ON (gαr)* CLOSED SETS IN TOPOLOGICAL SPACES

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INTRODUCTION

In 1970, Levine introduced the concept of generalized closed set and discussed the properties of sets, closed and open maps, compactness, normal and separation axioms. Later in 1996, the investigation on generalization of closed set has lead to significant contribution to the theory of separation axiom, generalization of continuity and covering properties. A.A.Omari and M.S.M. Noorani made an analytical study and gave the concepts of generalized closed set in topological spaces. In this paper, a new class of closed set called generalized α regular - star-closed set is introduced to prove that the class forms a topology. The notion of generalized α regular-star-closed set and its different characterizations are given in this paper. Throughout this paper (X, τ) and (Y, σ) represent the non – empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned.

Let A ⊆ X, the closure of A and interior of A will be denoted by cl(A) and int(A) respectively.

Preliminaries

Definition 2.1. Let A subset of A of a topological space (X, τ) is called

1. α-open set [9] if A ⊆ int(cl(int(A))).
2. generalized closed set (briefly g - closed) [5] if cl(A) ⊆ U whenever A ⊆ U and U is open.
3. weakly closed set(briefly w - closed) [13] if cl(A) ⊆ U whenever A ⊆ U and U is semi open.
4. generalized * closed set(briefly g * - closed) [15] if cl(A) ⊆ U whenever A ⊆ U and U is g - open.
5. generalized α - closed set (briefly g α - closed) [8] if αcl(A) ⊆ U whenever A ⊆ U and U is α open.
6. an α - generalized closed set (briefly a g - closed) [7] if αcl(A) ⊆ U whenever A ⊆ U and U is open.
7. generalized b - closed set (briefly gb - closed) [1] if bcl(A) ⊆ U whenever A ⊆ U and U is open in X.
8. semi generalized b - closed set (briefly sgb - closed) [4] if bcl(A) ⊆ U whenever A ⊆ U and U is semi open in X.
9. generalized α b closed set (briefly gαb - closed) [14] if bcl(A) ⊆ U whenever A ⊆ U and U is α open in X.
10. regular generalized b - closed set (briefly rgb - closed) [9] if bcl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
11. generalized pre regular closed set (briefly gpr - closed) [3] if pcl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
12. generalized α regular - closed set (briefly gαr - closed set)[16] if αcl(A) ⊆ U whenever A ⊆ U and U is regular open in X.

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13. a regular open set [17] if A = cl(int(A))
14. gsp-closed set [18] if spcl(A)⊆U whenever A⊆U and U is open in (X, τ).

**Generalized α Regular-star-Closed Sets**

In this section, we introduce generalized α regular–star-closed set and investigate some of its properties.

**Definition 3.1.** A subset A of a topological space (X, τ), is called generalized α regular-star-closed set (briefly αg*r* - closed set) if cl(A)⊆U whenever A⊆U and U is αg*r* open in X.

**Definition 3.2.** (gar)* -Open Set

A subset A of a topological space (X,τ) is called (gar)* - open set if and only if A* is (gar)* -closed in X.

**Theorem 3.3.** Every (gar)* closed set is g - closed set.

**Proof.** Let A be any (gar)* closed set in X such that A ⊆ U, where U is open. Therefore αcl(A) ⊆ U. Hence A is g -closed set in X.

The converse of above theorem need not be true as seen from the following example.

**Example 3.4.** Let X = {a, b, c} with τ = {X, φ, {a}, {a, b}}. The set {b} is g - closed set but not (gar)* - closed set.

**Theorem 3.5.** Every (gar)* closed set is gα - closed set.

**Proof.** Let A be any (gar)* - closed set in X. Let A ⊆ U and U is α - open set. Then U is gα open. Therefore αcl(A) ⊆ cl(A) ⊆ U. Hence A is gα - closed set.

The converse of above theorem need not be true as seen from the following example.

**Example 3.6.** LetX = {a, b, c} with τ = {X, φ, {a}, {a, b}}. The set {b} is gα - closed set but not (gar)* - closed set.

**Theorem 3.7.** Every (gar)* closed set is αg - closed set.

**Proof.** Let A be any (gar)* - closed set in X. Let A ⊆ U and U is open set. Then U is gα open. Therefore αcl(A) ⊆ cl(A) ⊆ U. Hence A is αg - closed set.

The converse of above theorem need not be true as seen from the following example.

**Example 3.8.** Let X = {a, b, c} with τ = {X, φ, {a}, {a, b}}. The set {a, c} is αg - closed set but not (gar)* - closed set.

**Theorem 3.9.** Every (gar)* - closed set is gpr - closed set.

**Proof.** Let A be any (gar)* - closed set in X and U be any regular open set containing A. Then pcl(A) ⊆ cl(A) ⊆ U. Therefore pcl(A) ⊆ U. Hence A is gpr - closed set.

The converse of above theorem need not be true as seen from the following example.

**Example 3.10.** Let X = {a, b, c, d} with τ = {X, φ, {a}, {b}, {a, b}, {a, b, c}}. The set {c} is gpr - closed set but not (gar)* - closed set.

**Theorem 3.11.** Every (gar)* - closed set is ggr - closed set.

**Proof.** Let A be any (gar)* - closed set in X and U be any regular open set containing A. Since every regular open set is ggr open, ucl(A) ⊆ cl(A) ⊆ U. Therefore ucl(A) ⊆ U. Hence A is ggr closed set.

The converse of above theorem need not be true as seen from the following example.

**Example 3.12.** Let X = {a, b, c, d} with τ = {X, φ, {a}, {b}, {a, b}, {a, b, c}}. The set {a, b} is ggr - closed set but not (gar)* - closed set.

**Theorem 3.13.** Every (gar)* - closed set is g*- closed set.

**Proof.** Let A be any (gar)* - closed set in X and U be any g*- open set containing A. Therefore cl(A) ⊆ U. Hence A is g* - closed set.

The converse of above theorem need not be true as seen from the following example.

**Example 3.14.** Let X = {a, b, c, d} with τ = {X, φ, {a}, {b}, {a, b}, {a, b, c}}. The set {a, d} is g* - closed set but not (gar)* - closed set.

**Theorem 3.15.** Every regular closed set is (gar)* - closed set.

**Proof.** Let A be any regular closed set in X such that A ⊆ U, where U is gαr open. Since A is regular closed set cl(int(A)) = A. Therefore cl(A) ⊆ cl(int(A)) = A ⊆ U. Therefore cl(A) ⊆ U. Hence A is (gar)* -closed set in X.

The converse of above theorem need not be true as seen from the following example.

**Example 3.16.** Let X = {a, b, c, d} with τ = {X, φ, {a}, {b}, {a, b}, {a, b, c}}. The set {c, d} is (gar)*-closed set but not regular closed set.

**Theorem 3.17.** Every (gar)* - closed set is gsp - closed set.

**Proof.** Let A be any (gar)* - closed set in X and U be any open set containing A. Since every open set is gαr open, spcl(A) ⊆ cl(A) ⊆ U. Therefore spcl(A) ⊆ U. Hence A is gsp closed set.

The converse of above theorem need not be true as seen from the following example.

**Example 3.18.** Let X = {a, b, c, d} with τ = {X, φ, {a}, {b}, {a, b}, {a, b, c}}. The set {a, c} is gsp - closed set but not (gar)* - closed set.

**Theorem 3.19.** Every (gar)* - closed set is gpr - closed set.

**Proof.** Let A be any (gar)* - closed set in X and U be any regular open set containing A. Since every regular open set is ggr open, pcl(A) ⊆ cl(A) ⊆ U. Therefore pcl(A) ⊆ U. Hence A is gpr closed set.

The converse of above theorem need not be true as seen from the following example.

**Example 3.20.** Let X = {a, b, c, d} with τ = {X, φ, {a}, {b}, {a, b}, {a, b, c}}. The set {a, b} is gpr - closed set but not (gar)* - closed set.
Characteristics of (gαr)* - Closed Sets

Theorem 4.1. If A and B are (gαr)* - closed sets in X then A ∪ B is (gαr)* - closed set in X.

Proof. Let A and B be (gαr)* - closed sets in X and U be any gαr-open set such that A∪B ⊆ U. Therefore cl(A) ⊆ U, cl(B) ⊆ U. Hence cl(A∪B) = cl(A) ∪ cl(B) ⊆ U. Therefore A ∪ B is (gαr)* - closed set in X.

Theorem 4.2. If a set A is (gαr)* - closed set then cl(A) − A contains no non empty gαr-closed set.

Proof. Let F be a gαr-closed set in X such that F ⊆ cl(A) − A. Then A ⊆ X − F. Since A is (gαr)* closed set and X − F is gαr-open then cl(A) ⊆ X − F. (i.e.) F ⊆ X − cl(A). So F ⊆ (X − cl(A)) ∩ (cl(A) − A). Therefore F = φ

Theorem 4.3. If A ⊆ Y ⊆ X and suppose that A is (gαr)* - closed set in X then A is (gαr)* - closed set relative to Y.

Proof. Given that A ⊆ Y ⊆ X and A is (gαr)* - closed set in X. To prove that A is (gαr)* - closed set relative to Y. Let us assume that A ⊆ Y ∩ U, where U is gαr-open in X. Since A is (gαr)* - closed set, A ⊆ U implies _cl(A) ⊆ U.

It follows that Y ∩ cl(A) ⊆ Y ∩ U. That is A is (gαr)* - closed set relative to Y.

Theorem 4.4. For x ∈ X, then the set X − {x} is a (gαr)* - closed set or gαr-open.

Proof. Suppose that X − {x} is not gαr-open, then X is the only gαr-open set containing X − {x}. (i.e.) cl(X − {x}) ⊆ X. Then X − {x} is (gαr)*-closed in X.

References


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