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Research Article

A SQUARE MULTIPLICATIVE LABELING FOR SOME FAMILIES OF GRAPHS

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ABSTRACT

A graph $G=(V,E)$ with p vertices and q edges is said to be a square multiplicative labeling, if there exists a bijection $f:V(G)\rightarrow\{1,2,.....,p\}$ such that the induced function $f:E(G)\rightarrow N$ is given by $f(uv)=f(u)^2 * f(v)^2$ for every $uv\in E(G)$ are all distinct. A graph which admits square multiplicative labeling is called square multiplicative graph. In this paper, we have investigated some families of graphs which admit square multiplicative labeling.

INTRODUCTION

In Mathematics, Graph Theory is the study of graphs, which are mathematical structures that are being used to model pair wise relationship between objects. A graph is made up of vertices, nodes or points which are connected by edges, arcs or lines. The graphs that are taken in this paper are finite and undirected. A linear graph $(or\ simply\ a\ graph)$ $G=(V,E)$ consists of set of object $V=\{v_1,v_2,.......\}$ called vertices and another set $E=\{e_1,e_2,.....\}$, whose elements are called edges, such that each edge e_k is identified with an unordered pairs (v_i,v_j) of vertices. The symbols $V(G)$ and $E(G)$ are denoted as vertex set and edge set of a graph G. The cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size G, denoted by q. Hence the graph is referred to as (p,q) graph. Graph Theory is applied in various fields such as Physics, Chemistry, Biology, Computer Science, Operations Research, Social Networks, etc., Graph labeling is the assignment of integers from its vertices or edges subject to some certain conditions. A dynamic survey on graph labeling is regularly updated by Gallian $[4]$ and it is published by electronic journal of combinatory. Some basic definitions and notations are taken from Bondy and Murthy $[3]$. Beineke and Hedge $[1]$ referred a graph with p vertices as strongly multiplicative if the vertices of G can be labeled with distinct integers $1,2,.....,p$ such that the labels induced on the edges by the product of the end vertices are distinct. Shalini and Paul Dhayabaran $[14]$ introduced a minimization of multiplicative graphs. Shalini, Visalatchi and Paul Dhayabaran [15] discussed square multiplicative labeling for disconnected graphs. In this paper, we studied some families of graphs which satisfy square multiplicative labeling.

Definition 1.1

A graph G is said to be **square multiplicative labeling** if there exists a bijection $f:V(G)\rightarrow\{1,2,.....,p\}$ such that the induced function $f:E(G)\rightarrow N$ given by

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$f(uv) = f(u)^2 * f(v)^2$ for every $uv \in E(G)$ are all distinct.

Definition 1.2

A graph G is said to be **square multiplicative graph** if it admit a square multiplicative labeling.

Definition 1.3

A **path** in a simple graph is a finite (or) infinite sequence of edges which connect a sequences of vertices are all distinct from one another. A path cannot both start and terminate at the same vertex.

Definition 1.4

The **double star** graph $(K_{1,n,n})$ is a tree obtained from the star $K_{1,n}$ by adding a new pendent edge of the exiting n pendant vertices. It consists of $2n+1$ vertices and $2n$ edges.

Definition 1.5

A **comb** is a graph obtained by joining a single pendent edge to each vertex of a path. The comb graph is defined as $P_n \odot K_1$. Let $P_n = \{v_1, v_2, \ldots\ldots, v_n\}$ be a path graph with n vertices. It consists of $2n$ vertices and $2n-1$ edges.

Definition 1.6

The graph (P_m, S_n) is obtained from m copies of the star graph S_n and the path P_m, $\{u_1, u_2, \ldots\ldots, u_m\}$ by joining u_j with the vertex v_n of the j^{th} copy of S_n by means of an edge, for $1 \le j \le m$.

Square Multiplicative Labeling

Theorem 2.1

A path is a square multiplicative graph for $n \ge 2$.

Proof

Let G be a graph of path P_n.

Let $\{v_1, v_2, \ldots\ldots, v_n\}$ be the vertices of P_n and $\{e_1, e_2, \ldots\ldots, e_{n-1}\}$ be the edges of P_n which are denoted as in the fig. 2.1.

The path consists of n vertices and $n-1$ edges.

If $G = P_n$ then $|V(G)| = n$

and $|E(G)| = n-1$.

Let us define a vertex labeling $f : V(G) \to \{1, 2, \ldots\ldots, p\}$ as follows

$$f(v_i) = i\,; \quad 1 \le i \le n.$$

And the edge function $f : E(G) \to N$ defined by $f(uv) = f(u)^2 * f(v)^2$ for every $uv \in E(G)$ are all distinct.

Then the edges labels are distinct.

Then G is a square multiplicative graph.

$(i.e)$ Hence, every path is a square multiplicative graph.

Example 2.1

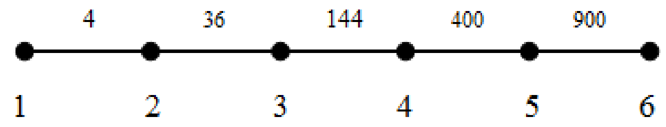

Figure 2.1 Path P_6

Theorem 2.2

Every double star $(K_{1,n,n})$ is a square multiplicative graph.

Proof

Let G be a graph of double star $(K_{1,n,n})$ with vertex set $\{u, u_i, v_i\ ;\ \ 1 \le i \le n\}$ obtained from the .

Let $\{u, u_1, u_2, \ldots\ldots, u_n\}$ and $\{v_1, v_2, \ldots\ldots, v_n\}$ be the vertices and $\{e_1, e_2, \ldots\ldots, e_{n-1}\}$ be the edges which are denoted as in the fig. 2.2.

The path consists of n vertices and $n-1$ edges.

If $G = (K_{1,n,n})$ then $|V(G)| = 2n+1$

and $|E(G)| = 2n$.

Let us define a vertex labeling $f : V(G) \to \{1, 2, \ldots\ldots, p\}$ as follows

$$f(u) = 1$$
$$f(u_i) = i+1\,; \quad 1 \le i \le n$$
$$f(v_i) = n+i+1\,; \quad 1 \le i \le n$$

And the edge function $f : E(G) \to N$ defined by $f(uv) = f(u)^2 * f(v)^2$ for every $uv \in E(G)$ are all distinct.

Then the edges labels are distinct.

Then G is a square multiplicative graph.

$(i.e)$ Hence, every double star is a square multiplicative graph.

Example 2.2

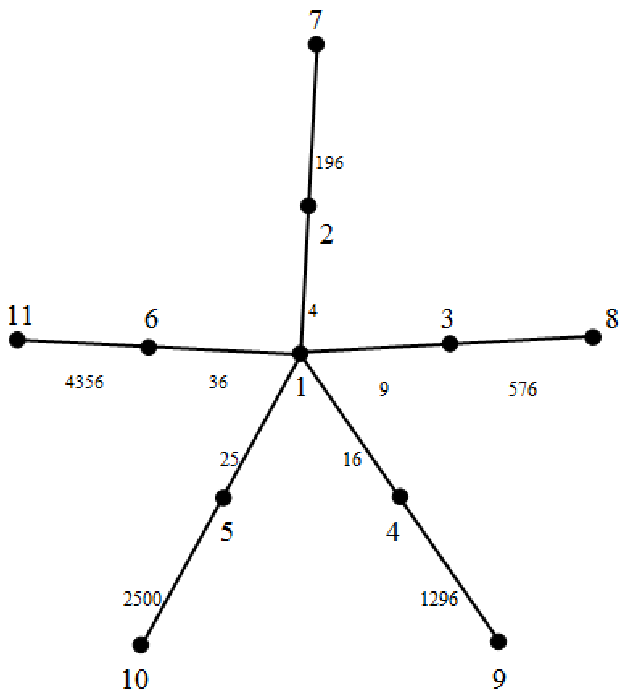

Figure 2.2 $K_{1,5,5}$

Theorem 2.3

A comb is a square multiplicative graph for $n \geq 3$.

Proof

Let G be a graph of comb with vertex set $\{u_i, v_i\ ;\ \ 1 \leq i \leq n\}$ obtained from the path .

Let $\{u_1, u_2, \ldots\ldots, u_n\}$ and $\{v_1, v_2, \ldots\ldots, v_n\}$ be the vertices and $\{e_1, e_2, \ldots\ldots, e_{n-1}\}$ be the edges which are denoted as in the fig. 2.3.

The path consists of n vertices and $n-1$ edges.
If $G = (P_n, S_1)$ then $|V(G)| = 2n$
and $|E(G)| = 2n-1$.

Let us define a vertex labeling $f : V(G) \to \{1, 2, \ldots.., p\}$ as follows

$$f(u_i) = i\ ;\quad 1 \leq i \leq n$$
$$f(v_i) = n + i\ ;\quad 1 \leq i \leq n$$

And the edge function $f : E(G) \to N$ defined by $f(uv) = f(u)^2 * f(v)^2$ for every $uv \in E(G)$ are all distinct.

Then the edges labels are distinct.
Then G is a square multiplicative graph.
$(i.e)$ Hence, every comb is a square multiplicative graph.

Example 2.3

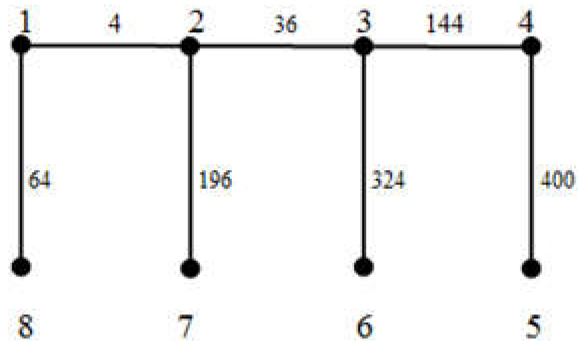

Figure 2.3 $P_4 \odot K_1$

Theorem: 2.4

Every P_n^2 is a square multiplicative graph for $n \geq 3$.

Proof

Let G be a graph of P_n^2. Let $\{v_1, v_2, \ldots\ldots, v_n\}$ be the vertices and $\{e_1, e_2, \ldots\ldots, e_{n+3}\}$ be the edges which are denoted as in the fig. 2.4.

The graph consists of n vertices and $n+3$ edges.

If $G = P_n^2$ then $|V(G)| = n$
and $|E(G)| = 2n-2$.

Let us define a vertex labeling $f : V(G) \to \{1, 2, \ldots.., p\}$ as follows

$$f(u_i) = i\ ;\quad 1 \leq i \leq n$$

And the edge function $f : E(G) \to N$ defined by $f(uv) = f(u)^2 * f(v)^2$ for every $uv \in E(G)$ are all distinct.

Then the edges labels are distinct.
Then G is a square multiplicative graph.
$(i.e)$ Hence, every P_n^2 is a square multiplicative graph.

Example 2.4

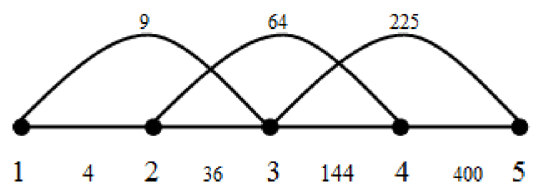

Figure: 2.4: P_5^2

CONCLUSION

In this paper, we investigated some families of graphs such as path, comb, double star, P_n^2 graphs which satisfy the formula $f(uv) = f(u)^2 * f(v)^2$ Finally, we conclude that, the above mentioned graphs are square multiplicative graphs.

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