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**Research Article**

# THE STRONG NON SPLIT DOMINATION OF A JUMP GRAPH

**Pratap Babu Rao N**

Department of Mathematics Veerasaiva Degree College Ballari

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**ABSTRACT**

A dominating set of a jump graph J (G)= (V, E) is a strong dominating set if the induced sub graph < V-D > is complete. The strong non split domination number $\gamma_{ns}(J(G))$ of J(G) is minimum cardinality of a strong non split dominating set. In this paper we relate this parameter to the parameters of jump graph J(G) and obtain its exact values for some standard graphs.



## INTRODUCTION

All the gaphs considered here are assumed to be finite, undirected, nontrivial and connected without loops or multiple edges. Any undefined term in this paper may be found in Haynes et.al.,[2]

Let J(G) be a jump graph. A set D <⊆ V is a dominating set of J(G) if every vertex in V-D is adjacent to some vertex in D. The domination number $\gamma$(J(G) of J(G) is the minimum cardinality of a dominating set. N.Pratap Babu Rao and Sweta. N introduced the concept of Non split domination in jump graphs.

A dominating set D of a graph $S_1$={1} is a non split dominating set if the induced sub graph <V-D> is connected. The non split dominating number $\gamma_{ns}(J(G))$ of J(G) is the minimum cardinality of a non split dominating set.

Dominating sets whose complements induces a complete sub graphs have a great diversity of applications one such application is the following.

In setting up the communication links in a network One might want a strong core group that can communicate with each other. Member of the core group and so that everyone in the group receives the message from someone outside the group and communicate it to every other in the group. This suggest the following definition.

Definition: A dominating set of vertices a $\gamma$-set if it is a dominating set with cardinality $\gamma(J(G))$. Similarly a $\gamma_{ns}$-set and a $\gamma_{sns}$-set are defined unless and otherwise stated, the graphs has p vertices and q edges.

## RESULTS

Theoem 1: For any graph J(G)

$$\gamma(J(G) \leq \gamma_{ns}(J(G)) \leq \gamma_{sns}(J(G))$$

Proof; This follows from the fact that every strong non split dominating set of J(G) is a non split dominating set and every non split dominating set is a dominating set.

The following characterization is easy to see hence we omit its proof,

Theorem 2: A strong non split dominating set D of J(G) is minimal if abnd only if for each vertex v ∈ D, one of the following conditions is satisfied.

1. There exists a vertex u ∈ V-D such that N(v) ∩ D = {v}
2. V is an isolated vertex in <D>
3. There exists a vertex w ∈ V-D such that w is not adjacent to v. Now, we obtain a relationship between $\gamma_{sns}(J(G))$ and $\gamma_{sns}(J(H))$ where J(H) is any spanning sub graph of J(G)

Theorem 3: For any spanning sub graph J(H) of J(G)
$\gamma_{sns}(J(G)) \leq \gamma_{sns}J(H))$

**Corresponding author:* **Pratap Babu Rao N**
Department of Mathematics Veerasaiva Degree College Ballari

Proof: Since every strong non split dominating set of J(H) is a strong non split dominating set of J(G),
Above inequality holds.
Next we obtain a lower bound in $\gamma_{sns}(J(G))$

Theorem 4: For any jump graph J(G)

$\beta_0(J(G)) \leq \gamma_{sns}(J(G))$ where $\beta_0(J(G))$ is the independent number of J(G)

***Proof:*** Let D be a $\gamma_{sns}(J(G))$-set of J(G) and S be an independent set of vertices in J(G), Then either $S \subseteq D$ or S contains at most one vertex for V-D and at most |D|-1 vertices from D.

This implies $\beta_0(J(G)) \leq \gamma_{sns}(J(G))$

The bound given above inequality is shows,
For example for a complete jump graph $J(K_p)$ $\gamma_{sns}(J(G)) = 1 = \beta_0(K_p)$
Now we prove the major result of this paper from which we can deduce the exact values of $\gamma_{sns}(J(G))$ for some standard graphs.

Theorem 5: For any graph J(G), $p - w(J(G)) \leq \gamma_{sns}(J(G))$
$\gamma_{sns}(J(G)) \leq p - w(J(G)) + 1$

Where w(J(G)) is a clique number of J(G)

Proof: Let D be a $\gamma_{sns}$-set og J(G) since V-D is complement
$w(J(G)) \geq |V\text{-}D|$ …………………..(A)
Let <S> be a complete graph with |S\= W(j(g)) Then for any vertex $u \in S$, <V-S> ∪ |u| is a strong non split dominating set of J(G) and
$\gamma_{sns}(J(G)) \leq p - w(J(G)) + 1$………………..(B)

From (A) and (B)
$p\text{-}w(J(G) \leq \gamma_{sns}(J(G)) \leq p - w(J(G)) + 1$
Hence the result.

Corollary 5.1: let J(G) be a graph with $w(J(G)) \geq \delta(J(G))$ then $\gamma_{sns}(J(G)) \leq p - \delta(J(G))$ where $\delta(J(G)))$ is the minimum degree of J(G). Further, the bound is attained if and only if one of the following is satisfied.

1. $W(J(G)) = \delta(J(G))$
2. $W(J(G)) = \delta(J(G)) + 1$ and every w-set S contains a vertex not adjacent to a vertex of V – S.

Proof: Suppose $W(J(G)) = \delta(J(G)) + 1$. Then from Theorem (4) and (5)

$\gamma_{sns}(J(G)) \leq p - w(J(G))$ suppose $w(J(G)) = \delta(J(G))$ and let S be a w-set of J(G).Then V – S is a strong non split dominating set oi J(G) and hence $\gamma_{sns}(J(G)) \leq p - \delta(J(G))$.
Now we prove that the second part
Suppose one of the given conditions is satisfied. Then from Theorems (4) AND (5) it is easy to see that
$\gamma_{sns}(J(G)) \leq p - \delta(J(G))$ or $\gamma_{sns}(J(G)) \leq p - \delta(J(G)) + 1$ suppose there exists a w-set S with
$|S| = \delta(J(G)) + 1$ such that every vertex in S is adjacent to some vertex in V – S. Then V-S is a strong non split dominating set of J(G), and hence $\gamma_{sns}(J(G)) \leq p - \delta(J(G)) - 1$, which is a contradiction.

Hence one of the given condition is satisfied.
In the next result we list the exact values of $\gamma_{sns}(J(G))$ for some standard graphs
Proposition 6:

1. For any complete graph $J(K_p)$ with $p \geq 2$ vertices $\gamma_{sns}(J(K_p)) = 1$
2. For any complete bipartite jump graph $K_{m,n}$ with $2 \leq m \leq n$ $\gamma_{sns}(J(K_{m,n})) = m + n - 2$
3. For any cycle $J(C_p)$ with $p \geq 3$ vertices $\gamma_{sns}(J(C_p)) = p - 2$
4. For any path $J(P_p)$ with $p \geq 4$ vertices $\gamma_{sns}(J(P_p)) = p - 2$
5. For any wheel $J(W_p))$ with $p \geq 4$ vertices $\gamma_{sns}(J(W_p)) = p - 3$.

A set D of vertices in a jump graph J(G) is a vertex set dominating set if for any set $S \subseteq V\text{-}D$, there exists a vertex v in D such that the induced sub graph $< S \cup \{v\} >$ is connected. The vertex set domination number $\gamma_{vs}(J(G)))$ of J(G) is the minimum cardinality of a vertex set dominating set [8]
Theorem 7: If a graph J(G) has independent strong non split dominating set then diam $(J(G)) \leq 3$ where diam (J(G)) is the diameter of J(G).
Proof: let D be an independent strong non split dominating set of G.

We consider the following cases;
Case i)let $u, v \in V - D$ then $d(u, v)=1$
Case ii) let $u \in D$ and $v \in V - D$ Since D is independent there exists a vertex $w \in V - D$ such that u is adjacent to w Thus $d(u, v) \leq d(u, w) + d(w, v) \leq 2$

Case iii) Let $u, v \in D$ As above there exists two vertices $w_1, w_2 \in V - D$ such that u is adjacent $w_1$ and v is adjacent to $w_2$ Thus $d(u, v) \leq d(u, w_1) + d(w_1, w_2) + d(w_2, v) \leq 3$
Thus for all vertices $u, v \in V$, $d(u, v) \leq 3$
Hence $diam(J(G)) \leq 3$.

Corollary 7.1 If $\gamma(J(G)) = \gamma_{sns}(G)$, then diam $(J(G)) \leq 3$
Proof: let D be a $\gamma_{sns}$-set of J(G). Since d is also a $\gamma$-set every vertex $v \in D$ is adjacent to at least one vertex $u \in V - D$. As in the proof of theorem 7 one can show the $d(u, v) \leq 3$ for all vertices $u, v \in V$.

Thus diam $(J(G)) \leq 3$
Theorem 8 : Let D be an independent set of vertices in J(G) if $|D| < 1 - \Delta(J(\bar{G}))$ then V – D is a strong non split dominating set of $J(\bar{G})$, Where $J(\bar{G})$ is the complement of J(G).

Proof: Since each vertex $v \in D$ is not adjacent to at least one vertex in V – D, it implies that V – D is a dominating set of $J(\bar{G})$ and further it is a strong non split dominating set as < D > is complete in $J(\bar{G})$.

A dominating set D of a connected graph J(G) is a split dominating set if the induced sub graph <V – D> is connected [4] . In [5] Kulli V. Rand B.Janakiram extended the concept of split domination to strong split domination as follows.

A dominating set D of a connected graph G is a strong split dominating set if induced sub graph <V-D> is totallty disconnected with at least two vertices. The strong split domination number $\gamma_{sns}(\bar{G})$ of G is minimum cardinality of a strong split dominating set.

Theorem 9: Let D be a $\gamma_{sns}$-set of J(G), Then by (1) and (5) V-D has at least two vertices Alsi by (ii) every vertex in V-D is not adjacent to at least one vertex in D. This impliesthat D is a dominating set of $J(\bar{G})$ and further it is a strong split

dominating set a <V-D> is totally disconnected with at least two vertices in $J(\bar{G})$.
Thus $\gamma_{ss}(J(\bar{G})) \leq \gamma_{sns}(J(G))$
Kulli V.R. and B.Janakiram [6] introduced then following concept.

A dominating set D of a graph G=(V,E) is a regular set dominating set if for any set

$I \subseteq V - D$, there exists a set $S \subseteq D$ such that the induced sub graph $< I \cup D >$ is regular. The regular set domination number $\gamma_{rs+}(G)$ of G is the minimum cardinality of a regular et dominating set.

Theorem 10: For any graph J(G)
$\gamma_{ns}(J(G)) \leq \gamma_{sns}(J(G)) + 1$.

Proof: let D be a $\gamma_{sns}$-set of J(G). Since <V-D> is complete for any vertex $u \in V\text{-}D$, $D \cup \{u\}$ is a regular set dominating set of J(G) This proves the result.

Theorem 11: If $diam(J(G)) \leq 3$ then
$\gamma_{ns}(J(G)) \leq p - m$. where m is the number of cut vertices of J(G)

Proof: If J(G) has no cut vertices, then the result is trivial. Let S be the set of all cut vertices with |S|=m, let $u,v \in S$ suppose u and v ae not adjacent. Sionce tere exists two vertices $u_1$ and $v_1$ such that $u_1$ is adjacent to u and $v_1$ is adjacent to v it implies that $d(u_1, v_1) \geq 4$a contradiction. Hence every two vertices in S are adjacent and every vertex in S is adjacent to at least one vertex in V-S. This proves that V-S is a strong non split dominating set of J(G).

Hence the result.

Theorem 12: Let J(G) be a jump graph such that every vertex of J(G) is either a cut vertex or an end vertex if V(j(G))= m then $\gamma_{ns}(J(G)) = \gamma_{sns}(J(G)) = p\text{-}m$ where m is the number of cut vertices of J(G).

Proof: Let S be the set of all cut vertices with |S|=m since w(G) = m it implies that every two vertices in S are adjacent and hence every vertex in S is adjacent to an end vertex This proves that V-S is a $\gamma_{ns}$-set of J(G) and further it is a $\gamma_{sns}(J(G))$-set as <S> is complete

Hence the result.

The following definition is used to prove our next result.

A dominating set D of a graph J(G) is an efficient dominating set if every vertex in V-D is adjacent to exactly one vertex in D. This concept was introduced by Cockayne *et.al*.,.

***Theorem13:*** Let J(G) be an n-regular graph with 2n vertices If D is an efficient dominating set of J(G)with n-vertices then both D and V-D are strong non split dominating sets of J(G).

***Proof:*** Since every vertex in V-D is adjacent to exactly one vertex in D. it implies that every two vertices in V-D and adjacent. As J(G) is n-regular every vertex in D uiss adjacent to some vertex in V-D. Suppose there exists a vertex $u \in D$ such that u is adjacent to two on more vertices in V-D. Then there exixts a vertex $v \in D$ such that deg $v \leq n\text{-}1$, a contradiction. Hence every vertex in D is adjacent to exactly one vertex in V-D. Thus as above every two vertices in D are adjacent. Hence D and V-D are strong non split dominating sets of J(G).

Theorem 14: Let J(G) be a gaph with $\Delta(J(G)) \leq p\text{-}2$ when Dbe a strong non split dominating set of G such that <D> is complete and $|D| \leq \delta(J(G))$. Then (1) D is minimal (2) V-D is also a minimal strong non split dominating set of J(G).

***Proof:*** Since <D> is complete, it implies that for each vertex v $\in$ D there exists a vertex $u \in V\text{-}D$ such that v is not adjacent to u. Then by theorem2 D is minimal.

$|D| \leq \delta(J(G))$, it implies that every vertex in D is adjacent to some vertex in V-D. Then V-D is strong non split dominating set of J(G) and further as above it is minimal.

***Theorem 15:*** If $\Delta(J(G)) < \alpha_0(J(G))$ then $\surd_{sns}(J(G)) = p\text{-}\ w(J(\bar{G}))$ where $\alpha_0(J(G))$ is the vertex covering number of J(G).
Proof: Let S be a vertex cover of J(G) with $|S| = \alpha_0(J(G))$. Since $\Delta(J(G)) < \alpha_0(J(G))$,

$J(G) \neq J(K_p)$ and V-S is an independent set with at least two vertices such that every vertex in V-S is not adjacent to at least one vertex in S This proves that S is a strong non split dominating set of $J(\bar{G})$
Thus $\surd_{sns}(J(\bar{G})) \leq |S|$
$\leq \alpha_0(J(G))$.
$\leq p - \beta_0(J(G))$
$\leq p - w(J(\bar{G}))$
Result follows from theorem 5.

Next we obtain Nordhus Gaddum type result [7]

Theorem 16:Let J(G) e a graph such that both J(G) and $J(\bar{G})$ are connected . $w(J(G)) \geq \delta(J(G))$ and $w(J(\bar{G})) \geq \delta(J(\bar{G}))$
$\surd_{sns}(J(G) + \surd_{sns}(J(\bar{G})) \leq p + 1 + \Delta(J(G)) - \delta(J(G))$
Proof: By corollary 5.1 $\surd_{sns}(J(G)) \leq p - \delta(J(G))$
$\surd_{sns}(J(\bar{G})) \leq p - \delta(J(\bar{G})) \leq 1 + \Delta(J(G))$
Hence the result.

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