm{int}((U,A) - ((U,A) - \tilde{s}pcl(F,A)))$

$= \tilde{s}p\,\mathrm{int}(\tilde{s}pcl(F,A))$

$(vi) \tilde{s}pExt(F,A)\tilde{\bigcap}\tilde{s}pExt(G,A) = \tilde{s}p\,\mathrm{int}((U,A)-(F,A))\tilde{\bigcap}\tilde{s}p\,\mathrm{int}((U,A)-(G,A))$

$\tilde{\subset} \tilde{s}p\,\mathrm{int}(((U,A)-(F,A)\tilde{\bigcap}((U,A)-(G,A)))$

$= \tilde{s}p\,\mathrm{int}((U,A) - ((F,A)\tilde{\bigcup}(G,A)))$

$= \tilde{s}pExt((F,A)\tilde{\bigcup}(G,A))$

Since (F,A) $\tilde{\subset}$ (F,A)$\tilde{\bigcup}$ (G,A) so

$\tilde{s}pExt((F,A)\tilde{\bigcup}(G,A)) \tilde{\subset} \tilde{s}pExt(F,A)$

$\tilde{s}pExt((F,A)\tilde{\bigcup}(G,A)) \tilde{\subset} \tilde{s}pExt(G,A)$

$\tilde{s}pExt((F,A)\tilde{\bigcup}(G,A)) \tilde{\subset} \tilde{s}pExt(F,A)\tilde{\bigcap}\tilde{s}pExt(G,A)$

$\tilde{s}pExt((F,A)\tilde{\bigcup}(G,A)) = \tilde{s}pExt(F,A)\tilde{\bigcap}\tilde{s}pExt(G,A)$

$(vi)\tilde{s}pExt((U,A)-\tilde{s}p\mathrm{int}(F,A))=\tilde{s}p\mathrm{int}((U,A)-((U,A)-\tilde{s}p\mathrm{int}(F,A)))$

$= \tilde{s}p\mathrm{int}(\tilde{s}pExt(F,A))$

$= \tilde{s}p\mathrm{int}(\tilde{s}p\mathrm{int}((U,A)-(F,A)))$

$= \tilde{s}p\mathrm{int}((U,A)-(F,A))$

$= \tilde{s}pExt(F,A)$

(viii) $\tilde{s}pExt(F,A)=\tilde{s}p\mathrm{int}((U,A)-(F,A))\,\tilde{\subseteq}\,(U,A)-(F,A)$

We have $\tilde{s}pExt((U,A)-(F,A))\,\tilde{\subseteq}\,\tilde{s}pExt(\tilde{s}pExt(F,A))$

This implies that $\tilde{s}p\mathrm{int}(F,A)\,\tilde{\subseteq}\,\tilde{s}pExt(\tilde{s}pExt(F,A))$

$(ix)\tilde{s}pExt(F,A)=\tilde{s}p\mathrm{int}((U,A)-(F,A))\,\tilde{\subseteq}\,(U,A)-(F,A)$

This implies that $(F,A)\,\tilde{\cap}\,\tilde{s}pExt(F,A)=(\text{¤},A)$

$(x)\tilde{s}pExt(F,A)=\tilde{s}p\mathrm{int}((U,A)-(F,A))\,\tilde{\subseteq}\,(U,A)-(F,A)$

and

$\tilde{s}p\mathrm{int}(F,A)\,\tilde{\subseteq}\,(F,A)\Rightarrow\tilde{s}pExt(F,A)\,\tilde{\cap}\,\tilde{s}p\mathrm{int}(F,A)=(\text{¤},A)$

$(xi)\tilde{s}pExt(F,A)=(U,A)-\tilde{s}pcl(F,A)$

$= (U,A)-(\tilde{s}p\mathrm{int}(F,A)\,\tilde{\cup}\,\tilde{s}pfr(F,A))$

$\Rightarrow (U,A)=\tilde{s}p\mathrm{int}(F,A)\,\tilde{\cup}\,\tilde{s}pExt(F,A)\,\tilde{\cup}\,\tilde{s}pfr(F,A)$

CONCLUSION

In this paper soft pre-frontier and soft pre-exterior are defined. Soft pre-limit point and soft pre-neighbourhood are established. Some results concerning these are also characterized in this paper.The results concerning pre-interior,pre-frontier and pre-exterior in general topology are true in soft topology also

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