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ABSTRACT

We have investigated Bianchi type VI cosmological model with the matter Wet Dark fluid coupled with electromagnetic field in general theory of relativity. To obtain the deterministic solution of Einstein’s field equation, by assuming a relation between metric potential $C = A^\gamma$ further some physical and geometrical properties of the model are also discussed.

INTRODUCTION

Bianchi type cosmological models are important in the sense that these models are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view, anisotropic universe has a greater generality than isotropic models. The simplicity of the field equations made Bianchi space time useful in constructing models of spatially homogeneous and anisotropic cosmologies. Hence, these models are to be known as very much suitable models of our universe. Therefore, study of these models creates much more interest.

As we know that observational data like la supernovae suggest that the universe is dominated by two dark components containing dark energy and dark matter. Dark energy with negative pressure is used to explain the present cosmic accelerating expansion while dark matter is used to explain galactic curves and large scale structure formation.

Origin of the dark energy and dark matter and their natures remains unknown.

The equation of state for wet Dark fluid is $p_{WDF} = \gamma(\rho_{WDF} - p_\gamma)$.
Where, the parameter $\gamma$ and $p_\gamma$ taken to be positive restrict ourselves to $0 \leq \gamma \leq 1$.
And we have energy conservation equation as $\rho_{WDF} + 3H(p_{WDF} + \rho_{WDF}) = 0$.

Using equation of state and $3H = \frac{\dot{V}}{V}$ in the above equation, we get
$$\rho_{WDF} = \frac{\gamma}{1 + \gamma} \rho_\gamma + \frac{c}{V(1 + \gamma)}$$
Where, $c$ is constant of integration and $V$ is volume expansion. Wet dark fluid (WDF) has two components: - one be - haves as cosmological constant and other as standard fluid with equation of state $p_{WDF} = \gamma\rho_{WDF}$.
If we take $c=0$ then this fluid will not violet the strong energy condition $p + \rho \geq 0$;
$$p_{WDF} + \rho_{WDF} = (1 + \gamma)\rho_{WDF} - \gamma\rho_\gamma$$
The Einstein field equation in the general relativity is given by

\[ (p_{WDF} + \rho_{WDF}) = (1+\gamma) \frac{c}{V(1+\gamma)} \]


In this paper, we have investigated Bianchi type VI_0 wet dark fluid coupled with electromagnetic field in general theory of relativity and obtained the solution of field equations. Further we have discussed the physical and geometrical behaviour of bianchi type VI_0 cosmological model.

**The Metric and Field Equations**

Here we consider the spatially homogeneous Bianchi type VI_0 metric in the form

\[ ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2m} dy^2 + C^2 e^{2m} dz^2 \]  

(2.1)

Where \( m \) is non-zero constant and A, B and C are function of \( t \) only.

The energy momentum tensor of the source wet dark fluid coupled with electromagnetic field is defined by

\[ T_{ij} = \left( p_{WDF} + \rho_{WDF} \right) u_i u_j - \rho_{WDF} + E_i^j \]  

(2.2)

Where, \( p_{WDF} \) is the isotropic pressure and \( \rho_{WDF} \) is the matter density and \( u_i \) is the flow vector of the fluid. The flow of the matter is taken orthogonal to the hyper surface of the homogeneity, so that \( g_{ij} u_i u_j = 1 \)  

(2.3)

In a co-moving system of coordinates, from (2.2) we find

\[ T_1^1 = T_2^2 = T_3^3 = -p_{WDF} \text{ and } T_4^4 = \rho_{WDF} \]  

(2.4)

Electromagnetic field is defined as

\[ E_i^j = -F_{ij} + \frac{1}{4} F_{ab} F^{ab} g_i^j \]  

(2.5)

Where, \( E_i^j \) is electromagnetic energy tensor and \( F_i^j \) is the electromagnetic field tensor.

We assume that \( F_{ij} \) is the only non-vanishing component of \( F_i^j \) which corresponds to the presence of magnetic field along y-direction.

The Einstein field equation in the general relativity is given by

\[ R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G \left( \rho_{WDF} + \frac{(F_{ij})^2}{2} e^{-2m} \right) \]  

(2.6)

Where, \( R_{ij} \) is known as Ricci tensor and \( R = g^{ij} R_{ij} \) is the Ricci scalar and \( T_{ij} \) is energy momentum tensor for matter

The field equations (2.6) together with the line element (2.1) with equations (2.2) we get

\[ \ddot{B} + \frac{\dot{B}}{B} + \dot{C} + \frac{\dot{BC}}{BC} + \frac{m^2}{A^2} = 8\pi G \rho_{WDF} + \frac{(F_{13})^2}{2} e^{-2m} \]  

(2.5)

\[ \ddot{A} + \frac{\dot{A}}{A} + \frac{\dot{AC}}{AC} - \frac{m^2}{A^2} = 8\pi G \rho_{WDF} + \frac{(F_{13})^2}{2} e^{-2m} \]  

(2.6)

\[ \ddot{A} + \frac{\dot{A}}{A} + \frac{\dot{BC}}{BC} - \frac{m^2}{A^2} = 8\pi G \rho_{WDF} + \frac{(F_{13})^2}{2} e^{-2m} \]  

(2.7)

\[ \dot{AB} + \dot{BC} + \dot{AC} - \frac{m^2}{AC} = -8\pi G \rho_{WDF} + \frac{(F_{13})^2}{2} e^{-2m} \]  

(2.8)

\[ m \left( \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0 \]  

(2.9)

From equation (2.9) we get

\[ \frac{\dot{B}}{B} = \frac{\dot{C}}{C} \]  

(2.10)

After integration, we have

\[ B = kC \]  

(2.11)

Where \( k \) is a constant of integration.

Here the sake of simplicity, we consider \( B = C, k = 1 \) without loss of generality.

**Solution of the Field Equations**

We assume that the expansion is proportional to the shear which is physical conditions. This condition leads to

\[ C = A^n \text{, where } n \text{ is a constant and } B = C \]  

(3.1)

From equation (2.5) and (2.7) with (3.1) we obtain

\[ \frac{\dot{C}}{C} + \frac{\dot{A}}{A} + \frac{\dot{AC}}{AC} - \frac{2m^2}{AC} = 0 \]  

(3.2)

Using the equation (3.1) in equation (3.2) we get

\[ \frac{\dot{A} + 2n}{A} \frac{A^2}{(n-1)} = \frac{-2m^2}{A} \]  

(3.3)

Now assuming \( A = f(A) \), equation (3.3) takes the from

\[ \frac{d}{dA} (f^2) + \frac{4n}{A} f^2 = \frac{-4m^2}{(n-1)} \]  

(3.4)

Equation (3.4) has the general solution...
Where $k_i$ is the constant of integration.

The Bianchi type $-VI_0$ model in this case reduces to the form

$$ds^2 = \left(\frac{dt}{\sqrt{-m^2 + k_i T^{-4n}}}\right)^2 + T^{-2} dX^2 + T^2 \left[ e^{-2nu} dY^2 + e^{2nu} dZ^2 \right]$$  \hspace{1cm} (3.6)

Without loss of generality, after substituting $A=T$, $x=X$, $y=Y$, $z=Z$ in (3.6) we have

$$ds^2 = \left[ \frac{dT^2}{-m^2 + k_i T^{-4n}} \right] + T^{-2} dX^2 + T^2 \left[ e^{-2nu} dY^2 + e^{2nu} dZ^2 \right]$$  \hspace{1cm} (3.7)

### Physical and Geometrical Behaviour of the Model

Form the equation (3.7) we get

$$\rho_{WDF} = \frac{1}{8\pi G} \left[ \frac{(2n+1)m^2}{(n-1)T^2} - \frac{n(n-2)k_i}{T^{4n+2}} \right]$$  \hspace{1cm} (3.8)

Wet dark fluid (WDF) has two components: - one behaves as cosmological constant and other as standard fluid with equation of state $p_{WDF} = \gamma \rho_{WDF}$  \hspace{1cm} (3.9)

By using (3.8) in (3.9) we get

$$p_{WDF} = \gamma \frac{1}{8\pi G} \left[ \frac{(2n+1)m^2}{(n-1)T^2} - \frac{n(n-2)k_i}{T^{4n+2}} \right]$$  \hspace{1cm} (3.10)

The expansion scalar and shear scalar are given by

$$\theta = \frac{2n+1}{T} \left[ \frac{-m^2}{n(n-1)} + \frac{k_i}{T^{4n}} \right]^{\frac{1}{2}}$$  \hspace{1cm} (3.11)

$$\sigma^2 = \frac{(1-n)^2}{3T^2} \left[ \frac{-m^2}{n(n-1)} + \frac{k_i}{T^{4n}} \right]$$  \hspace{1cm} (3.12)

Where

$$\frac{\sigma^2}{\theta^2} = \frac{(1-n)^2}{3(2n+1)^2}$$  \hspace{1cm} (3.13)

Since $\lim_{T \to \infty} \frac{\sigma}{\theta} = \text{constant}$, the anisotropy is maintained for all time. It can be seen that the model is irrotational. Therefore, the model describes a continuously expanding, shearing and non rotating universe with big-bang.

### CONCLUSION

In the present study, we have investigated the effect of wet dark fluid with electromagnetic field in bianchi type $VI_0$ Universe. Einstein’s field equations have been solved exactly suitable physical and kinematical parameter since $\lim_{T \to \infty} \frac{\sigma}{\theta} = \text{constant}$, which gives us the anisotropy is maintained for all time and it can be seen that the model is irrotational. Therefore, the model describes a continuously expanding, shearing and non rotating universe with big-bang in general theory of relativity.

### References

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