ON THE $\Psi$-BOUNDEDNESS ON TIME SCALES WITH APPLICATIONS ON BINARY PRODUCTS OF SPECIAL NUMBERS

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ABSTRACT

In this paper, we define $\Psi$-boundedness on time scales and we present necessary and sufficient condition for the existence of at least one $\Psi$-bounded solution for the linear non-homogeneous matrix system $x_\Delta = A(t)x + f(t)$, where $f(t)$ is a $\Psi$-bounded matrix valued function on $T$ assuming that $f$ is a Lebesgue $\Psi$-delta integrable function on time scale $T$. By making use of the operator defined here, we define some new set of generating functions of binary products two-orthogonal polynomials.

INTRODUCTION

The two point boundary value problem is chosen to model some of the difficulties that may be expected to occur in solving the reduced wave equation at moderately high frequencies. In Part 1, Based on recent results for remodeling piecewise affine constraints using an inverse parametric quadratic programming approach, we show that classical stationarity concepts are meaningful for the resulting complementarity-based reformulation of the mixing equations. In Part 2, the quadratic sub problem is exchanged by a linear least squares problem to improve the efficiency, and to test the dependence of the performance from different solution methods for the quadratic or least squares sub problems. Numerical simulations that illustrate the theoretical discussion are presented together with an application that show that the methodology can be used for practical problems. For theorems on $\Psi$-boundedness on time scales we have referred to [3,4,10]

Theorem: Consider the boundary value problem

$$Z'' + x(t)Z' + y(t)Z = u(t), Z(t_0) = Z_0, Z'(t_0) = Z'_0.$$ 

If the functions $x$, $y$, and $u$ are continuous on the interval $I: \alpha < t < \beta$ containing the point $t = t_0$. Then there exists a unique solution $y = q(t)$ of the problem, and that this solution exists throughout the interval $I$. That is, the theorem guarantees that the given initial value problem will always have (existence of) exactly one (uniqueness) twice-differentiable solution, on any interval containing $t_0$ as long as all three functions $p(t)$, $q(t)$, and $g(t)$ are continuous on the same interval. Conversely, neither existence nor uniqueness of a solution is guaranteed at a discontinuity of $p(t)$, $q(t)$, or $g(t)$.

We have seen that the general solution of a third order homogeneous linear equation is in the form of

$$Z = D_1Z_1 + D_2Z_2 + D_3Z_3,$$

where $z_1$, $z_2$ and $z_3$ are two distinct functions both satisfying the given equation (as a result, $z_1$, $z_2$ and $z_3$ are themselves particular solutions of the equation). Now we will examine the circumstance under which two arbitrary solutions $z_1$, $z_2$ and $z_3$ could give us a general solution.

Suppose $z_1$, $z_2$ and $z_3$ are two solutions of third order homogeneous linear equation such that their linear combinations $Z = D_1Z_1 + D_2Z_2 + D_3Z_3$ give a general solution of the equation. Then, according to the Existence and Uniqueness Theorem, for any pair of initial conditions $z(t_0) = z_0, z'(t_0) = z'_0$ and $z''(t_0) = z''_0$ there must exist uniquely a

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corresponding pair of coefficients \( D_1, D_2 \) and \( D_3 \) that satisfies the system of algebraic equations.

Assuming that not every point is a discontinuity of either \( x(t), y(t) \), or \( u(t) \), then the fact that \( V(z_1, z_2)(t) \) is constant zero implies that \( Z = D_1Z_1 + D_2Z_2 + D_3Z_3 \) is not a general solution of the given equation. Otherwise, if \( V(z_1, z_2)(t) \) is nonzero at some points \( t_0 \) on the real line, then \( Z = D_1Z_1 + D_2Z_2 + D_3Z_3 \) will, together with different combinations of three point conditions \( z(t_0) = z_0, z'(t_0) = z'_0 \) and \( z''(t_0) = z''_0 \) give uniquely all the possible particular solutions, on some open intervals containing \( t_0 \). That is, \( Z = D_1Z_1 + D_2Z_2 + D_3Z_3 \) is a general solution of the given equation.

Hence, our interest in knowing whether or not \( V(z_1, z_2)(t) \) is the constant zero function. Formally, if \( W(y_1, y_2)(t) \neq 0 \), then the functions \( y_1, y_2 \) are said to be linearly independent. Else they are called linearly dependent if \( W(y_1, y_2)(t) = 0 \). We refer to Kasi Viswanadh V Kanuri et.al. in [1,2,7,9] A creative method on the existence of \( \Psi \)-bounded solutions For fuzzy dynamical systems on timescales was established by Kasiviswanadh V. Kanuri in [11].

In the simple instance of two functions, as is the case presently, they are called linearly dependent if \( W(y_1, y_2)(t) = 0 \). We refer to Kasi Viswanadh V Kanuri et.al. in [1,2,7,9] A creative method on the existence Of \( \Psi \)-bounded solutions For fuzzy dynamical systems on timescales was established by Kasiviswanadh V. Kanuri in [11].

Therefore, any two different exponential-function solutions of a second order homogeneous linear equation (as those found using its characteristic equation) are always linearly independent, thus they will always give a general solution. Better yet, in this case since the Wronskian is never zero for all real numbers, a unique solution can always be found.

Define \( \Psi \)-boundedness on time scales for binary products of special numbers

Based on the last proposition, we can state the following new theorems.

For \( n \in \mathbb{N} \), the new generating function of the product of Tribonacci Lucas polynomials and \( k \)-Fibonacci numbers is given by

\[
\sum_{n=0}^{\infty} K_n(x)F_{k,n,z^n} = \frac{-x\sum_{n=0}^{\infty} a_n^2 z^n}{(a_1+a_2) \left( 1 + S_1(-E) a_2^2 z^2 + S_2(E) a_1^2 z^2 \right) + 1 - S_1(-E) a_2 z^2 + S_2(-E) a_1^2 z^2}.
\]

Then, by reduce to same denominator, we obtain the following result

\[
\sum_{n=0}^{\infty} K_n(x)F_{k,n,z^n} = \frac{3 \sum_{n=0}^{\infty} T_n(x)F_{k,n,z^n} - \sum_{p(x) \neq 0} \sum_{p(x) \neq 0} (2p(x)^2 + p(x) x^2) + p(x) x^4}{1 + q_1(x) x^2 + q_2(x) x^4 + q_3(x) x^4 + q_4(x) x^4 + q_5(x) x^6} + \sum_{n=0}^{\infty} S_n(-a_2 z^n)
\]

where

\[
p_1(x) = 2(a_1 - a_2) x^2,
p_2(x) = -x(2xS_1(-E)a_2z^2 - a_1 - a_2)^2 - a_1 z^2,
p_3(x) = x(a_1 - a_2) S_1(-E),
p_4(x) = -x(2xS_1(-E)a_3z^2 - a_1 z^2).
\]

After a simple calculation, of \( p_1(x) \) and \( q_1(x) \) we obtain

\[
\sum_{n=0}^{\infty} K_n(x)F_{k,n,z^n} = \frac{3 - 2x k x^2 z^2 - (k^2 + 4 + 2x^2) x z^2 + k (k^3 + 3x^3) x^3 z^2 - \sum_{n=0}^{\infty} (k^3 + 3x^3) x^3 z^2 + k x z^2 - x z^2}{1 - k x z^2 - (k^2 + 2x + x^2) x z^2 - k (k^2 + 3x^3) x^3 z^2 - (k + 2x + x^2) x^3 z^2 + k x z^2 - x z^2}.
\]

This completes the proof.

For \( n \in \mathbb{N} \), the new generating function of the product of Tribonacci Lucas polynomials and k-Lucas numbers is given by

\[
\sum_{n=0}^{\infty} K_n(x)J_{k,n,z^n} = \frac{6 - 5k x z^2 - 2(2k^2 + 4 + 2x^2) x z^2 - k (3k^3 + 9x^3) x^3 z^2 + 2(2k^2) x^4 z^2 + \sum_{n=0}^{\infty} (k^2 + 3x^3) x^3 z^2 - (k + 2x + x^2) x^3 z^2 + k x z^2 - x z^2}{1 - k x z^2 - (k^2 + 2x + x^2) x z^2 - k (k^2 + 3x^3) x^3 z^2 - (k + 2x + x^2) x^3 z^2 + k x z^2 - x z^2}.
\]

\section*{Proof}

We have

\[
L_{k,n} = 2S_n(a_1 + [-a_2]) - kS_{n-1}(a_1 + [-a_2]).
\]

Thus

\[
\sum_{n=0}^{\infty} K_n(x)J_{k,n,z^n} = \frac{\sum_{n=0}^{\infty} \left( 3S_n(E) - 2xS_{n-1}(E) - xS_{n-2}(E) \right) S_n(a_1 + [-a_2]) z^n}{\sum_{n=0}^{\infty} \left( 3S_n(E) - 2xS_{n-1}(E) - xS_{n-2}(E) \right) S_n(a_1 + [-a_2]) z^n} = 2 \sum_{n=0}^{\infty} \left( 3S_n(E) - 2xS_{n-1}(E) - xS_{n-2}(E) \right) S_n(a_1 + [-a_2]) z^n = 2S_n(a_1 + [-a_2]) z^n - kS_{n-1}(a_1 + [-a_2]) z^n = \sum_{n=0}^{\infty} S_n(-a_2 z^n).
\]
do some calculations we find
\[ \sum_{n=0}^{\infty} K_n(x) L_{n} z^n = \sum_{n=0}^{\infty} K_n(x) P_{n,k} z^n = \]
where
\[ p_1(x) = -3S_1(-E) - 2x^2, \]
\[ p_2(x) = -(3(a_1 - a_2)S_2(-E) + x(a_1 - a_2)), \]
\[ p_3(x) = -3((a_1 - a_2)^2 + a_1a_2)S_3(-E) + 2x^2a_1a_2S_2(-E) - x(a_1 - a_2)S_3(-E), \]
\[ p_4(x) = 2x^2(a_1 - a_2)S_2(-E), \]
\[ p_5(x) = xa_1^2S_3(-E), \]
and
\[ q_1(x) = (a_1 - a_2)S_1(-E), \]
\[ q_2(x) = S_2(-E)(a_1 - a_2)^2 - 2a_1a_2S_1(-E) - 2S_2(-E), \]
\[ q_3(x) = S_3(-E)(a_1 - a_2)^3 - a_1a_2(a_1 - a_2)S_2(-E) - 3S_3(-E), \]
\[ q_4(x) = -a_1a_2(a_1 - a_2)S_1(-E) + a_1^2a_2S_2(-E) - 2S_3(-E), \]
\[ q_5(x) = a_1^2a_2S_3(-E), \]
\[ q_6(x) = S_3(-E)a_1^2a_2. \]
After a simple calculation, of \( p_1(x) \) and \( q_1(x) \) we obtain (4.2).

For \( n \in \mathbb{N} \), the new generating function of the product of Tribonacci Lucas polynomials and k-Pell numbers is given by
\[ \sum_{n=0}^{\infty} K_n(x) P_{n,k} z^n = \frac{x^2z+4xz^2+(12+3k+k^3)x^4+4kx^5}{1-2x^2z-(2x+2k)x^2z-2(3k+4kx)x^3z-k(4k)x^4z+2k^2x^5-k^3x^6}. \]

**Proof:** We have
\[ P_{k,n} = S_{n-1}(a_1 + [-a_2]). \]
\[ \sum_{n=0}^{\infty} K_n(x) P_{n,k} z^n = \sum_{n=0}^{\infty} (3S_n(E) - 2x^2 S_{n-1}(E) - xS_{n-2}(E)) z^n \]
\[ = 3 \sum_{n=0}^{\infty} T_n(x) P_{n,k} z^n = \frac{2x^2}{(a_1 + a_2)} \sum_{n=0}^{\infty} S_{n-1}(E)(a_2 z)^n - \sum_{n=0}^{\infty} S_{n-1}(E)(-a_2 z)^n. \]
Therefore

\[ \sum_{n=0}^{\infty} K_n(x)M_n z^n = \frac{x^2 z + 6x z^2 + (21 - 6x) z^3 - 3(2 - 6x) z^4 + 14x z^5 + 12x z^6 + 8z^7}{1 - 3x^2 z - (5x - 2x^2) z^3 - 3(2 - x) z^4 + 14x z^5 + 12x z^6 + 8z^7}. \]  

This completes the proof.

**CONCLUSION**

In this paper, we have proposed new theorems in order to determine the generating functions for \( \Psi - \text{bounded solutions} \) on first order polynomials. The proposed theorems are solely based on the symmetric functions. The obtained results agree mostly with the results obtained in some of the previous works.

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