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Research Article

APPLICATION OF MATRIX IN STUDY OF NAVIER-STOKES EQUATIONS

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ABSTRACT

In present paper we are transforming well known Navier-Stokes equations from linear system to it's matrix form. We are discussing some of the basic properties using properties of matrices such as determinant, eigen value, trace. Using Hassian matrix, we are finding critical points of motion of fluid.

INTRODUCTION

The well-known equations of motion of fluid in R^3 are the Naiver-Stokes equations. They are second ordered non-linear partial differential equations. Let the Euclidean space be co-ordinatising with space co-ordinates (x, y, z). Assume that fluid is moving in R^3 has a fluid element S with position vector (x, y, z) at given specific time. Let U =(u_x, u_y, u_z) represents the velocity of particle S and p denotes pressure exerted by fluid on S due to it's motion. Let ρ denote the density of fluid, Re denotes is Reynolds number, E_t denotes total energy ,P_r denotes Prandtl number and τ denote amount of flux transport through unit area having nine components along all three directions. Here x, y, z and t are free variables and six unknown quantities as P=pressure, ρ = density, T= temperature, u_x = velocity component along x direction, u_y = velocity component along y direction, u_z =velocity component along z direction. So due to conservation of mass, momentum and energy and using Newton's II law we can write Navier-Stokes Equations in R^3 as

$$\frac{\partial(\rho u_x)}{\partial t} + \frac{\partial(\rho u_x^2)}{\partial x} + \frac{\partial(\rho u_x u_y)}{\partial y} + \frac{\partial(\rho u_x u_z)}{\partial z} = -\frac{\partial p}{\partial t} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right] \quad \text{(along x-axis)}$$

$$\frac{\partial(\rho u_y)}{\partial t} + \frac{\partial(\rho u_x u_y))}{\partial x} + \frac{\partial(\rho u_y^2)}{\partial y} + \frac{\partial(\rho u_y u_z)}{\partial z} = -\frac{\partial p}{\partial t} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right] \quad \text{(along y-axis)}$$

$$\frac{\partial(\rho u_z)}{\partial t} + \frac{\partial(\rho u_x u_z)}{\partial x} + \frac{\partial(\rho u_y u_z)}{\partial y} + \frac{\partial(\rho u_z^2)}{\partial z} = -\frac{\partial p}{\partial t} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right] \quad \text{(along z-axis)}$$

These are three equations and unknown are more so for unique and smooth solution, add two more constraints 1) continuity of conservation of momentum and 2) conservation of energy equation given as below

$$\frac{\partial(\rho)}{\partial t} + \frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\rho u_y)}{\partial y} + \frac{\partial(\rho u_z)}{\partial z} = 0 \quad \text{i.e.} \quad \nabla.(\rho.U) = 0.$$

(Continuity equation)

And

$$\frac{\partial(E_t)}{\partial t} + \frac{\partial(\rho u_x E_t)}{\partial x} + \frac{\partial(\rho u_y E_t)}{\partial y} + \frac{\partial(\rho u_z E_t)}{\partial z} = -\frac{\partial(u_x p)}{\partial x} - \frac{\partial(u_y p)}{\partial y} - \frac{\partial(u_z p)}{\partial z} - \frac{1}{R_{e_r} P_{r_r}}\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right] + \frac{1}{R_{r_r}}\left[\frac{\partial(u_x\tau_{xx} + u_y\tau_{xy} + u_z\tau_{xz})}{\partial x} + \frac{\partial(u_x\tau_{xy} + u_y\tau_{yy} + u_z\tau_{yz})}{\partial y} + \frac{\partial(u_x\tau_{xz} + u_y\tau_{yz} + u_z\tau_{zz})}{\partial z}\right] \quad \text{(Energy Equation)}$$

Let us write the system of first four of these equations into matrix form as

$$R_{e_r}\left(\left[\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z} \quad \frac{\partial}{\partial t}\right]\begin{bmatrix} \rho u_x & \rho u_y & \rho u_z & \rho \\ \rho u_x^2 + p & \rho u_x u_y & \rho u_x u_z & \rho u_x \\ \rho u_x u_y & \rho u_z^2 + p & \rho u_y u_z & \rho u_y \\ \rho u_x u_z & \rho u_y u_z & \rho u_z^2 + p & \rho u_z \end{bmatrix}\right)$$

$$= \left(\left[\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z} \quad \frac{\partial}{\partial t}\right]\begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} & 0 \\ \tau_{xy} & \tau_{yy} & \tau_{yz} & 0 \\ \tau_{xz} & \tau_{yz} & \tau_{zz} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right.$$

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Now the momentum matrix of the above equation is

$$\begin{bmatrix} \rho u_x & \rho u_y & \rho u_z & \rho \\ \rho u_x^2 + p & \rho u_x u_y & \rho u_x u_z & \rho u_x \\ \rho u_x u_y & \rho u_y^2 + p & \rho u_y u_z & \rho u_y \\ \rho u_x u_z & \rho u_y u_z & \rho u_z^2 + p & \rho u_z \end{bmatrix} =$$

$$\rho . \begin{bmatrix} u_x & u_y & u_z & 1 \\ u_x^2 & u_x u_y & u_x u_z & u_x \\ u_x u_y & u_y^2 & u_y u_z & u_y \\ u_x u_z & u_y u_z & u_z^2 & u_z \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ p & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \end{bmatrix}$$

So determinant of velocity matrix is = det

$$(\rho . \begin{bmatrix} u_x & u_y & u_z & 1 \\ u_x^2 & u_x u_y & u_x u_z & u_x \\ u_x u_y & u_y^2 & u_y u_z & u_y \\ u_x u_z & u_y u_z & u_z^2 & u_z \end{bmatrix})$$

$$= \rho^4 . u_x . u_y . u_z . \det(\begin{bmatrix} u_x & u_y & u_z & 1 \\ u_x & u_y & u_z & 1 \\ u_x & u_y & u_z & 1 \\ u_x & u_y & u_z & 1 \end{bmatrix})$$

$$= 0$$

Here we can conclude that all the points where fluid motion takes placed are critical points. So the turbulence for motion can occur even if very small external force is applied. Also the divergence matrix is applied on the velocity matrix to get acceleration. So there is scope to study entire description motion of fluid i. e. extreme values, change of nature of path using eigen values and trace of the acceleration matrix. Hassian matrix can be used to find extreme values of motion.

CONCLUSION

The Nevier-Stokes equations are converted into matrix form. Also all points of the motion through which the fluid is passing are critical points. So the turbulence for motion can occur even if very small force is applied.

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