ISSN: 0976-3031

Available Online at http://www.recentscientific.com

CODEN: IJRSFP (USA)

International Journal of Recent Scientific Research
Vol. 10, Issue, 02(C), pp. 31364-31371, March, 2019

International Journal of Recent Scientific Research

DOI: 10.24327/IJRSR

Research Article

THE RECOMPANYING OF CRACKS IN ISOTROPIC ENVIRONMENT WITH PERIODIC SYSTEM OF CIRCULAR HOLES FILLED WITH RIGIDINCLUSIONS WITH TRANSVERSE SHEAR

Mehtiyev R.K*

Azerbaijan Technical University

DOI: http://dx.doi.org/10.24327/ijrsr.2019.1002.3245

ARTICLE INFO

Article History:
Received 6th December,, 2018
Received in revised form 15th January, 2019
Accepted 12th February, 2019
Published online 28th March, 2019

Key Words:

doubly periodic lattice, fiber-binder, average stresses, linear algebraic equations, singular equations, cohesive cracks, transverse shear, stresses in bonds.

ABSTRACT

In this paper we consider an elastic medium weakened by a doubly periodic system of circular holes filled with absolutely rigid inclusions. The medium (binder) is weakened by two doubly periodic systems of rectilinear cracks with connections between the shores in the end zones. General concepts are constructed that describe a class of problems with a doubly periodic distribution of stresses outside circular holes and cracks under transverse shear. An analysis of the ultimate equilibrium of cracks in the framework of the model of the end zone is performed on the basis of a nonlocal fracture criterion with a force condition for the advance of the crack tip and a deformation condition for determining the advance of the edge of the end zone of the crack.

INTRODUCTION

As the external load increases in this plane, zones of increased stresses are formed around the holes, the location of which is periodic in nature. In areas of increased stress, cracks may occur. The problem of the initiation of a crack is an important problem in the mechanics of damage[1, 5, 10,11, 12]. The statement of this problem significantly expands the original concept of A. Grief fits, according to which the material always has a large number of tiny cracks. As the intensity of the external load increases near the holes, pre-destruction zones appear which are modeled by regions with weaker interparticle bonds in the material. The interaction of the shores of these zones is modeled by the introduction of links between the banks of the pre-destruction zone with a given deformation diagram. The physical nature of such bonds and the size of the pre-destruction zones depend on the type of material. Since these zones are small in comparison with the rest of the isotropic medium weakened by the doubly periodic system of circular holes, they can be mentally removed by replacing them with cuts whose surfaces interact with each other according to some law corresponding to the action of the remote material.

Corresponding author:* **Mehtiyev R.K
Azerbaijan Technical University

Taking these effects into account in the problems of fracture mechanics is an important but very difficult task.

Formulation of the Problem

Just imagine that there be an isotropic medium weakened by a periodic system of circular holes having radiuses $\lambda \left(\lambda < 1\right)$ and centers at points.

$$P_{mn} = m\omega_1 + n\omega_2,\ \omega_1 = 2,\ \omega_2 = \omega_1 \cdot he^{i\alpha},\quad h > 0,\ \operatorname{Im}\omega_2 > 0,\ \left(m = 0, \pm 1, \pm 2, ...\right).$$

The circular openings of the medium are filled with absolutely rigid inclusions welded along the contour.The considered surface is subjected to transverse shear by forces τ_{xy}^{∞} (fig. 1).

As the external load increases in this plane, zones of increased stresses are formed around the holes, the location of which is periodic in nature. In areas of increased stress, cracks may occur. The problem of the initiation of a crack is an important problem in the mechanics of damage. The statement of this problem significantly expands the original concept of A. Griffiths, according to which the material always has a large number of tiny cracks. The formation (initiation) of a crack under load corresponds to the data of friction graphic observations. As the intensity of the external load increases near the holes, pre-destruction zones appear, which are modeled by regions with weaker inter particle bonds in the material. The interaction of the shores of these zones is modeled by the introduction of links between the banks of the pre-destruction zone with a given deformation diagram. The physical nature of such bonds and the size of the pre-destruction zones depend on the type of material. Since these zones (layers of over-stressed material) are small in comparison with the rest of the isotropic medium weakened by the periodic system of circular holes, they can be mentally removed by replacing slits whose surfaces interact with each other according to some law corresponding to the action of the remote material.

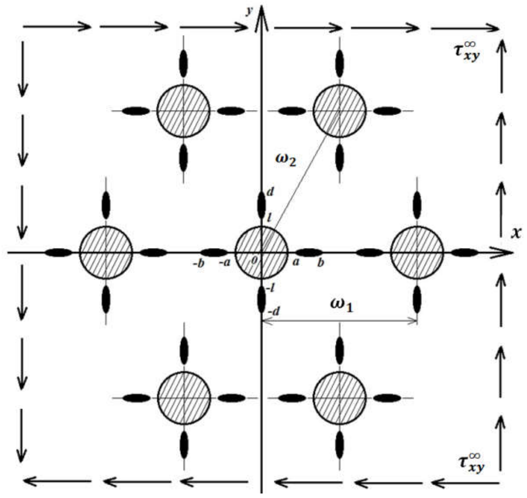

Fig 1 The calculation scheme for the problem of the initiation of cracks in a mediumwith rigid inclusions at transverse shear

Taking these effects into account in the problems of fracture mechanics is an important but very difficult task.

In the case under researching, the appearance of an embryonic fissure in a medium weakened by a periodic system of circular holes is the process of transition of the prefracture region to the region of broken bonds between the surfaces of the material. In this case, the size of the pre-destruction zone is unknown in advance and is to be determined.

Studies of the appearance of regions with a disturbed structure of the material indicate that in the initial stage the pre-destruction zones are a narrow elongated layer, and then, as the load increases, a secondary system of zones appears suddenly with material with partially broken connections.

For a mathematical description of the nucleation of a crack in an isotropic medium weakened by a periodic system of circular holes filled with rigid inclusions in the case under consideration, we arrive at the problem of the theory of elasticity for a medium when there are pre-destruction zones in the medium. The pre-destruction zones are oriented in the direction of the maximum tangential stresses. It is assumed that in an isotropic medium there are two periodic systems of rectilinear prefracture zones collinear with the abscissa and ordinate axes (fig. 1) of unequal length. Interaction of the shores of the pre-destruction zone (connection between the banks) restrains the initiation of a crack. For a mathematical description of the interaction of the shores of the pre-destruction zone, it is assumed that there are links between them, the deformation law of which is given.

A model of crack initiation in composites with a periodic structure is proposed, based on the checking of the zone of the cracking process.

It is believed that the zone of the cracking process is a layer of definite length containing material with partially broken bonds between individual structural elements.The present of links between the shores of the pre-destruction zone (zones of weakened inter particle bonds of the material) is modeled by applying to the surface of the pre-destruction zone the cohesion forces caused by the presence of bonds. The analysis of the limiting equilibrium of the pre-destruction zone in the case of transverse shear is performed on the basis of the criterion for limiting the shear of the bonds of the material and includes: 1) establishing the dependence of the adhesion forces on the shift of the shores of the pre-destruction zone; 2) evaluation of the stress pasted near the pre-destruction zone, taking into account external loads and adhesion forces, as well as the location of rigid inclusions; 3) determination of the dependence of critical external loads on the geometric parameters of the compound medium, under which a crack appears.

When an additional load $q_x(x)$ and $q_y(y)$ acts on the complex body in the bonds that connect the shores of the pre-destruction zones, tangential forces arise, respectively. These stresses are unknown in advance and are to be determined from the solution of the boundary value problem of fracture mechanics by boundary conditions expressing the absence of elastic displacements along the circumference of circular holes and conditions on the shores of the pre-destruction zones, respectively [2, 6, 7]

$u + i\upsilon = 0$ on contours of circular holes

on the shores of pre-destruction zones (1)

$\sigma_y - i\tau_{xy} = -iq_x(x)$ collinear axis of abscissae

(2)

$\sigma_x - i\tau_{xy} = -iq_y(y)$ collinear axis of ordinates

The basic relations of the task should be supplemented by the relations connecting the shift of the shores of the pre-destruction zones and the tangential forces in the bonds. Without loss of generality, we represent these relations in the form

$$u^+(x, 0) - u^-(x, 0) = Q(x, q_x(x))q_x(x), \tag{3}$$

$$\upsilon^+(0, y) - \upsilon^-(0, y) = Q(y, q_y(y))q_y(y), \tag{4}$$

where the functions $Q(x, q_x(x))$ and $Q(y, q_y(y))$ are the effective compliances of the constraints; $(u^+ - u^-)$–shift of the banks of the pre-destruction zones of the collinear axis of abscissas; $(\upsilon^+ - \upsilon^-)$–shift of the banks of the pre-destruction zones of the collinear axis of ordinates.

To determine the limiting value of the external load at which the crack nucleation occurs, it is necessary to supplement the formulation of the problem with the condition (criterion) for the appearance of a crack (breaking of interparticle bonds in the material). As such a condition, we take criterion for the critical shift of the shores of the pre-destruction zone

$$u^{+}-u^{-}=\delta_{IIc} \text{ in } L_1$$

$$\upsilon^{+}-\upsilon^{-}=\delta_{IIc} \text{ in } L_2, \quad (5)$$

where δ_{IIc} – is the characteristic of the resistance of the material of the medium to cracking; L_1 – a set of zones of pre-destruction, collinear axis of abscissas; L_2 – a set of zones of pre-destruction, collinear axis of ordinates.

METHOD FOR SOLVING THE PROBLEM

To solve the problem, the method developed in the solution of the periodic elastic problem, with the method constructing in explicit form the Kolossov–Muskhelishvili potentials surrounding to unknown tangential displacements along the pre-destruction zones, is naturally combined.

Voltage and displacement in the straight elastic theory we can describe[9] it with the two analytic functions $z=x+iy$ $\Phi(z)$ and $\Psi(z)$ in the help of the formule of Kolossova-Muschelishvili:

$$\sigma_y+\sigma_x=\sigma_r+\sigma_\theta=2\left[\Phi(z)+\overline{\Phi(z)}\right], \quad (6)$$

$$\sigma_y-\sigma_x+2i\tau_{xy}=e^{-2i\theta}\left(\sigma_\theta-\sigma_r+2i\tau_{r\theta}\right)=2\left[\bar{z}\Phi'(z)+\Psi(z)\right],$$

$$2\mu(u+i\upsilon)=\kappa\varphi(z)-z\overline{\Phi(z)}-\overline{\psi(z)},$$

$$\varphi'(z)=\Phi(z), \qquad \psi'(z)=\Psi(z),$$

where μ –moduleofthedisplacematerial;ν – coefficentofPuasson; $\kappa=3-4\nu$ – for elastic deformation; $\kappa=(3-\nu)/(1+\nu)$ –fro elastic voltage condition; r,θ – polar coordinats.

On the basis of formulas (6) and boundary conditions on the contours of circular holes (1) and pre-destruction zones (2), the problem reduces to the determination of two functions analytic in the domain D and from the boundary conditions (*t* and t_1 are the affixes of the points of the banks of the pre-destruction zones collinear with the abscissa axes and ordinate, respectively).

$$\varepsilon\overline{\Phi(\tau)}+\Phi(\tau)-\left[\bar{\tau}\Phi'(\tau)+\Psi(\tau)\right]e^{2i\theta}=0, \quad (7)$$

$$\Phi(t)+\overline{\Phi(t)}+t\overline{\Phi'(t)}+\overline{\Psi(t)}=-iq_x(t), \quad (8)$$

$$\Phi(t_1)+\overline{\Phi(t_1)}+t_1\overline{\Phi'(t_1)}+\overline{\Psi(t_1)}=-iq_y(t_1),$$

where; $\tau=\lambda e^{i\theta}+m\omega_1+n\omega_2$, $m,n=0,\pm1,\pm2,\ldots$,.

The statement of the problem covers simultaneously cases of rigid inclusions ($\varepsilon=-k$) and free holes $(\varepsilon=1)$.

The solution of the boundary value problem (7) – (8) is sought in the form

$$\Phi(z)=\Phi_1(z)+\Phi_2(z)+\Phi_3(z), \quad (9)$$

$$\Psi(z)=\Psi_1(z)+\Psi_2(z)+\Psi_3(z),$$

$$\Phi_1(z)=\frac{1}{2\omega}\int_{L_1} g(t)\,ctg\frac{\pi}{\omega}(t-z)\,dt, \quad (10)$$

$$\Psi_1(z)=-\frac{\pi z}{2\omega^2}\int_{L_1} g(t)\sin^{-2}\frac{\pi}{\omega}(t-z)\,dt,$$

$$\Phi_2(z)=\frac{i}{2\omega}\int_{L_2} g_1(t_1)\,ctg\frac{\pi}{\omega}(it_1-z)\,dt_1,$$

$$\Psi_2(z)=-\frac{i}{2\omega}\int_{L_2}\left\{g_1(t_1)\left[2ctg\frac{\pi}{\omega}(it_1-z)+\frac{\pi}{\omega}(2t_1+iz)\sin^2\frac{\pi}{\omega}(it_1-z)\right]\right\}dt_1,$$

$$\Phi_3(z)=i\tau_{xy}^{\infty}+i\sum_{k=0}^{\infty}\alpha_{2k+2}\frac{\lambda^{2k+2}\rho^{(2k)}(z)}{(2k+1)!}+i\alpha_0, \quad (11)$$

$$\Psi_3(z)=i\tau_{xy}^{\infty}+i\sum_{k=0}^{\infty}\beta_{2k+2}\frac{\lambda^{2k+2}\rho^{(2k)}(z)}{(2k+1)!}-i\sum_{k=0}^{\infty}\alpha_{2k+2}\frac{\lambda^{2k+2}S^{(2k+1)}(z)}{(2k+1)!},$$

Where

$$\rho(z)=\left(\frac{\pi}{\omega}\right)^2\sin^{-2}\left(\frac{\pi}{\omega}z\right)-\frac{1}{3}\left(\frac{\pi}{\omega}\right)^2, \quad S(z)=$$

$$\sum_m{}'\left[\frac{P_m}{(z-P_m)^2}-\frac{2z}{P_m}-\frac{1}{P_m}\right]$$

the prime of the sum means that the summation excludes the index m = 0,the integrals in (10) are taken along the lines;

$$L_1=\{[-b,-a]\cup[a,b]; \qquad L_2=[-d,-l]\cup[l,d]\},$$

$g(t)$ и $g_1(t_1)$ – the required functions characterizing the shift of the banks of the pre-destruction zones

$$g(x)=-\frac{2\mu i}{1+\kappa}\frac{d}{dx}\left[u^{+}(x,0)-u^{-}(x,0)\right] \text{in} L_1, \quad (12)$$

$$g_1(y)=\frac{2\mu}{1+\kappa}\frac{d}{dy}\left[\upsilon^{+}(0,y)-\upsilon^{-}(0,y)\right] \text{in} L_2.$$

To the relations (9) –(12) we must add additional conditions that follow from the physical meaning of the problem

$$\int_{-b}^{-a} g(t)\,dt = 0\,; \qquad \int_{a}^{b} g(t)\,dt = 0\,; \int_{-d}^{-l} g_1(t_1)\,dt_1 = 0\,;$$

$$\int_{l}^{d} g_1(t_1)\,dt_1 = 0\,. \tag{13}$$

We give the dependencies that the coefficients of expressions (9) – (11) must satisfy.

From the anti-symmetric conditions with respect to the coordinate axes, we find that

$$\operatorname{Im}\alpha_{2k} = 0, \quad \operatorname{Im}\beta_{2k} = 0, \quad (k = 1, 2, ...).$$

From the condition of constancy of the principal vector of all forces acting on the arc joining two congruent points in *D*, it follows that [8]

$$\alpha_0 = \frac{\pi^2}{24}\beta_2\lambda^2$$

It is easy to verify that the functions (9)–(11), under the condition (13), determine the class of problems by a periodic stress distribution.

The unknown functions $g(x)$ and $g_1(y)$, constants α_{2k} and β_{2k} must be determined from the boundary conditions (7) and (8). In view of the fulfillment of the periodicity conditions, the system of boundary conditions (7) is replaced by a single functional equation, for example, on the contour $\tau = \lambda e^{i\theta}$, and the system of boundary conditions (8) – boundary conditions on the contours L_1 and L_2.

To form equations α_{2k}, β_{2k} with respect to the coefficients $\Phi_3(z)$ and $\Psi_3(z)$ functions and represent the boundary condition (7) in the form

$$\varepsilon\overline{\Phi_3(\tau)} + \Phi_3(\tau) - \left[\overline{\tau}\Phi_3'(\tau) + \Psi_3(\tau)\right]e^{2i\theta}$$
$$= f_1(\theta) + if_2(\theta) + \varphi_1(\theta) + i\varphi_2(\theta), \tag{14}$$

$$f_1(\theta) + if_2(\theta) = -\varepsilon\overline{\Phi_1(\tau)} - \Phi_1(\tau) + e^{2i\theta}\left[\overline{\tau}\Phi_1'(\tau) + \Psi_1(\tau)\right], \tag{15}$$

$$\varphi_1(\theta) + i\varphi_2(\theta) = -\varepsilon\overline{\Phi_2(\tau)} - \Phi_2(\tau) + e^{2i\theta}\left[\overline{\tau}\Phi_2'(\tau) + \Psi_2(\tau)\right].$$

With respect to the functions u $f_1(\theta) + if_2(\theta)$ and $\varphi_1(\theta) + i\varphi_2(\theta)$ we will assume that they are expanded in Fourier series. Because of the anti-symmetric, these series have the form

$$f_1(\theta) + if_2(\theta) = \sum_{k=-\infty}^{\infty} A_{2k}e^{2ik\theta}\,, \operatorname{Re}A_{2k} = 0\,, \tag{16}$$

$$A_{2k} = \frac{1}{2\pi}\int_0^{2\pi}\left[f_1(\theta) + if_2(\theta)\right]e^{-2ik\theta}d\theta\,,$$
$$(k = 0, \pm 1, \pm 2, ...),$$

$$\varphi_1(\theta) + i\varphi_2(\theta) = \sum_{k=-\infty}^{\infty} B_{2k}e^{2ik\theta}\,, \qquad \operatorname{Re}B_{2k} = 0\,,$$

$$B_{2k} = \frac{1}{2\pi}\int_0^{2\pi}\left[\varphi_1(\theta) + i\varphi_2(\theta)\right]e^{-2ik\theta}d\theta\,,$$
$$(k = 0, \pm 1, \pm 2, ...).$$

Exacting here the relations (12) and changing the order of integration, after calculating the integrals using the theory of residues, we find

$$A_{2k} = -\frac{1}{2\omega}\int_{L_1} g(t)f_{2k}(t)dt\,, \qquad f_0(t) = (1+\varepsilon)\gamma(t)\,,$$

$$\gamma(t) = ctg\frac{\pi}{\omega}t\,, \;\; f_2(t) = -\frac{\lambda^2}{2}\gamma^{(2)}(t),$$

$$f_{2k}(t) = -\frac{\lambda^{2k}(2k-1)}{(2k)!}\gamma^{(2k)}(t) + \frac{\lambda^{2k-2}}{(2k-3)!}\gamma^{(2k-2)}(t)\,,$$

$(k = 2, 3, \ldots)$,

$$f_{-2k}(t) = \frac{\varepsilon\lambda^{2k}}{(2k)!}\gamma^{(2k)}(t) \qquad (k=1,2,\ldots),$$

$$B_{2k} = -\frac{i}{2\omega}\int_{L_2} g_1(t_1)\varphi_{2k}(it_1)dt_1\,,$$

$$\varphi_0(it_1) = \frac{1}{2}(1+\varepsilon)\left[\delta(it_1) - \overline{\delta(it_1)}\right], \;\; \delta(it_1) = \operatorname{ctg}\frac{\pi}{\omega}(it_1),$$

$$\varphi_2(it_1) = -\frac{\lambda^2}{2}\delta^{(2)}(it_1) + 2\left[\delta(it_1) - it\delta'(it_1)\right],$$

$$\varphi_{2k}(it_1) = \frac{(1-2k)\lambda^{2k}}{(2k)!}\delta^{(2k)}(it_1) + \frac{2\lambda^{2k-2}}{(2k-2)!}\left[\kappa\delta^{(2k-2)}(it_1) - it_1\delta^{(2k-1)}(it_1)\right],$$

$(k = 2, 3, \ldots)$,

$$\varphi_{-2k}(it_1) = -\frac{\varepsilon\lambda^{2k}}{(2k)!}\overline{\delta^{(2k)}(it_1)}, \quad (k = 1, 2, \ldots).$$

Substituting the Fourier series (14)instead of $\Phi_3(\tau)$, $\overline{\Phi_3(\tau)}$, $\Phi_3'(\tau)$ and $\Psi_3(\tau)$ the Fourier series (14) their occuptions in the surrounding is *z=0* line of Loran in the right side (14) row Fure (16) and in the same levels $e^{i\theta}$ comparing coefficients we get two infinitive algebraic systems in the measurement of α_{2k} and β_{2k}.

$$i\alpha_{2j+2}=\sum_{j=0}^{\infty}ia_{j,k}\alpha_{2k+2}+b_j\,,\quad \left(j=0,1,2,...\right),\tag{17}$$

$$a_{j,k}=\frac{1}{\varepsilon}\left(2j+1\right)\gamma_{j,k}\lambda^{2k+2j+2},$$

$$\gamma_{0,0}=\frac{3}{8}g_2\lambda^2+\sum_{i=1}^{\infty}\frac{\left(2i+1\right)g_{i+1}^2\lambda^{4i+2}}{2^{4i+4}},$$

$$\gamma_{j,k}=-\frac{\left(2j+2k+2\right)!g_{j+k+1}}{\left(2j+1\right)!\left(2k+1\right)!2^{2j+2k+2}}+\frac{\left(2j+2k+4\right)!g_{j+k+2}\lambda^2}{\left(2j+2\right)!\left(2k+2\right)!2^{2j+2k+4}}+$$

$$+\frac{g_{j+1}g_{k+1}\lambda^2}{2^{2j+2k+4}}\left[1+\frac{\left(1+\varepsilon\right)^2K_2\lambda^2}{1-\left(1+\varepsilon\right)K_2\lambda^2}\right]+$$

$$+\varepsilon\sum_{i=0}^{\infty}\frac{\left(2j+2k+1\right)!\left(2k+2i+1\right)!g_{j+k+1}g_{k+i+1}\lambda^{4i+2}}{\left(2j+1\right)!\left(2k+1\right)!\left(2i+1\right)!\left(2i\right)!2^{2j+2k+4i+4}},$$

$\left(j,\,k=1,\,2,\,...\right)$,

$$\varepsilon b_j=A'_{2j+2}-\frac{\left(2j+1\right)g_{j+1}\lambda^{2k+2}}{\left(1-\left(1+\varepsilon\right)K_2\right)2^{2j+2}}A'_0-\sum_{k=0}^{\infty}\frac{\left(2j+2k+3\right)!g_{j+k+2}\lambda^{2j+2k+4}}{\left(2j\right)!\left(2k+3\right)!2^{2j+2k+4}}A'_{-2k+2}$$

$$g_j=2\sum_{m=1}^{\infty}\frac{1}{m^{2j}},\qquad \left(j=1,2,...\right),\qquad K_2=\frac{\pi^2}{24},$$

$$A'_0=A_0+B_0-2i\tau_{xy}^{\infty},$$

$$A'_2=A_2+B_2+i\tau_{xy}^{\infty},\quad A'_{2k}=A_{2k}+B_{2k}$$

$$\left(k=-1,\pm 2,...\right).$$

The coefficients β_{2k} are determined by this method

$$i\beta_{2k}=\frac{1}{1-\left(1+\varepsilon\right)K_2\lambda^2}\left[-A'_0+\left(1+\varepsilon\right)\sum_{k=1}^{\infty}\frac{g_{k+1}\lambda^{2k+2}i\alpha_{2k+2}}{2^{2k+2}}\right],\tag{18}$$

$$i\beta_{2j+4}=i\left(2j+3\right)\alpha_{2j+2}+\varepsilon\sum_{k=0}^{\infty}\frac{\left(2j+2k+3\right)!\,g_{j+k+2}\lambda^{2j+2k+4}}{\left(2j+2\right)!\,\left(2k+1\right)!\,2^{2j+2k+4}}i\alpha_{2k+2}-A'_{-2j-2}\,.$$

Requiring that the functions (9) – (11) satisfy the boundary condition on the shore of the pre-destruction zone L_1, we obtain a singular integral equation with [5] respect to $g\left(x\right)$:

$$\frac{1}{\omega}\int_{L_1}g\left(t\right)ctg\frac{\pi}{\omega}\left(t-x\right)dt+H(x)=-iq_x(x)\,,\tag{19}$$

$$H(x)=\Phi_s(x)+\overline{\Phi_s(x)}+x\overline{\Phi'_s(x)}+\overline{\Psi_s(x)}\,,$$

$$\Phi_s(x)=\Phi_2(x)+\Phi_3(x)\,,\ \ \Psi_s(x)=\Psi_2(x)+\Psi_3(x)\,.$$

Similarly, satisfying the boundary condition on the line L_2, after some transformations we obtain one more singular integral equation with respect to the required function $g_1\left(y\right)$:

$$-\frac{\pi}{\omega^2}\int_{L_2}g_1\left(t\right)\left[\left(t-y\right)sh^{-2}\frac{\pi}{\omega}\left(t-y\right)\right]dt+N(y)=-iq_y(y)\tag{20}$$

$$N(y)=\Phi_0(iy)+\overline{\Phi_0(iy)}+\left(iy\right)\overline{\Phi'_0(iy)}+\overline{\Psi_0(iy)}\,,$$

$$\Phi_0(z)=\Phi_1(z)+\Phi_3(z)\,,\ \ \Psi_0(z)=\Psi_1(z)+\Psi_3(z)\,.$$

Systems (17) and (18), together with the singular integral equations (19) and (20), are the basic resolving equations of the problem that allow us to determine the functions g(x) and g_1(y)coefficients α_{2k}, β_{2k}.

Method of Numerical Solution and Analysis

Using the expansion $ctg\frac{\pi}{\omega}z$, $sh^{-2}\frac{\pi}{\omega}z$ of the functions, in the main period band, and also using a change of variables, after some transformations singular equations are reduced to the standard form. Using quadrature formulas, we reduce the basic resolving equations (17), (18) – (20) to the set of two infinite systems of linear algebraic equations and to two finite algebraic systems

$$p_k^0=g(\eta_k)\,,\left(k=1,\,2,\,...,\,M\right),\ R_v^0$$

$\left(v=1,\,2,\,...,\,M\right)$with respect to approximate values of the unknown functions at the node points.

$$\sum_{v=1}^{M}A_{m,v}p_v^0-\frac{1}{2}H_*\left(\eta_m\right)=-iq_x\left(\eta_m\right),\ (m=1,\,2,\,\ldots,\,M-1),\tag{21}$$

$$\sum_{v=1}^{M}A_{m,v}\frac{p_k^0}{\sqrt{1/2\left(1-\lambda_1^2\right)\left(\tau_v+1\right)+\lambda_1^2}}=0\,,$$

$$\sum_{v=1}^{M}B_{m,v}R_v^0+\frac{1}{2}N_*\left(\eta_m\right)=-iq_y\left(\eta_m\right)(m=1,\,2,\,\ldots,\,M-1).\tag{22}$$

Here

$$A_{m,v}=\frac{1}{2M}\left[\frac{1}{\sin\theta_m}\mathrm{ctg}\frac{\theta_m+\left(-1\right)^{|m-v|}\theta_v}{2}+B\left(\eta_m,\tau_v\right)\right],$$

$$\theta_m=\frac{2m-1}{2M}\pi\,,\quad \left(m=1,2,...,M\right),\qquad \tau_m=\cos\theta_m\,,$$

$$\eta_m=\tau_m\,,\ \lambda_1=\frac{a}{\ell}\,,$$

$$B\left(\eta,\tau\right)=-\frac{1-\lambda_1^2}{2}\sum_{j=0}^{\infty}g_{j+1}\left(\frac{\ell}{2}\right)^{2j+2}\cdot u_0^jA_j\,;$$

$$A_j=\left[(2j+1)+\frac{(2j+1)(2j)(2j-1)}{1\cdot 2\cdot 3}\left(\frac{u}{u_0}\right)+\ldots+\right.$$
$$\left.\frac{(2j+1)(2j)(2j-1)\ldots[(2j+1)-(2j+1-1)]}{1\cdot 2\ldots(2j+1)}\cdot\left(\frac{u}{u_0}\right)^j\right],$$
$$u=\frac{1-\lambda_1^2}{2}(\tau+1)+\lambda_1^2,\qquad u_0=\frac{1-\lambda_1^2}{2}(\eta+1)+\lambda_1^2,$$
$$B_{mv}=\frac{1}{2M}\left[\frac{1}{\sin\theta_m}\mathrm{ctg}\frac{\theta_m+(-1)^{|m-v|}\theta_v}{2}+B_*(\eta_m,\tau_v)\right],$$
$$B_*(\eta,\tau)=-\frac{1-\lambda_2^2}{2}\sum_{j=0}^{\infty}(-1)^j(2j+1)g_{j+1}\left(\frac{r}{2}\right)^{2j+2}\cdot u_1^jA_j'$$
,
$$A_j'=\left\{(2j+1)+\frac{(2j+1)(2j)(2j-1)}{1\cdot 2\cdot 3}\left(\frac{u_1}{u_2}\right)+\ldots+\left(\frac{u_1}{u_2}\right)^j\right\},$$
$$u_1=\frac{1-\lambda_2^2}{2}(\tau+1)+\lambda_2^2,$$
$$u_2=\frac{1-\lambda_2^2}{2}(\eta+1)+\lambda_2^2,\qquad \lambda_2=\frac{B}{r}.$$

The right part of the systems $q_x(\eta_m)$ and $q_y(\eta_m)$ obtained includes unknown voltages at the nodal points belonging to the pre-destruction zones. Using the solution obtained, we represent the equation (12) in the form

$$g(x)=-\frac{2\mu i}{1+\kappa}\frac{d}{dx}\left[C(x,q_x(x))q_x(x)\right],$$
$$g_1(y)=\frac{2\mu}{1+\kappa}\frac{d}{dy}\left[C(y,q_y(y))q_y(y)\right]. \tag{23}$$

These equations serve to determine the effort in the links between the shores of the pre-destruction zones. To construct the missing equations, we require that the conditions (2) be satisfied at the node points belonging to the pre-destruction zones. In doing so, we use the finite difference method. As a result, we obtain two more systems of *M* equations each to determine the approximate values $q_x(\eta_m)$,(m = 1, 2, ..., M) and $q_y(\eta_m)$,(*m = 1, 2, ..., M*). Since the stresses are bounded in the perforated body, the solution of the singular integral equations should be sought in the class of everywhere bounded functions. Consequently, to the systems (21)–(22), we must add the conditions for the boundedness of the stresses at the vertices of the pre-destruction zones

$$\sum_{k=1}^{M}(-1)^{k+M}p_k^0 tg\frac{\theta_k}{2}=0,\qquad \sum_{k=1}^{M}(-1)^k p_k^0 ctg\frac{\theta_k}{2}=0,$$

$$\sum_{v=1}^{M}(-1)^{v+M}R_v^0 tg\frac{\theta_v}{2}=0,\quad \sum_{v=1}^{M}(-1)^v R_v^0 ctg\frac{\theta_v}{2}=0 \tag{24}$$

The resulting systems of equations (17), (18), (21) – (24) completely determine the solution of the problem. For the numerical realization of the above method, calculations were performed. Each of the infinite systems was cut to five equations. In numerical calculations, *M = 30* was assumed, which corresponds to a partition of the integration interval into 30 chebyshev nodes. Since the dimensions of the pre-destruction zones are unknown, the resolving algebraic system of equations (17), (18), (21) – (24) of the problem is non-linear even for linear constraints. To solve it, we use the method of successive approximations, the essence of which is as follows: we solve the combined algebraic system for certain definite values of the dimensions of the pre-destruction zones relative to the remaining unknowns. The remaining unknowns enter the resolving system in a linear fashion. The accepted values of the dimensions of the pre-destruction zones and the corresponding values of the remaining unknowns will not, in general, satisfy the conditions for the bounded stresses at the vertices of the pre-destruction zones. Therefore, choosing the values of the dimensions of the pre-destruction zones, we will repeatedly repeat the calculations until the conditions for the boundedness of the stresses (24) are satisfied with a given accuracy. In the case of a nonlinear law of deformation of bonds, an iterative algorithm similar to the method of elastic solutions was used to determine the tangential forces in the prefracture zones [9]. It is believed that the law of deformation of interparticle bonds in the pre-destruction zone is linear when $(u^+-u^-)\le u_*$ and $(\upsilon^+-\upsilon^-)\le\upsilon_*$. The first step of the iterative counting process is to solve the system of equations for linear-elastic constraints. The following iterations are performed only in the event that on the part of the pre-destruction zone there is an inequality $(u^+-u^-)>u_*$ or $(\upsilon^+-\upsilon^-)>\upsilon_*$. For such iterations, a system of equations is solved in each approximation for quasi-elastic bonds with the pre-destructive zone varying along the shores and the force-dependent forces in the effective compliance bonds, which was computed at the previous step of the calculation. The calculation of effective compliance is similar to the definition of the secant modulus in the variable elasticity method [4]. The process of successive approximations ends when the forces along the pre-destruction zone obtained at two consecutive iterations are practically unchanged. The nonlinear part of the bond strain curve was approximated by a bilinear dependence [8], the ascending segment of which corresponded to the deformation of bonds $\left(0<(u^+-u^-)\le u_*\right)$ with their maximum bonding force. When $(u^+-u^-)>u_*$ the law of deformation was described by a nonlinear dependence, determined by the points (u_*,τ_*) and (δ_c,τ_c), with an increasing $\tau_c\ge\tau_*$ linear dependence (linear hardening corresponding to the elastoplastic deformation of the bonds).

To determine the limiting equilibrium state of the medium at which a crack appears, we use condition (5). Using the solution obtained, the following conditions are determined by the conditions determining the ultimate external load [2, 3]:

$$Q(r, q_y(r))q_y(r) = \delta_{IId}\,, \quad Q(r^*, q_x(r^*))q_x(r^*) = \delta_{IId}\,. \qquad (25)$$

Here, $x = \pm d$ и $x = \pm d^*$ the coordinates of the points where the crack is formed, respectively.

As a result of the numerical calculation, the length of the pre-destruction zones, the forces in the bonds and the shift of the opposite banks of the pre-destruction zones from the loading parameter τ_{xy}^{∞}.

In fig. 2 shows the plot of the relative length of the pre–destruction zone $b_* = (b-a)/\lambda$ from the dimensionless value of external loading $\tau_{xy}^{\infty}/\tau_*$ for different values of the radius of the holes (curves 1 – 4); $1 - \lambda = 0{,}2$; $2 - \lambda = 0{,}3$; $3 - \lambda = 0{,}4$; $4 - \lambda = 0{,}5$.

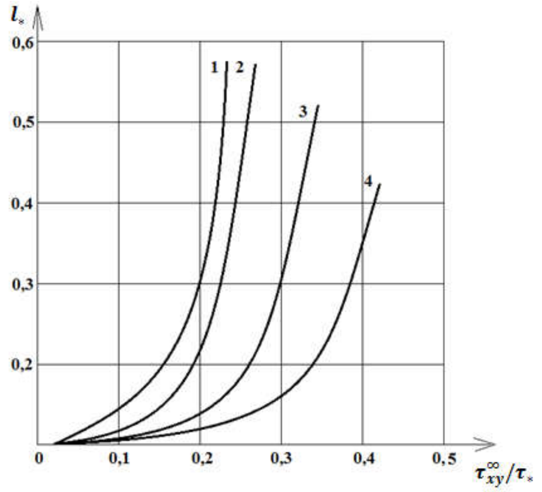

Fig 2 Dependences of the relative length of the pre–destruction zone $b_* = (b-a)/\lambda$ on the dimensionless value of external loading $\tau_{xy}^{\infty}/\tau_*$ for certain values of the radius of the holes $\lambda = 0{,}2 \div 0{,}5$ (curves 1 – 4)

In fig. 3 shows the dependence of forces q_x/τ_{xy}^{∞} in the bonds along the pre-destruction zone on the dimensionless coordinate $x = (\ell + a)/2 + x'(\ell - a)/2$ for different values of the hole radius: $\lambda = 0{,}2 \div 0{,}5$ (curves 1 – 4).

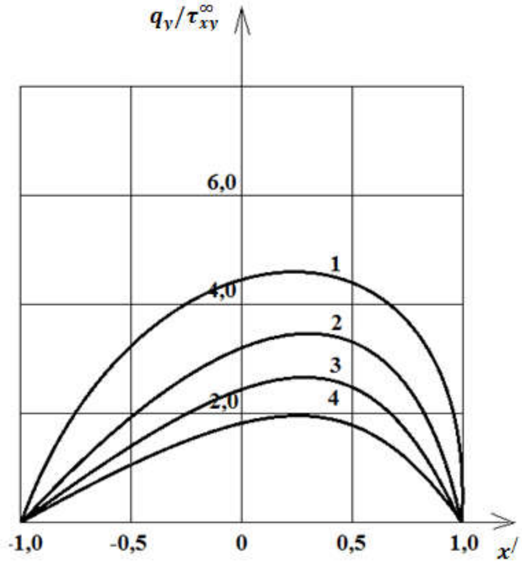

Fig 3 Dependences of the distribution of tangential stresses q_x/τ_{xy}^{∞} in bonds along the pre-destruction zone for different values of the hole radius: $\lambda = 0{,}2 \div 0{,}5$ (curves 1 – 4)

The joint solution of the resolving algebraic system and the conditions (25) makes it possible (with the given characteristics of the crack resistance of the material) to determine the critical value of the external load, the size of the prefracture zones for the state of limiting equilibrium at which a crack appears.
On the basis of the numerical results obtained in fig. 4 slots of the dependence of the critical load $\tau^* = \tau_{xy}^{\infty}/\tau_*$ on distance for the zone of prefracture, collinear axes of abscissas are plotted $\lambda = 0{,}3$.

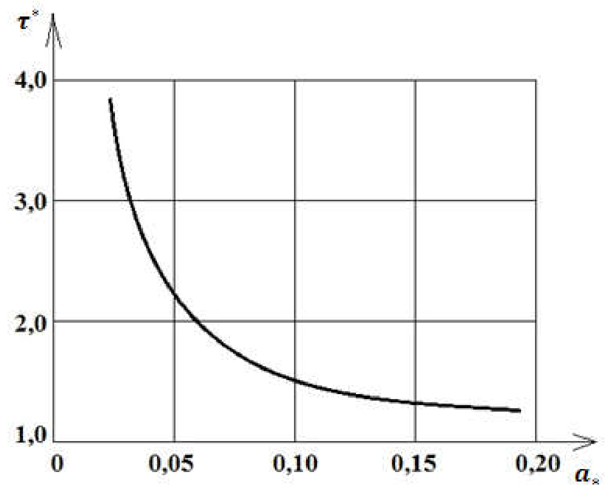

Fig 4 Dependence of the critical load $\tau^* = \tau_{xy}^{\infty}/\tau_*$ on the distance $a_* = a - \lambda$ at $\lambda = 0{,}3$

In fig. 5 shows the dependence of the critical load τ^{*a} upon changing the length of the prefracture $b_* = b - a$ zone for $\lambda = 0{,}3$, $a_* = 0{,}05$

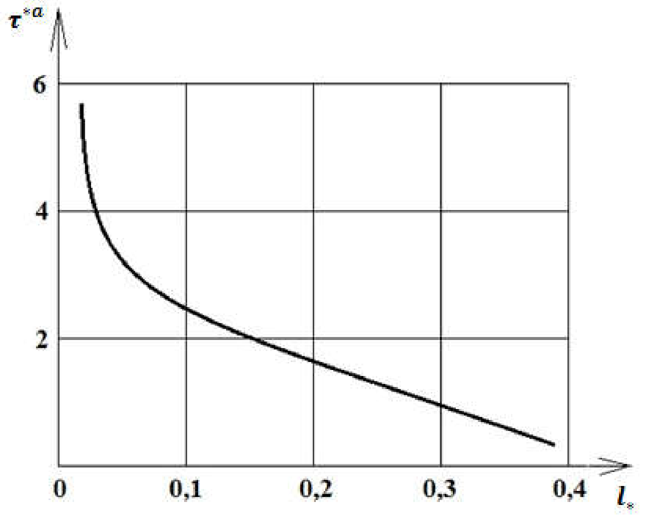

Fig 5 The dependence of the critical load $\tau^{*a} = \tau_{xy}^{\infty} / \tau_*$ upon changing the length of the prefracture zone $b_* = b - a$ at, $\lambda = 0{,}3$, $a_* = 0{,}05$

Analysis of the equilibrium state of a body with a periodic system of rigid inclusions and shores of pre-destruction zones with connections between the shores for transverse shear reduces to a parametric study of the resolving algebraic system (17), (18), (21), (22) – (24) and the deformation criterion of destruction (25) for various laws of deformation of interparticle bonds of a material, elastic constants and geometric characteristics of a perforated body. Directly from the solution of the algebraic systems obtained, the efforts in the connections and the shift of the shores of the pre-destruction zones are determined. The model of the crack initiation with connections between the banks makes it possible to investigate the basic laws of the distribution of forces in bonds under various laws of their deformation, analyze the ultimate equilibrium of the medium with the pre-destruction zone taking into account the deformation condition of the crack initiation, and evaluate the critical external stress and fracture toughness of the material.

The obtained relations make it possible to investigate the limiting-equilibrium state of a medium with a periodic system of circular holes filled with absolutely rigid inclusions welded along the contour and weakened by rectilinear pre-destruction zones with links between the collinear axes of abscissa and ordinates of unequal length under transverse shear.

CONCLUSIONS

The obtained relationships make it possible to investigate the limiting-equilibrium state of a medium with a doubly periodic system of circular holes filled with absolutely rigid inclusions welded along the contour and weakened by rectilinear prefracture zones with links between the collinear axis of absciss and ordinate of unequal length under transverse shear.

References

1. Birger I.A. General algorithms for solving problems in the theory of elasticity, plasticity, and creep // Uspekhi mekhaniki deformirovykh sred (Successes in the mechanics of deformable media), Moscow: Nauka, 1975.- C. 51-73.
2. Bolotin, V. V. Mechanics of nucleation and initial development of fatigue cracks // Fizikohim. mechanics of materials. - 1986. - P. 22, No. 1. - P. 18-23.
3. Goldstein, R.V., Modeling of the Crack Resistance of Composite Materials, by R. V. Goldshtein, M. N. Perelmuter // Comput. mechanics of continuous media. - 2009. - T. 2, No. 2. - P. 22-39.
4. Ilyushin, A.A. Plasticity / A.A. Ilyushin. - Moscow: LOGO, 2004. - 376 p.
5. Ladopoulos, E. G. Singular integral Equations, Linear and Non-Linear Theory and its Applications in Science and Engineering / E. G. Ladopoulos. - New York, Berlin: Springer Verlag. 2000. - 547 p.
6. Mamedov A.T., Mekhtiev R.K. Modeling of a fibrous composite reinforced with unidirectional orthotropic fibers weakened by rectilinear cracks under longitudinal shear / Mamedov A.T., Mekhtiev R.K. / / Mechanics of composite materials and structures. October-December 2017, VOLUME 23, No. 4, p. 579-591
7. Mekhtiev R.K. Dzhafarova S.A. Abdulazimova E.A. Interaction of a doubly periodic system of orthotropic inclusions and rectilinear cracks under transverse shear Miedzynarodowe czasopismo naukowe, Colloquium-journal, No. 2 (13), 2018 Czesc 1 Warszawa, Polska
8. Mirsalimov, V.M. Non-dimensional elastic-plastic problems / V.M. Mirsalimov. - Moscow: Science. 1987. -256 p.
9. Muskhelishvili, NI Some basic problems of the mathematical theory of elasticity / NI Muskhelishvili .- Moscow: Nauka. 1966. - 707 p.
10. Panasyuk V.V., Savruk M.P., Datsyshin A.P. Distribution of stresses near cracks in plates and shells. Kiev, Naukova Dumka, 1976.-443p.
11. Vaghari, A. R. Nucleation of cracks in a perforated heart- releasing material with temperature-dependent elastic properties. A. R. Vaghari, V. M. Mirsalimov, J. Appl. Mech. Tech. Phys. - 2012. - Vol. 53, No. 7. - P. 589-598.
12. Zolgharnein, E. Nucleation of a crack under the influence of cylindrical bodies E. Zolgharnein, V. M. Mirsalimov // Acta Polytechnica Hungarica. - 2012. - Vol. 9, No. 2. - P. 169-183.
13. Mehtiyev R.K. The longitudinal shift of bodies with a complex structure weakened by straight–line cracks // Construction mechanics and the calculation of structures issn 0039–2383№ 5 2017. Pp. 69–72.

How to cite this article:

Mehtiyev R.K., 2019,The Recompanying of Cracks in Isotropic Environment with Periodic System of Circular Holes Filled with Rigidinclusions with Transverse Shear. *Int J Recent Sci Res.* 10(03), pp. 31364-31371.
DOI: http://dx.doi.org/10.24327/ijrsr.2019.1003.3245

****