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Research Article

CONNECTED DOMINATION IN SUBDIVISION OF A BLOCK GRAPH OF GRAPHS

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ABSTRACT

For any graph G, ***block graph*** B(G) is a graph whose set of vertices is the union of the set of blocks of G in which two vertices are adjacent if and only if the corresponding blocks of G are adjacent. A ***subdivision graph*** of a block graph is obtained from B(G) by subdividing each edge of B(G). A dominating set D is called connected dominating set of a subdivision of a block graph is the induced subgraph $\langle D\rangle$ is connected. The connected domination number $\gamma_c[S(B(G))]$ of a subdivision graph of $B(G)$ is the minimum cardinality of a connected dominating set in $S(B(G))$. In this paper, we obtain many bonds on $\gamma_c[S(B(G))]$ in terms of vertices, edges, blocks and different parameters of G and not the members of $S(B(G))$. Further we determine its relationship with other domination parameters.

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INTRODUCTION

All graphs considered here are simple, finite, nontrivial, separable, undirected and connected. As usual, p, q and n denote the number of vertices, edges and blocks of a graph G respectively. For graph theoretic terminology we refer F.Harary [3]. Hedetniemi and Laskar in [5] studied connected domination and further connected domination number of a graph is studied by Sampatkumar and Walikar in [9]. As usual, the maximum degree of a vertex in G is denoted by $\Delta(G)$. A vertex v is called a cut vertex if removing it from G increases the number of components of G. For any real number x, $\lceil x\rceil$ denotes the smallest integer not less than x and $\lfloor x\rfloor$ denotes the greatest integer not greater than x. A graph G is called trivial if it has no edges. If G has at least one edge then G is called a nontrivial graph. A nontrivial connected graph G with at least one cut vertex is called a separable graph, otherwise a non-separable graph.

A vertex cover in a graph G is a set of vertices that covers all edges of G. The vertex covering number $\alpha_0(G)$ is a minimum cardinality of a vertex cover in G. An edge cover of a graph G without isolated vertices is a set of edges of G that covers all vertices of G. The edge covering number $\alpha_1(G)$ of a graph G is the minimum cardinality of an edge cover of G. A set of vertices in a graph G is called an independent set if no two vertices in the set are adjacent. The vertex independence number $\beta_0(G)$ of a graph G is the maximum cardinality of an independent set of vertices in G. The edge independence number $\beta_1(G)$ of a graph G is the maximum cardinality of an independent set of edges.

A nontrivial connected graph with no cut vertex is called a block. A subdivision of an edge uv is obtained by removing an edge uv, adding a new vertex w and adding edges uw and wv. For any (p, q) graph G, a subdivision graph $S(G)$ is obtained from G by subdividing each edge of G. Here, a subdivision graph S(B(G)) is obtained from B(G) by subdividing each edge of B(G).

A set $D \subseteq V(G)$ of a graph $G = (V, E)$ is a dominating set if every vertex in $V - D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of a minimal dominating set in G. A dominating set D is a total dominating set if the induced subgraph $\langle D\rangle$ has no isolated vertices. The total domination number $\gamma_t(G)$ of a graph G is the minimum cardinality of a total dominating set in G. This concept was introduced by Cockayne, Dawes and Hedetniemi in [2].

A set F of edges in a graph $G(V, E)$ is called an edge dominating set of G if every edge in $E - F$ is adjacent to at least one edge in F. The edge domination number $\gamma'(G)$ of a graph G is the minimum cardinality of an edge dominating set of G. Edge domination number was studied by S.L. Mitchell and Hedetniemi in [7].

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A dominating set D is called connected dominating set of G if the induced subgraph $\langle D\rangle$ is connected. The connected domination number $\gamma_c(G)$ of a graph G is the minimum cardinality of a connected dominating set in G. The connected domination number $\gamma_c[S(B(G))]$ is the minimum cardinality of a connected dominating set in $S(B(G))$. For any connected graph G with $\Delta(G) < p-1, \gamma(G) \leq \gamma_t(G) \leq \gamma_c(G)$.

In this paper, many bonds on $\gamma_c[S(B(G))]$ were obtained in terms of vertices, edges, blocks and other parameters of G. Also, we obtain some results on $\gamma_c[S(B(G))]$ with other domination parameters of G.

RESULTS

Initially we present the exact value of connected domination number of a block graph of a non separable graph G.

Theorem 1: For any non separable graph $G, \gamma_c[B(G)] = 1$.
The following are the results on some standard graphs.

Theorem 2: For any Star graph $K_{1,m}$,

$$\gamma_c\left[S\left(B(K_{1,m})\right)\right] = \begin{cases} 1, if\ p = 3 \\ p, if\ p = 4 \\ > p, if\ p \geq 5 \end{cases}$$

Theorem 3: For any Path graph P_p,

$$\gamma_c\left[S\left(B(P_p)\right)\right] \begin{matrix} < p, if\ p = 3,4 \\ \geq p, if\ p \geq 5 \end{matrix}$$

The following theorem relates between $\gamma_c[S(B(G))]$ and number of blocks of G.

Theorem 4: For any separable graph G, $\gamma_c[S(B(G))] \geq \left\lfloor\frac{n}{2}\right\rfloor$, where n is the number of blocks of G. Equality holds for P_3.

Proof: Let G be a separable graph. Since the number of vertices in a block graph of G are equal to number of blocks in G, by subdivision of each edge in a block graph of G, we get more number of vertices in S(B(G)) than number of blocks of G. Hence, $\gamma_c[S(B(G))] \geq \left\lfloor\frac{n}{2}\right\rfloor$.

In the following theorem, we relate $\gamma_c[S(B(T))]$ and $p(T)$.

Theorem 5: For any tree T, $\gamma_c[S(B(T))] \geq \left\lfloor\frac{p-1}{2}\right\rfloor$, where p is the number of vertices in T. Equality holds for P_3.

Proof: Let T be a tree. Then T contains $p-1$ blocks in it. From Theorem 4, we have $\gamma_c[S(B(G))] \geq \left\lfloor\frac{n}{2}\right\rfloor$. Since, in a tree T we have $n(T) = p(T) - 1$, we get

$$\gamma_c[S(B(T))] \geq \left\lfloor\frac{p-1}{2}\right\rfloor.$$

The following lower bound is a relationship between $\gamma_c[S(B(T))]$ and number of edges of T.

Theorem 6: For any tree T, $\gamma_c[S(B(T))] \geq \left\lfloor\frac{q(T)}{2}\right\rfloor$, where $q(T)$ is the number of edges in T. Equality holds for P_3.

Proof: Suppose G is a tree then $q(T) = p(T) - 1 = n(T)$. From Theorem 5, we have $\gamma_c[S(B(T))] \geq \left\lfloor\frac{p-1}{2}\right\rfloor$. Hence, we get $\gamma_c[S(B(T))] \geq \left\lfloor\frac{q(T)}{2}\right\rfloor$.

The following upper bound is a relationship between $\gamma_c[S(B(G))]$, number of blocks $n(G)$ and number of vertices $p(G)$.

Theorem 7: If G is a (p,q) graph, $\gamma_c[S(B(G))] \leq n(G) + p(G)$.

Proof: Let D be a connected dominating set in $S(B(G))$. Then D must contain at least one vertex from each block of G. Let $b_1, b_2, \dots, b_n$ are the block vertices of $B(G)$ corresponding to the blocks $B_1, B_2, \dots, B_n$ of G. Since $|V(S(B(G))| > |V(B(G))|$ and $n(G) < p(G)$, clearly

$$\gamma_c[S(B(G))] = |D| \leq n(G) + p(G).$$

We thus have a result, due to Ore [8].

Theorem A [8]: If G is a (p,q) graph with no isolated vertices, then $\gamma(G) \leq \frac{p}{2}$.

In the following Theorem we obtain the relation between $\gamma_c[S(B(G))], \gamma(G), n(G)$ and $p(G)$.

Theorem 8: For any connected (p,q) graph G, $\gamma_c[S(B(G))] + \gamma(G) \leq \frac{3p}{2} + n(G)$.

Proof: From Theorem 7 and Theorem A, $\gamma_c[S(B(G))] + \gamma(G) \leq n(G) + p(G) + \frac{p(G)}{2} = \frac{3p}{2} + n(G)$. Hence, $\gamma_c[S(B(G))] + \gamma(G) \leq \frac{3p}{2} + n(G)$.

We have a following result due to Harary [3].

Theorem B [3, P.95]: For any nontrivial (p, q) connected graph G,

$$\alpha_0(G) + \beta_0(G) = p = \alpha_1(G) + \beta_1(G).$$

The following theorem relates between $\gamma_c[S(B(G))]$, $n(G)$, $\alpha_0(G)$, $\beta_0(G)$, $\alpha_1(G)$ and $\beta_1(G)$.

Theorem 9: If G is a (p,q) graph, then

$$\gamma_c[S(B(G))] \leq n(G) + \alpha_0(G) + \beta_0(G) = n(G) + \alpha_1(G) + \beta_1(G).$$

Proof: From Theorem 7 and Theorem B, we get

$$\gamma_c[S(B(G))] \leq n(G) + \alpha_0(G) + \beta_0(G) = n(G) + \alpha_1(G) + \beta_1(G).$$

The following Theorem is due to V.R.Kulli [6].

Theorem C [6, P.19]: For any graph $G, \gamma(G) \leq \beta_0(G)$.
In the following Theorem, we develop the relation between $\gamma_c[S(B(G))]$, $\gamma(G), \alpha_0(G), \beta_0(G)$ and $n(G)$.

Theorem 10: For any connected (p, q) graph G, $\gamma_c[S(B(G))] + \gamma(G) \leq n(G) + \alpha_0(G) + 2\beta_0(G)$.

Proof: From Theorem 9 and Theorem C, we get

$$\gamma_c[S(B(G))] + \gamma(G) \leq n(G) + \alpha_0(G) + 2\beta_0(G)$$

T.W.Haynes *et al*. [4] establish the following result.

Theorem D [4, P.165]: For any connected graph G, $\gamma_c(G) \leq 2\beta_1(G)$.
In the following Theorem, we develop the relation between $\gamma_c[S(B(G))]$, $\gamma_c(G), \alpha_1(G), \beta_1(G)$ and $n(G)$.

Theorem 11: For any connected (p, q) graph G, $\gamma_c[S(B(G))] + \gamma_c(G) \leq n(G) + \alpha_1(G) + 3\beta_1(G)$.

Proof: From Theorem 9 and Theorem D,

$$\gamma_c[S(B(G))] + \gamma_c(G) \leq n(G) + \alpha_1(G) + 3\beta_1(G)$$

The following upper bound was given by V.R.Kulli[6].

Theorem E[6, P.44]: If G is connected (p, q) graph and $\Delta(G) < p - 1$, then
$\gamma_t(G) \leq p - \Delta(G)$.

We obtain the following result.

Theorem 12: If G is a connected (p, q) graph and $\Delta(G) < p - 1$,
$\gamma_c[S(B(G))] + \gamma_t(G) \leq 2p + n(G) - \Delta(G)$.

Proof: From Theorem 7 and Theorem E, we get
$\gamma_c[S(B(G))] + \gamma_t(G) \leq 2p + n(G) - \Delta(G)$.
The following Theorem is due to S.Arumugam *et al*. [1].

Theorem F[1]: For any (p, q) graph G, $\gamma'(G) \leq \left\lfloor \frac{p}{2} \right\rfloor$. The equality is obtained for $G = K_p$.
Now we establish the following upper bound.

Theorem 13: For any (p, q) graph G, $\gamma_c[S(B(G))] + \gamma'(G) \leq n(G) + 3\left\lfloor \frac{p}{2} \right\rfloor$.

Proof: From Theorem 7 and Theorem F, the result follows.

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