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ANALYSIS OF THE METHODOLOGY FOR SOLVING LORENZ CHAOTIC SYSTEM

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ABSTRACT

In this paper, with the aid of the Wolfram Mathematica or Matlab software the powerful, the Adomian decomposition method (ADM) is used to found the numerical solution of Lorenz chaotic system. It is shown that the method is straightforward and effective mathematical tool for solving this system. We make a comparison between our method and another method to illustrate the accuracy of this method. Finally, we submit comprehensive conclusions.

INTRODUCTION

The Chaotic Lorenz system is a system of ordinary differential equations first studied by Edward Lorenz (1963) see [1]. It is notable for having chaotic solutions for certain parameter values and initial conditions. In particular, the Lorenz attractor is a set of chaotic solutions of the Lorenz system which, when plotted, resemble a butterfly or figure eight.

The Chaotic Lorenz system also arises in simplified models for lasers [2], electric circuits [3], chemical reactions [4], forward osmosis [5] and so on.

From a technical standpoint, the Chaotic Lorenz system is nonlinear, non-periodic, three-dimensional and deterministic. The Lorenz equations have been the subject of hundreds of research articles, and at least one book-length study [6].

The Adomian decomposition method (ADM) is a semi-analytical method for solving ordinary and partial nonlinear differential equations. The method was developed from the 1970s to the 1990s by George Adomian, chair of the Center for Applied Mathematics at the University of Georgia [7, 8].

This paper is organized as follows: In Section 2, we introduce the Chaotic Lorenz system. In Section 3, we analysis the proposed method. In Section 4, comparison between results for our method and results obtained from fourth- Order Runge-Kutta Technique (RK4). In Section 5, numerical results are discussed. Finally, the conclusion of this study is given in section 5.

The Lorenz Chaotic System

We consider famous Lorenz system as (see[9]),

$$\frac{dx}{dt} = \sigma(x - y), \tag{1}$$

$$\frac{dy}{dt} = Rx - y - xz, \tag{2}$$

$$\frac{dz}{dt} = xy - bz, \tag{3}$$

where x, y and z are respectively to the convective velocity, the temperature difference between descending and ascending flows, and the mean convective heat flow, and σ, b are called bifurcation parameter which are real constants. Throughout this paper, we set $\sigma = 10, b = -8/3$ and the parameter R it is well-known that chaos sets in around the critical parameter value $R = 28$ where the system exhibits chaotic behavior.

Analysis of the methodology

ADM for solving chaotic Lorenz's system

In this subsection, we present direct application of the ADM to the Lorenz system (1)-(3). First, note that the Lorenz system is a special case of a more general (homogeneous) system of

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ODEs where the nonlinear term occurs as products of two of the dependent variables. This general system, which was previously studied by Vadas P. (see [10,11]) is given by:

$$x_i^1 \sum_{j=1}^{n} a_{ij} X_j + \sum_{p=1}^{n}\sum_{q=1}^{n} a_{ipq} X_p X_q, \quad \forall i = 1,2,\dots,n. \quad (4)$$

Where the derivatives are with respect to time t. If we denote the linear term (the first term on the r. h. s.) as $R_{i1}$and the nonlinear term (the second term) as R_{i2}there we can write the above system of equations in this form:

$$LX_i = R_{i1} + R_{i2}, \quad \forall i = 1,2,\dots,n. \quad (5)$$

Where L is the differential operator $d/_{dt}$.

Now we applying the inverse (integral) operatorL^{-1} to (5) we obtain

$$X_i(t) = X_i(t^*) + L^{-1}R_{i1} + L^{-1}R_{i2}, \quad \forall i = 1,2,\dots,n. \quad (6)$$

Here we have assumed that the general system (4) (or equivalently (5)) is an initial-value problem so that it's solution is uniquely determined via the information$X_i(t^*)$ $\forall i = 1,2,\dots,n.$ According to the ADM(see [8]), the solution $X_i(t)$ is given by the series,

$$X_i(t) = \sum_{m=0}^{\infty} X_{im}(t), \quad \forall i = 1,2,\dots,n. \quad (7)$$

Then, the linear term R_{i1} then becomes

$$R_{i1} = \sum_{j=1}^{n}\sum_{m=0}^{\infty} a_{ij} X_{im}, \quad (8)$$

Thus,$L^{-1}R_{i1}$ is given by

$$L^{-1}R_{i1} = \sum_{j=1}^{n}\sum_{m=0}^{\infty} a_{ij} \int_{t^*}^{t} X_{im} \; dt, \quad \forall i = 1,2,\dots,n. \quad (9)$$

The nonlinear term $R_{i2}$is decomposed as,

$$R_{i2} = \sum_{p=1}^{n}\sum_{q=1}^{n} a_{ipq} \sum_{m=0}^{\infty} A_{im,p,q}.$$

where the$A_{im,p,q}$ are the so called Adomian polynomials. In this case, it is given by the formula,

$$A_{im,p,q} = \frac{1}{m!}\frac{d^m}{d\lambda^m}\left[M\left(\sum_{k=0}^{\infty}\lambda^k X_{kp}, \sum_{k=0}^{\infty}\lambda^k X_{k0}\right)\right]_{\lambda=0}$$

Where$M(x,y) = xy$for each $m = 0,1,2,\dots$ Moreover $L^{-1}R_{i2}$is given by

$$L^{-1}R_{i2} = \sum_{p=1}^{n}\sum_{q=1}^{n} a_{ipq} \sum_{m=0}^{\infty} \int_{t^*}^{t} A_{im,p,q} \; dt$$

Substituting (7),(9),(12)into (6) we then have for each $i = 1,2,\dots,n$

$$\sum_{m=0}^{\infty} X_{im}(t) = X_i(t^*) + \sum_{j=1}^{n}\sum_{m=0}^{\infty} a_{ij} \int_{t^*}^{t} X_{im} dt + \sum_{p=1}^{n}\sum_{q=1}^{n} a_{ipq} \sum_{m=0}^{\infty} \int_{t^*}^{t} A_{im,p,q} \; dt (13)$$

Therefore, we have for each $i = 1,2,\dots,n$

$X_{i0} = X_i(t^*)$,

$X_{i1} = \sum_{j=1}^{n} a_{ij} \int_{t^*}^{t} X_{i0} dt + \sum_{p=1}^{n}\sum_{q=1}^{n} a_{ipq} \int_{t^*}^{t} A_{i0,p,q} dt$,

$X_{i2} = \sum_{j=1}^{n} a_{ij} \int_{t^*}^{t} X_{i1} dt + \sum_{p=1}^{n}\sum_{q=1}^{n} a_{ipq} \int_{t^*}^{t} A_{i1,p,q} dt$,

$X_{i,(m+1)} = \sum_{j=1}^{n} a_{ij} \int_{t^*}^{t} X_{i,m} dt + \sum_{p=1}^{n}\sum_{q=1}^{n} a_{ipq} \int_{t^*}^{t} A_{i,m,p,q} dt$,

After calculating the Adomian polynomials (11) and integrating, one then has for all$t \geq t^*$,$X_i(t) = \sum_{m=0}^{\infty} d_{im} \frac{(t-t^*)^m}{m!}$, $\forall\, i = 1,2,\dots n.$

Where the coefficients $d_{im}$are given by

$d_{i0} = X_i(t^*)$,

$d_{im} = \sum_{j=1}^{n} a_{ij} d_{j(m-1)} + (m-1)! \sum_{p=1}^{n}\sum_{q=1}^{n}\sum_{k=0}^{m-1} a_{ipq} \frac{d_{qk}}{k!} \frac{d_{p(m-k-1)}}{k!(m-k-1)!}$ $m \geq 1$ (20)

Hence, from (18) – (20), the explicit solution to the Lorenz system (1) – (3) is

$X = \sum_{m=0}^{\infty} a_m \frac{(t-t^*)^m}{m!}$,

$y = \sum_{m=0}^{\infty} b_m \frac{(t-t^*)^m}{m!}$,(22)

$z = \sum_{m=0}^{\infty} C_m \frac{(t-t^*)^m}{m!}$,

where the coefficients are given by the recurrence relations,

$a_0 = X(t^*)$, $b_0 = y(t^*)$, $c_0 = z(t^*)$,

$a_m = -\sigma a_{m-1} + \sigma b_{m-1}, m \geq 1$

$b_m = Ra_{m-1} - b_{m-1} - (m-1)! \sum_{k=0}^{m-1} \frac{a_k c_{m-k-1}}{k!(m-k-1)!}$, $m \geq 1$

$c_m = bc_{m-1} + (m-1)! \sum_{k=0}^{m-1} \frac{a_k b_{m-k-1}}{k!(m-k-1)!}$, $m \geq 1$

Then, by using the Mathematica program we obtain that:

$X_1 = -15.8 - 16.80t + 774.960t^2 + 2170.52533t^3$

$y_1 = -17.48 - 138.192t + 1426.1160t^2 + 7682.957638t^3$

$z_1 = 35.64 + 181.1440t - 1186.410134t^2 - 11745.60804t^3$

Matlab code of RKU for solving Lorenz equations

We construct a code of RK4 for get a numerical solution of the Lorenz's system as follow:

```
t(1) = 0; %initializing x,y,z,t
x(1) = 1;
y(1) = 1;
z(1) = 1;
sigma = 10; %value of constants
rho = 28;
beta = (8.0/3.0);
h = 0.01; %step size
t = 0:h:20;
f = @(t,x,y,z)sigma*(y - x); %ode
g = @(t,x,y,z)x*rho - x.* z - y;
p = @(t,x,y,z)x.*- y - beta*z;
for i = 1:(length(t) - 1)%loop
k1 = f(t(i),x(i),y(i),z(i));
l1 = g(t(i),x(i),y(i),z(i));
m1 = p(t(i),x(i),y(i),z(i));
k2 = f(t(i) + h/2, (x(i) + 0.5*k1*h), (y(i) + (0.5*l1*h)), ((z(i) + (0.5*m1*h)));
l2 = g(t(i) + h/2, (x(i) + 0.5*k1*h), (y(i) + (0.5*l1*h)), ((z(i) + (0.5*m1*h)));
m2 = p(t(i) + h/2, (x(i) + 0.5*k1*h), (y(i) + (0.5*l1*h)), ((z(i) + (0.5*m1*h)));
k3 = f(t(i) + h/2, (x(i) + 0.5*k2*h), (y(i) + (0.5*l2*h)), ((z(i) + (0.5*m2*h)));
l3 = g(t(i) + h/2, (x(i) + 0.5*k2*h), (y(i) + (0.5*l2*h)), ((z(i) + (0.5*m2*h)));
m3 = p(t(i) + h/2, (x(i) + 0.5*k2*h), (y(i) + (0.5*l2*h)), ((z(i) + (0.5*m2*h)));
k4 = f(t(i) + h, (x(i) + k3*h), (y(i) + l3*h), (z(i) + m3*h));
l4 = g(t(i) + h, (x(i) + k3*h), (y(i) + l3*h), (z(i) + m3*h));
m4 = p(t(i) + h, (x(i) + k3*h), (y(i) + l3*h), (z(i) + m3*h));
x(i + 1) = x(i) + h*(k1 + 2*k2 + 2*k3 + k4)/6; %final equations
y(i + 1) = y(i) + h*(l1 + 2*l2 + 2*l3 + l4)/6;
z(i + 1) = z(i) + h*(m1 + 2*m2 + 2*m3 + m4)/6;
end
plot3(x,y,z)
```

(10) (11) (12) (14) (15) (16) (17)

When we execute the above program, we can confirm the famous figure "Butterfly effect".

(18)

(19)

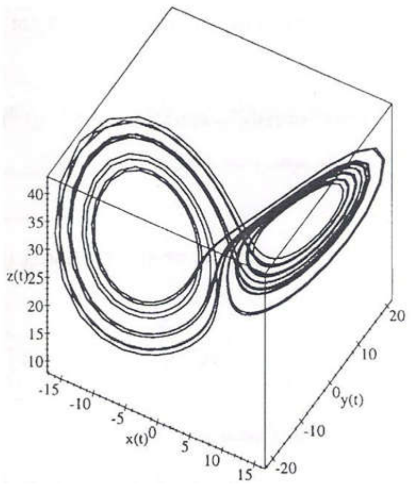

Fig 1 The Lorenz's butterfly effect obtained by the RK4 technique

Comparison between ADM and RK4

After that, we treat the ADM solutions as an algorithm for approximating the dynamical response in a sequence of time interval $[0, t_1), [t_1, t2), \ldots\ldots [t_{m-1}, T)$ which applied on the solution [12-14], we notice that the three projections of the curves obtained by the Runge-Kutta technique (see figure 1) are similar to those obtained by Adomian method (see figures 2-4)

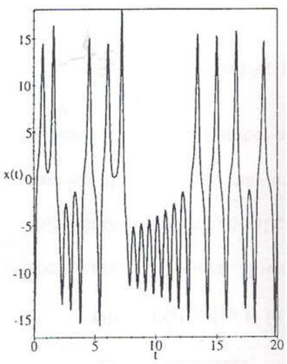

Fig. (2): Solution X(t) for T = 20

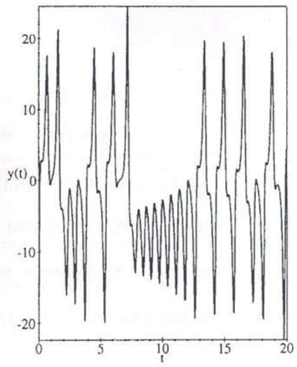

Fig. (3): Solution Y(t) for T = 2,

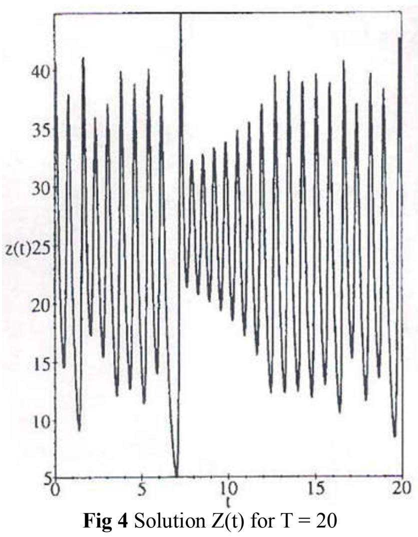

Fig 4 Solution Z(t) for T = 20

So now we can compare the accuracy of the ADM on the chosen time step $\Delta t = 0.01$with the RK4 method on the chosen time step $\Delta t = 0.001$. We choose this time step since a smaller one's computationally costly. (see [15-17]).

We present the absolute error between the 10-term ADM solution at time step $\Delta t = 0.01$ and the RK4 solutions on $\Delta t = 0.001$which given in table (1).

Table 1 Differences between 10-term decomposition and RK4 solutions for R=28

t	$\Delta = \lvert ADM_{0.01} - RK4_{0.001} \rvert$		
	Δx	**Δy**	**Δz**
2	3.128E-08	5.292E-09	1.127E-09
4	1.231E-08	6.274 E-09	8.715E-8
6	1.191 E-07	2.416 E-06	1.941E-08
8	4.077 E-07	6.118 E-07	1.709E-07
10	3.182 E-06	2.948 E-05	2.226E-05
12	2.023 E-07	1.970 E-05	1.191E-05
14	2.173E-04	4.625 E-05	3.497E-04
16	2.112 E-04	2.188 E-04	2.733E-04
18	1.514 E-03	2.526 E-04	2.188E-04
20	1.819 E-02	4.059 E-02	2.019E-02

NUMERICAL RESULTS AND DISCUSSION

To demonstrate the accuracy of the ADM against RK4, the simulations were done in this paper for the time $t \in [0, 20]$. In this paper, we decide to use 10-terms in the Adomian decomposition series solutions. We note that increasing the number of terms improves the accuracy of the ADM solutions, but the expense of increased computational efforts.

CONCLUSION

The decomposition technique has been applied to solving the system of Lorenz. It gives a simple and powerful tool for obtaining the solution of differential systems. The three projections of the curves obtained by the Runge-Kutta technique (see Figure 4) are similar to those obtained by Adomian method (see Figures 1-3). It is a consequence of the theoretical properties of the decomposition method proved in Hashim. I (see [18-32]). Furthermore, we used 400 subdivisions of $[0, T]$ for the classical Rung-Kutta method and only 20 subdivisions of $[0, T]$ are sufficient for giving the same solution by the Adomian method. The convergence of the Adomian method is faster than Runge-Kutta technique. Unlike numerical Runge-Kutta method, Adomian's technique gives an exact solution in each subdivision without discretization of time.

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