Available Online at http://www.recentscientific.com

CODEN: IJRSFP (USA)

International Journal of Recent Scientific Research
Vol. 9, Issue, 5(F), pp. 26901-26906, May, 2018

ISSN: 0976-3031

International Journal of Recent Scientific Research

DOI: 10.24327/IJRSR

Research Article

E-CORDIALITY OF TAIL C4RELATED ONE POINT UNION GRAPHS

Mukund V. Bapat*

Department of Mathematics, Shri Kelkar College, Devgad. Dist.: Sindhudurg, Maharashtra, India 416613

DOI: http://dx.doi.org/10.24327/ijrsr.2018.0905.2147

ARTICLE INFO

Article History:

Received 10th February, 2018
Received in revised form 6th March, 2018
Accepted 24th April, 2018
Published online 28th May, 2018

Key Words:

E-cordial, tail graph, one point union, C_4, invariance.
Subject Classification: 05C78

ABSTRACT

In this paper we obtain e-cordial labeling of one point union of k copies of tail graph namely $G^{(k)}$ where G = tail $C_4(P_t)$. We have taken different values of t as 2,3,4. If we change point of union on G then different structures of $G^{(k)}$ are obtained. Of these we have taken pairwise non isomorphic structures of $G^{(k)}$ and have proved that they all are e-cordial under certain conditions. We have also considered the case that more than one tails are attached to G such that sum of edges is t-1 for given t as above and the family is e-cordial.

INTRODUCTION

In 1997 Yilmaz and Cahit introduced weaker version of edge graceful labeling E-cordial labeling [4]. Let G be a (p,q) graph. f:E→{0,1}be a function. Define f on V by f(v) $=\sum\{f(vu)(vu)\in E(G)\}(mod\ 2)$.The function f is called as E-cordial labeling if $|v_f(0)-v_f(1)|\leq 1$ and $|e_f(0)-e_f(1)|\leq 1$ where $v_f(i)$ is the number of vertices labeled with i =0,1. And $e_f(i)$ is the number of edges labeled with i= 0,1,We follow the convention that $v_f(0,1)$ =(a, b) for $v_f(0)$=a and $v_f(1)$=b further $e_f(0,1)$=(x,y)for $e_f(0)$=x and $e_f(1)$=y. A graph that admits E-cordial labeling is called as E-cordial graph. Yilmaz and Cahit prove that trees T_n are E-cordial iff for n not congruent to 2(mod 4), K_n are E-cordial iff n not congruent to 2(mod 4), Fans F_n are E-cordial iff for n not congruent to 1(mod 4).Yilmaz and Cahit observe that A graph on n vertices cannot be E-cordial if n is congruent to 2 (mod 4).One should refer Dynamic survey of graph labeling by Joe Gallian [2] for more results on E-cordial graphs.

The graphs we consider are finite, undirected, simple and connected. For terminology and definitions we refer Harary[3] and Dynamic survey of graph labeling by Joe Gillian [2]. 3. Preliminaries:

Fusion of vertex. Let G be a (p,q) graph. Let u≠v be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has at least p-1vertices and q-1 edges.[5] 3.2$G^{(K)}$ it is One point union of k copies of G is obtained by taking k copies of G and fusing a fixed vertex of each copy with same fixed vertex of other copies to create a single vertex common to all copies. If G is a (p, q) graph then $|V(G_{(k)}|$ = k(p-1)+1and |E(G)|= k.q3.3 A tail graph (also called as antenna graph) is obtained by fusing a path p_k to some vertex of G. This is denoted by tail(G, P_k). If there are t number of tails of equal length say (k-1) then it is denoted by tail(G, tp_k). If there are two or more tails attached at same vertex of G we denote it by tail G(P_t,P_k..) If G is a (p,q) graph and a tail P_k is attached to it then tail(G, P_k) has p+k-1 vertices and q+k-1 edges.

MAIN RESULTS

Theorem: All structures of one point union of k copies ofG = tail (C_4,P_2)i.e. $G^{(K)}$ are e-cordial for all k =1,2,..

Proof: The different structures are due to the common vertex on G is changed. It follows from figure 4.1 that there are four non-isomorphic structures possible and are at verices 'a', 'b', 'c', or 'd' of G.

Corresponding author:* **Mukund V. Bapat
Department of Mathematics, Shri Kelkar College, Devgad. Dist.: Sindhudurg, Maharashtra, India 416613

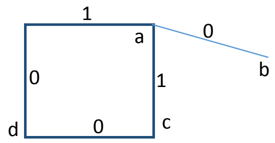

Fig.4.1 common vertex can be ‘a’, ‘b’, and ‘c’ or ‘d’ . $v_f(0,1)= (3,2)$, $e_f(0,1)= (3,2)$

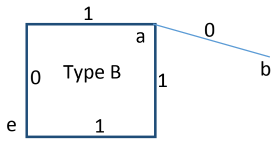

Fig.4.2 common vertex can be ‘a’, ‘b’, and ‘c’ or ‘d’ . $v_f(0,1)= (3,2)$, $e_f(0,1)= (2,3)$

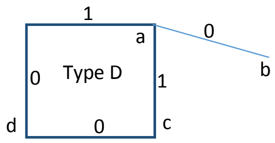

Fig.4.3 common vertex can be ‘a’, ‘b’, and ‘c’ or ‘d’ . $v_f(0,1)= (3,2)$, $e_f(0,1)= (3,2)$

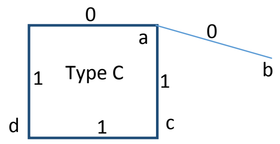

Fig.4.4 common vertex can be ‘a’, ‘b’, and ‘c’ or ‘d’ . $v_f(0,1)= (3,2)$, $e_f(0,1)= (2,3)$

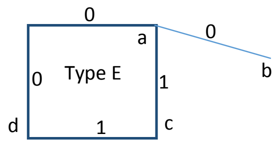

Fig.4.5 common vertex can be ‘a’, ‘b’, and ‘c’ or ‘d’ . $v_f(0,1)= (3,2)$, $e_f(0,1)= (3,2)$

Define a function$f:E((G)^{(k)})\rightarrow\{0,1\}$. This gives us four types of labeling as Type E, type B, type C and Type D as shown above. In $G^{(K)}$ when k =1 the copy in figure 4.1 will work.

Structure 1: is obtained when one point union at vertex ‘b’ is taken.This is done by fusing type B label with type D label at vertex ‘b’ on it.Type B is used when k≡ 1 (mod 2) and type D is used when k≡ 0(mod 2).

Structure 2: is obtained when one point union at vertex ‘a’ is taken.This is done by fusing type B label with type D label at vertex ‘a’ on it.Type B is used when k≡ 1 (mod 2) and type D is used when k≡ 0 (mod 2).

Structure 3: is obtained when one point union at vertex ‘c’ is taken.This is done by fusing type C label with type E label at vertex ‘c’ on it.Type C is used when k≡ 1 (mod 2) and type E is used when k≡ 0 (mod 2).

Structure 4: is obtained when one point union at vertex ‘d’ is taken.This is done by fusing type C label with type D label at vertex ‘d’ on it.Type C is used when k≡ 1 (mod 2) and type D is used when k≡ 0 (mod 2).

The label number distribution for all structures is as follows:

For k = 2x, x=1,3,5,..we have $v_f(0,1)= (5+4x,4+4x)$, $e_f(0,1)= (5x,5x)$ and label of common vertex ‘0’.

For k = 2x+1 , x=0, 1, 2, 3,,..we have $v_f(0,1)= (3+4x, 2+4x)$, $e_f(0,1)= (2+5x,3+5x)$ and label of common vertex ‘0’.Thus the graph is e-cordial.

Theorem: All structures of one point union of k copies of G = tail (C_4, $2P_2$)i.e. $G^{(K)}$ are e-cordial for all k not congruent to 1(mod 4).

Proof: There are four possible structures on $G^{(k)}$ which are pairwise non-isomorphic depending on common point ‘e’ ‘a’, ‘d’ or ‘c’. This is clear from fig 4.6.The function $f:E(G^{(K)})\rightarrow\{0,1\}$ gives following four types of labels namely type A , type B, type C, and type E. We combine it to obtain a labeled copy of $G^{(K)}$.

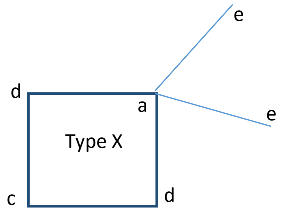

Fig.4.6 common vertex can be ‘a’ or ‘c’ or ‘d’ or ‘e’

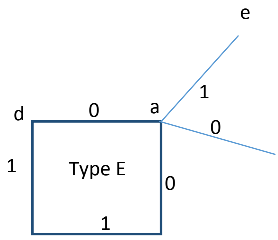

Fig.4.7. $v_f(0,1)= (2,4)$, $e_f(0,1)= (3,3)$

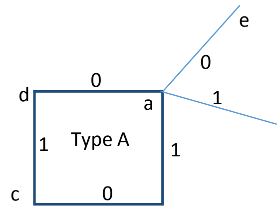

Fig.4.8. $v_f(0,1)= (2,4)$, $e_f(0,1)= (3,3)$

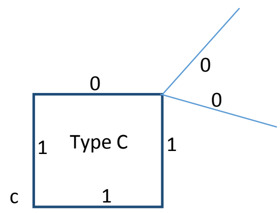

Fig.4.9. $v_f(0,1)= (2,4)$, $e_f(0,1)= (3,3)$

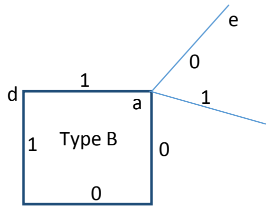

Fig.4.10 $v_f(0,1)$= (4,2), $e_f(0,1)$= (3,3)

In structure 1 we fuse Type E and Type B label at vertex 'a'. For k≡1, 2 (mod2) we use Type E label and Type B label for k ≡0 ,3 (mod2).

In structure 2 we fuse Type E and Type B label by fusing at vertex 'e'. For k≡1,2 (mod4) we use Type E label and Type B label for k ≡0,3 (mod 4).

In structure 3 we fuse Type E and Type B label by fusing at vertex 'd'. For k≡1,2 (mod 4) we use Type E label and Type Blabel for k ≡0,3 (mod4).

In structure 4 we fuse Type A and Type C label by fusing at vertex 'c'. For k≡1,2 (mod4) we use Type A label and Type C label for k ≡0,3 (mod 4).

The resultant label numbers are $v_f(0,1)$= (5+10x,6+10x), $e_f(0,1)$= (3k,3k)) for k ≡2(mod 4) or k=4x+2 such that x=0, 1, 2 …and $v_f(0,1)$= (8+10x), 8+10x)), $e_f(0,1)$= (3k, 3k) for or k ≡3(mod 4) or k =4x+3 such that x= 0,1, 2 …and $v_f(0,1)$= (1+10x, 10x), $e_f(0,1)$= (3k,3k) for k ≡0(mod 4) or k =4x such that x= 1, 2 … Thus for all values of k except for k ≡ 1 (mod 4)we have the graph is e-cordial.

Theorem: All structures of one point union of k copies of G = tail (C_4, P_3)i.e. $G^{(K)}$ are e-cordial for all k not congruent to (1mod 4).

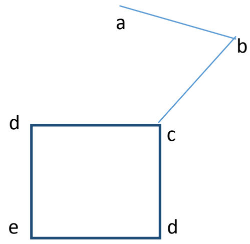

Fig.4.11 common vertex can be 'a' or 'c' or 'd' or 'e'

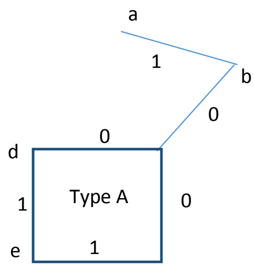

Fig.4.12 $v_f(0,1)$= (2,4), $e_f(0,1)$= (3,3)

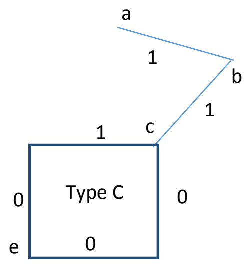

Fig.4.13 $v_f(0,1)$= (4,2), $e_f(0,1)$= (3,3)

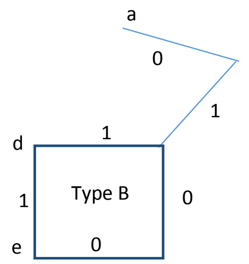

Fig.4.14 $v_f(0,1)$= (4,2), $e_f(0,1)$= (3,3)

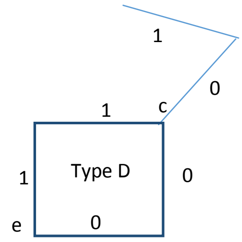

Fig.4.15 $v_f(0,1)$= (2,4), $e_f(0,1)$= (3,3)

Proof: There are five possible structures on $G^{(k)}$ which are pairwise non-isomorphic depending on common point 'e' 'a', 'b', 'd' or 'c'. This is clear from fig 4.11. The function $f:E(G^{(K)})\rightarrow\{0,1\}$ gives following four types of labels namely type A , type B, type C, and type D. We combine it to obtain a labeled copy of $G^{(K)}$.

In structure 1 we fuse Type A and Type B label at vertex 'a'. For k≡1,0 (mod4) we use Type A label and Type B label for k ≡2, 3 (mod4).

In structure 2 we fuse Type A and Type C label by fusing at vertex 'b'. For k≡1, 0 (mod4) we use Type E label and Type B label for k ≡2, 3 (mod 4).

In structure 3 we fuse Type D and Type C label by fusing at vertex 'c'. For k≡1, 0 (mod 4) we use Type D label and Type C label for k ≡2, 3 (mod4).

In structure 4 we fuse Type A and Type B label by fusing at vertex 'd'. For k≡ 1, 0 (mod4) we use Type A label and Type B label for k ≡2, 3 (mod 4).

In structure 5 we fuse Type D and Type C label by fusing at vertex 'e'. For k≡1, 0 (mod4) we use Type D label and Type C label for k ≡2, 3 (mod 4).

The resultant label numbers are $v_f(0,1)$= (5+10x,6+10x), $e_f(0,1)$= (3k, 3k)) for k ≡2(mod 4) or k=4x+2 such that x=0, 1, 2 …and $v_f(0,1)$= (8+10x), 8+10x)), $e_f(0,1)$= (3k, 3k) for or k ≡3(mod 4) or k =4x+3 such that x= 0,1, 2 …and $v_f(0,1)$=

(1+10x, 10x), $e_f(0,1)$= (3k,3k) for k ≡0(mod 4) or k =4x such that x= 1, 2 …Thus for all values of k except for k ≡ 1 (mod 4)we have the graph is e-cordial.

Theorem: All structures of one point union of k copies of G = tail (C_4, P_4)i.e. $G^{(K)}$ are e-cordial for all k=1, 2, ..

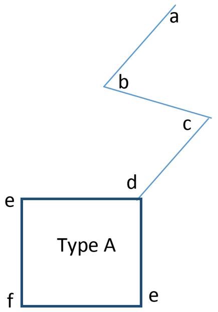

Fig.4.16 Six Points For Six

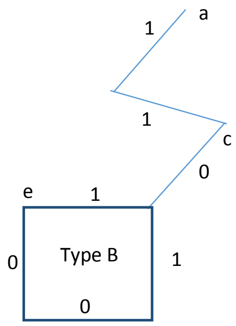

Fig.4.17 $v_f(0,1)$= (3,4), $e_f(0,1)$= (3,4)

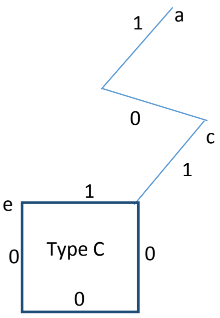

Fig.4.18 $v_f(0,1)$= (3,4), $e_f(0,1)$= (4,3)

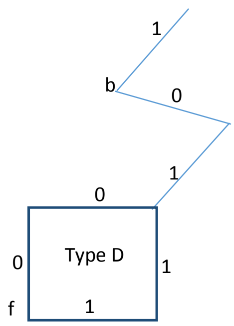

Fig.4.19 $v_f(0,1)$= (3,4), $e_f(0,1)$= (3,4)

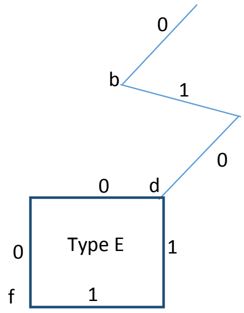

Fig.4.20 $v_f(0,1)$= (3,4), $e_f(0,1)$= (4,3)

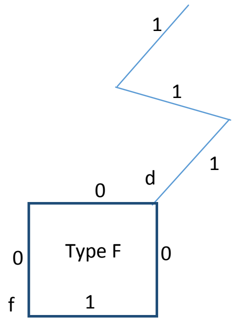

Fig.4.21 $v_f(0,1)$= (3,4), $e_f(0,1)$= (3,4)

Proof: There are six possible structures on $G^{(k)}$ which are pairwise non-isomorphic depending on common point 'e','f', 'a', 'b', 'd' or 'c'.This is clear from fig. 4.16. The function f:E($G^{(K)}$)→{0,1} gives following five types of labels namely type E , type B, type C, and type D, type F. We combine it to obtain a labeled copy of $G^{(K)}$.

In structure 1 we fuse Type B and Type C label at vertex 'a'. For k≡ 1(mod 2) we use Type B label and Type C label for k ≡0 (mod 2).

In structure 2 we fuse Type D and Type E label by fusing at vertex 'b'. For k≡ 1(mod 2) we use Type D label and Type E label for k ≡0 (mod 2).

In structure 3 we fuse Type B and Type C label by fusing at vertex 'c'. For k≡ 1 (mod 2) we use Type B label and Type C label for k ≡2 (mod 0).

In structure 4 we fuse Type F and Type E label by fusing at vertex 'd'. For k≡ 1(mod 2) we use Type F label and Type E label for k ≡0 (mod 2).

In structure 5 we fuse Type B and Type C label by fusing at vertex 'e'. For k≡ 1 (mod 2) we use Type B label and Type C label for k ≡0 (mod 2).

In structure 6 we fuse Type D and Type E label by fusing at vertex 'f'. For k≡ 1 (mod 2) we use Type D label and Type E label for k ≡0 (mod 2).

The resultant label numbers are $v_f(0,1)$= (3+6x,4+6x), $e_f(0,1)$= (3+7x, 4+7x)) for k ≡2x+1, x = 0,1 ,2, .. $v_f(0,1)$= (1+6x,6x), $e_f(0,1)$= (7x, 7x)) for k ≡2x; x = 1 ,2, ..

Thus for all values of k we have the graph is e-cordial.

Theorem: All structures of one point union of k copies of G = tail (C_4,P_2,P_3)i.e. $G^{(K)}$ are e-cordial for all k=1, 2, ..

Proof: From fig 4.22 it follows that we can get 6 different structures for one point union of G. Define $f:E(G^{(K)}) \rightarrow \{0,1\}$ gives following four types of labels namely type E , type B, type C, and type D, type F. We combine it to obtain a labeled copy of $G^{(K)}$.

In structure 1 we fuse Type B and Type C label at vertex 'a'. For k≡ 1(mod 2) we use Type B label and Type C label for k ≡0(mod4).

In structure 2 we fuse Type B and Type E label by fusing at vertex 'b'. For k≡ 1(mod 2) we use Type B label and Type E label for k ≡0 (mod 2).

In structure 3 we fuse Type B and Type C label by fusing at vertex 'c'. For k≡ 1 (mod 2) we use Type B label and Type C label for k ≡2 (mod 0).

In structure 4 we fuse Type D and Type E label by fusing at vertex 'd'. For k≡ 1(mod4) we use Type Dlabel and Type E label for k ≡0 (mod 2).

In structure 5 we fuse Type D and Type E label by fusing at vertex 'e'. For k≡ 1 (mod4) we use Type D label and Type E label for k ≡0 (mod 2).

In structure 6 we fuse Type B and Type C label by fusing at vertex 'f'. For k≡ 1 (mod4) we use Type B label and Type C label for k ≡0 (mod 2).

The resultant label numbers are $v_f(0,1)= (3+6x,4+6x)$, $e_f(0,1)= (3+7x, 4+7x))$ for k ≡2x+1, x = 0,1 ,2, ..$v_f(0,1)= (1+6x,6x)$, $e_f(0,1)= (7x, 7x))$ for k ≡2x; x = 1 ,2, ..

Thus for all values of k we have the graph is e-cordial.

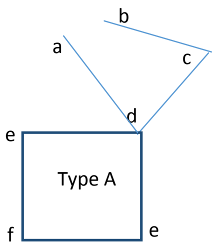

Fig. 4.22 Six Points for Six Structures

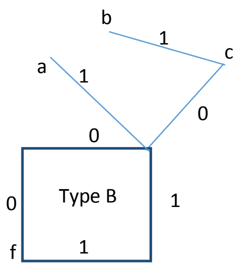

Fig.4.23 $v_f(0,1)= (3,4)$, $e_f(0,1)= (3,4)$

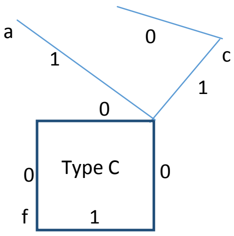

Fig.4.24 $v_f(0,1)= (3,4)$, $e_f(0,1)= (4,3)$

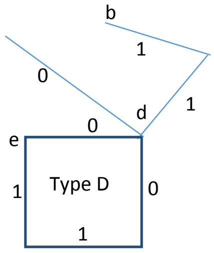

Fig.4.25 $v_f(0,1)= (3,4)$, $e_f(0,1)= (3,4)$

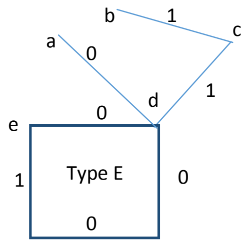

Fig.4.26 $v_f(0,1)= (3,4)$, $e_f(0,1)= (4,3)$

Theorem: All structures of one point union of k copies of G = tail $(C_4,3P_2)$i.e. $G^{(K)}$ are e-cordial for all k=1, 2, ..

Proof

From fig 4.27 it follows that we can get 4 different structures for one point union of G. Define $f:E(G^{(K)}) \rightarrow \{0,1\}$ gives following four types of labels namely type B, type C, and type D, type E. We combine it to obtain a labeled copy of $G^{(K)}$.

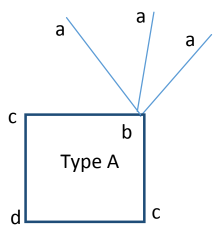

Fig. 4.27 Six Points for Six Structures

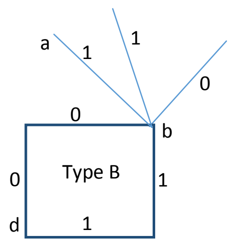

Fig.4.28 $v_f(0,1)= (3,4)$, $e_f(0,1)= (3,4)$

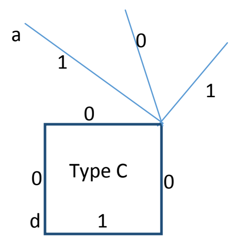

Fig.4.29 $v_f(0,1)= (3,4)$, $e_f(0,1)= (4,3)$

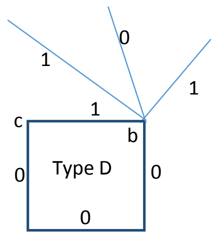

Fig.4.30 $v_f(0,1)=(3,4)$, $e_f(0,1)=(4,3)$

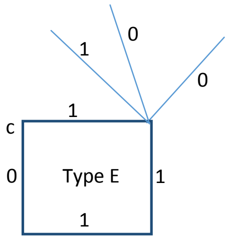

Fig.4.31 $v_f(0,1)=(3,4)$, $e_f(0,1)=(3,4)$

In structure 1 we fuse Type B and Type C label at vertex 'a'. For k≡ 1(mod 2) we use Type B label and Type C label for k ≡0 (mod4).

In structure 2 we fuse Type B and Type D label by fusing at vertex 'b'. For k≡ 1(mod 2) we use Type B label and Type D label for k ≡ 0 (mod 2).

In structure 3 we fuse Type E and Type D label by fusing at vertex 'c'. For k≡ 1 (mod 2) we use Type E label and Type D label for k ≡ 2 (mod 0).

In structure 4 we fuse Type B and Type C label by fusing at vertex 'd'. For k≡ 1(mod4) we use Type B label and Type C label for k ≡0 (mod 2).

The resultant label numbers are $v_f(0,1)=(3+6x,4+6x)$, $e_f(0,1)=(3+7x, 4+7x))$ for k ≡2x+1, x = 0,1 ,2, .. $v_f(0,1)=(1+6x,6x)$, $e_f(0,1)=(7x, 7x))$ for k ≡2x; x = 1 ,2, ..

Thus for all values of k we have the graph is e-cordial.

CONCLUSIONS

In this paper we show that 1) All structures of one point union of k copies of G = tail (C_4, P_2)i.e. $G^{(K)}$ are e-cordial. 2) All structures of one point union of k copies of G = tail (C_4, $2P_2$)i.e. $G^{(K)}$ are e-cordial for all k not congruent to 1(mod 4). 3) All structures of one point union of k copies of G = tail (C_4, P_3)i.e. $G^{(K)}$ are e-cordial for all knot congruent to (1mod 4).4) All structures of one point union of k copies of G = tail (C_4, P_4)i.e. $G^{(K)}$ are e-cordial for all k=1, 2,..5) All structures of one point union of k copies of G = tail (C_4, P_4)i.e. $G^{(K)}$ are e-cordial for all k=1, 2, .. 6)All structures of one point union of k copies of G = tail (C_4, $3P_2$)i.e. $G^{(K)}$ are e-cordial for all k=1, 2, ..

References

1. Bapat M.V. Ph.D. thesis "equitable and other types of graph labeling", University Of Mumbai, 2004
2. Joe Gallian Dynamic survey of graph labeling 2016
3. Harary, Graph Theory, Narosa publishing, New Delhi
4. Yilmaz, Cahit, E-cordial graphs, Ars combina, 46,251-256.
5. Introduction to Graph Theory by D. WEST, Pearson Education Asia.

1 Mukund V. Bapat, Hindale, Devgad, Sindhudurg, Maharashtra, India: 416630 mukundbapat@yahoo.com

How to cite this article:

Mukund V. Bapat.2018, E-Cordiality of Tail C4 Related One Point Union Graphs. *Int J Recent Sci Res.* 9(5), pp. 26901-26906. DOI: http://dx.doi.org/10.24327/ijrsr.2018.0905.2147

****