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## Research Article

# SOLUTION TO ESTIMATE THE TURBULENT DRAG COEFFICIENT FOR TWO-PHASE FLUID FLOW IN PIPELINES

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#### Key Words:

$v_l, v_g$  -velocity for the liquid and gas, m/s,  
 $m_l, m_g$  -mass of the liquid and gas,  $D$  -  
pipe diameter,  $A_l, A_g$  - area for the liquid  
and gas phases,  $\epsilon$  - the void fraction,  $L$  -  
the estimated distance,  $f_l, f_g, f_{lg}$  - drag  
factor assuming the entire flow as liquid,  
as gas and as two-phases,  $\mu_l, \mu_g$  - dynamic  
viscosity, Ns/m,  $Q$  - flow rate,  $\bar{V}$  -mean  
specific volume

### ABSTRACT

To improve the capacity of the two-phase fluid movement through the horizontal pipes it is necessary to determine as correctly as possible the turbulent drag coefficient, to estimate the associated energetic balance. For modeling was considered the hydrodynamic flow, in turbulent regime. From the analytical known methods are selected the homogeneous, the separated-flow, and the mechanistic models, considered more accurate and suitable for the dedicated applications. This two-phase flow is important in a large variety of applications from engineering, such as natural gas production, oil transportation, drilling, the food processing, polymer processing industry, pharmaceutical domains, etc. The Present paper, is dedicated especially for long pipes of transportation. To better model the reality, is considered the flow between liquid and gas, with different flow rates for each of them. As first step, in laboratory was modified the gas flow rate. For the Reynolds number attached to the flow the range values are from 6000 to 140000. In laboratory were realized around 200 measurements points, tested for each selected models. The analyzed cases allowed the estimation in a proper manner of the accuracy of the drag turbulent factor, by calculating all 10 statistical parameters, for pipes up to 80 cm.

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### INTRODUCTION

A hydrodynamic flow regime, into horizontal pipes with interfacial gas-liquid distribution, may have different possible forms, resulting in various flow patterns, due to the flow rate of the fluid and gas, Fig.1.

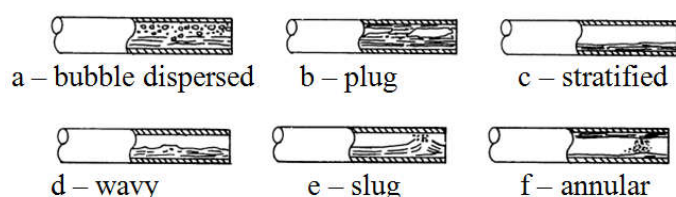


Fig 1 Flow patterns in horizontal pipelines

Such type of two-phase fluid flow is important for the transportation of oil and petroleum products, especially in the long pipelines. As an example in Europe: the Pan-European Pipeline, the Baltic Pipelines System, the Odessa-Brody pipeline (the Sarmatia pipeline) or in the North America & Canada, the Enbridge Pipeline System, the Keystone Pipeline, the Trans-Alaska Pipeline System, etc.

Moreover, the dimensionless pressure gradients are usually expressed, as drag factors. In time, some previous analytical methods have described the two-phase flows in the horizontal tubes, but the errors are considerable (around 8-10%). The relation between the pressure gradient and the mass flow is also calculated in dimensionless form, ordinarily, as a relation between the drag factor and the Reynolds number.

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If there is a single phase of flow, the shear forces on the wall, which creates friction, known as linear hydraulic losses are expressed from the Moody diagram. They take into account, depending on the flow regime only of the Reynolds number, both of the Reynolds number and the surface roughness, or only of the pipeline roughness.

For the two-phase flow, a completely analytical model universally accepted, for all cases of the two-phase pressure drop, has not been yet developed. There are some semi-analytical solutions and some empirical correlations adapted to the specific domain of applications. Until now, there are twenty-two prediction methods, used for the different cases of the two-phase flow, in pipelines. From the analytical solutions, in previous research were studied four models capable to describe, as good as possible, the conditions encountered in the two-phase flow through horizontal, long pipelines: the homogeneous model [1], the separated-flow model [2], the mechanistic models [3], and the drift-flux model [4].

The first three models were selected in the present paper, because they are more adequate for the concrete dedicated applications. The pressure gradient depends on the flow type, and the prediction of the drag factor, for each model is represented by specific boundary conditions. In the case of the two-phase flow, an additional interaction appears between the phases, having as consequence a supplementary difficulty in evaluation of the pressure drop. For each model the variables were made dimensionless, to enable generalization at different solutions of horizontal tubes, or pipelines, with different diameters. The accuracy of the developed correlations from this paper is evaluated by comparing the predictions of previous calculations and correlations with the measured and obtained results, and with the data from the technique literature. The relation between the pressure gradient and mass flow is also expressed in dimensionless form, as a relation between the drag factor and the Reynolds number, considered for the two-phase flow. This one was correlated with the generalized Reynolds number, with values from 6000 to 140000. In the case of flow for a single phase, the shear forces on the wall create friction, losses, followed by a pressure decreasing, known as linear hydraulic losses. In the case of the two-phase flows, an additional interaction appears between the two phases, having as consequence a supplementary difficulty in the evaluation of the pressure drop. The gas-liquid interfacial distribution may have different possible forms, with effect in various flow patterns, due to the different flow rates of the fluid and gas. For the mentioned three models is realized a combination as to numerically modeling and determination of the drag factor, depending on the Reynolds number, based on around 3000 measurements made partially in the Romanian laboratories, and partially taken from the literature. Accordingly, to the area of interest, there are selected only measurements made with oil and gas, at different combinations of the flow rate. It is estimated a relative velocity between the two-phase flow, as to be calculated in a proper manner the type of the coefficient for the hydraulic losses, taking into account the different ways, types of flow through pipes: dispersed bubble, slug, stratified, and annular flow, Fig.1. For each one, the variables were dimensionless, in order to allow the generalization at different types of horizontal tubes, pipelines, with different diameters.

All the ten, main statistical parameters are calculated and presented, as the obtained results. The accuracy of the developed correlations from this paper is evaluated by comparing it with the predictions from previous calculations, experimental measurements and correlations, with predictions of correlations from others authors [5], [6], [7] and models, available in the literature [8], [9].

## MATERIALS AND METHODS

### Theoretical aspects and dimensionless parameters

The main equations used in all three tested models, based on Bernoulli equation (energy conservation), became:

$$dP + d(G_g v_g + G_l v_l) + \rho_{TP} g dL + dP_f = 0 \quad (1)$$

The local void fraction is the time averaged volumetric fraction of the gas, in the two-phase flow. The entire void fraction is given by a cross section average, of the local void:

$$\varepsilon = \frac{1}{A} \int \varepsilon_{local} dA$$

The time-averaged area occupied by the gas phase in total area:

$$\varepsilon = \frac{A_g}{A_g + A_l} \quad (2)$$

To assure the continuity equation, each phase of the flow must be in movement. The mass and the volume of the flow rates are, in this condition:

$$\begin{aligned} W_l &= \rho_l A_l v_l; \\ W_g &= \rho_g A_g v_g \\ Q_l &= v_l A_l \\ Q_f &= v_f A_f \end{aligned} \quad (3)$$

Dividing Eq.(3) to the cross-section we obtain the mass velocities:

$$\begin{aligned} G_l &= v_l \rho_l (1 - \varepsilon) \\ G_g &= v_g \rho_g \varepsilon \end{aligned}$$

In the two-phase flow are defined several terms, describing the flow characteristics:  $x_g$ - mass fraction of the gas in the fluid flow,  $\varepsilon$  - the space quantities in flow, due to the gas presence, and the slip velocities ratio S.

$$\begin{aligned} x_g &= m_g / (m_g + m_l) \\ \varepsilon &= (h_g - h_l) / h_l \\ S &= v_g / v_l \end{aligned} \quad (4)$$

The two-phase flow appears in different forms in horizontal pipes, depending on the liquid and gas pressure, on the velocities of the two phases, and of the volume flow rate. In Fig.1, are presented the most usual types of the flow patterns in horizontal pipes.

By transforming the Bernoulli equation for a steady two-phase flow, the pressure gradient is represented by a sum of the pressure gradients, due to drag and acceleration (in horizontal pipes the component due gravity may be neglected). Then:

$$\frac{dP}{dL} = \left( \frac{dP}{dL} \right)_f + \left( \frac{dP}{dL} \right)_a \quad (5)$$

Where

$$\left(\frac{dP}{dL}\right)_f = -\frac{2f_{TP}G^2\bar{V}}{gD} \quad (6)$$

$$\left(\frac{dP}{dL}\right)_a = -\frac{G^2}{g} \frac{d\bar{V}}{dL}$$

In Eq.6 the unknowns are  $f_{TP}$  and  $\bar{V}$ . Generally, it is difficult to estimate from the beginning the two-phase drag pressure,  $f_{TP}$  coefficient. As a first step, it is estimated as a single phase, the drag reduction, noted  $f_{fo}$  assuming that there is only liquid, and  $f_{go}$  if there is only gas. It is multiplied by an appropriate function of the flow rate, and the gas-liquid parameters.

$$f_{fo}^2 = \frac{\left(\frac{dP}{dL}\right)_f}{\left(\frac{dP}{dL}\right)_{fo}}; \quad f_{go}^2 = \frac{\left(\frac{dP}{dL}\right)_f}{\left(\frac{dP}{dL}\right)_{go}}$$

With these notations, the relations (6) became for liquid only, for two-phases, and for gas only.

$$\left(\frac{dP}{dL}\right)_{fo} = -\frac{2f_{fo}G^2v_f}{gD}; \quad \left(\frac{dP}{dL}\right)_f = -\frac{2f_fG_f^2v_f}{gD} \quad (7)$$

$$\left(\frac{dP}{dL}\right)_g = -\frac{2f_gG_g^2v_g}{gD}$$

Based on relations (7), Fanning estimate the drag reduction factor for a gas-liquid mixture, where the  $v_{TP}$  is the mixed gas-liquid velocity,  $v_{TP} = V_g + v_l$ .

$$f = \frac{\frac{dp}{dL} D}{2\rho_{TP}v_{TP}^2} \quad (8)$$

The pressure  $\frac{dp}{dL}$  is related to the wall shear stress  $\tau = \frac{dp}{dL} \frac{D}{4}$  and with the two-phase density,  $\rho_{TP} = \rho_l \cdot \varepsilon + \rho_g(1 - \varepsilon)$ .

The Fanning drag factor must be correlated with the Reynolds number attached the two-phase flow.

$$Re = \frac{v_{TP} D}{\nu_l}$$

## RESULTS

### The tested two-phase flow models

#### The Homogeneous model

This model is based on the hypothesis that, both liquid and gas phases have the same velocity, and the slip factor is equal to unity. The equations in this model reduce the void fraction, homogeneous density and  $\bar{V}$  at:

$$\varepsilon = \frac{x}{x + (1-x)\rho_g / \rho_l} \quad (9)$$

$$\rho_{TP} = \varepsilon \cdot \rho_g + (1 - \varepsilon)\rho_l \quad (10)$$

$$\bar{V} = xv_g + (1-x)v_f$$

Despite its small applicability, the homogeneous model is still the most widely accepted in the numerical modeling, for predicting the two-phase pressure drop.

The final form of the pressure drop, after few calculations became:

$$\frac{dp}{dL} = -\frac{2G^2 f_{TP} \cdot v_f}{gD} \left[ 1 + x \left( \frac{v_{TP}}{v_f} \right) \right] - \frac{G^2 \cdot v_f}{g} \left( \frac{v_{TP}}{v_f} \right) \frac{dx}{dL} \quad (11)$$

The Eq.11 may be directly integrated for flows with simple pressure gradient.

#### The Separated flow model

This model is based on the assumption that the two phases have different values of the velocities. The liquid velocity and the slip ratio are defined as (4). From Eq.4 we may deduce the void fraction  $\varepsilon$ . Then, the velocity and the slip ratio became:

$$v_l = \frac{(1-x)G_{gl}}{1-f_l} \quad (12)$$

$$S = \frac{x}{1-x} \frac{v_g}{v_l} \frac{1-f_g}{f_l}$$

The final form of the pressure drop is:

$$\frac{dP}{dL} = \left(\frac{dp}{dL}\right)_f - \frac{G^2}{g} \frac{d}{dL} \left[ \frac{x^2 v_g}{\varepsilon} + \frac{(1-x)^2}{1-\varepsilon} v_f \right] \quad (13)$$

In Eq.13 we must know the void fraction and the two-phase friction. If the slip factor is equal to 1, we obtain the homogeneous model. The model may be directly integrated, by estimation the drag reduction, with some empirical correlations and can be applied and to the annular flow.

#### The Mechanistic model

This model is adequate for some types of flows: liquid phase viscous and gas phase turbulent or liquid phase turbulent and gas phase viscous is more adapted to the proposed studied applications.

The drag reduction is:

$$f = 0.053 \cdot v_g^{*0.23} \cdot v_l^{*0.202} D - 0.46L^{0.076} \quad (14)$$

Where:

$$v_g^* = v_g \sqrt[4]{\frac{\rho_l}{g \cdot \sigma}}$$

$$v_l^* = v_l \sqrt[4]{\frac{\rho_l}{g \cdot \sigma}} \quad (15)$$

Independent of the adopted model, for the gas mass and for the liquid stream film, the force balance may be written:

$$-A_v \frac{dP}{dL} - \tau_{lS} l - gA_v \rho_v = 0$$

$$-A_l \frac{dP}{dL} - \tau_{lS} l - \tau_{TP} s l - gA_l \rho_l = 0 \quad (16)$$

For flows of high viscosity oils in horizontal pipes is very important to evaluate, as accurately as possible, the void fraction, the two-phase drag and the pressure gradient.

**The numerical model**

As a data base for the numerical modeling were selected around 3000 measurements made partially into the Hydraulics Laboratory of the Department of Hydraulics, Hydraulic machinery, and Environmental Engineering, Faculty of Energy from University Politehnica of Bucharest, on a dedicated stand, but were also selected another data from the scientific literature, presented in Table 1.

**Table 1** Data set measured and collected from literature

Source	Nr. exp.	Fluid	$\eta$ (mPa's)	$V_l$ (m/s)	$V_g$ (m/s)	D (m)
Rivero (1995)	750	Air-oil	1-200	0.02-0.19	0.61-11.89	0.0508
Ortega (2000)	500	Air-oil	500	0.1-2.77	0.02-38.24	0.0508
Cabello	240	Air-kerosene	1	0.11-4.52	0.77-45.65	0.0508
Ortega (2001)	350	Air-oil	1200	0.01-0.8	0.23-24.39	0.0508
Mata	320	Air-oil	100	0.58-2.68	0.26-12.91	0.078
Radulescu (2012-2014)	800	Air-oil	800	0.4-2.2	0.4-14.5	0.04

There were tested more than 2900 possibilities for the two-phase flow, based on the evaluation of the coefficients, realized by different authors, for all of the three mentioned models. Were calculated the all ten statistical parameters, which characterize the flow, using the Riply range factor, obviously necessary when the prediction level of confidence must be higher than 90%. The estimated uncertainties, using the root-sum-square method in the drag factor estimation and the Reynolds number is more or less 5%.

$$R = \exp \left[ t_{0,01} E_{10} \sqrt{1 + \frac{1}{n}} \right] \quad (17)$$

The observed hydrodynamic entrance was selected at 70 diameters, and the air admission is considered at 24 diameters before the analyzed zone. The flow rate for oil was maintained between  $3.8-4.2 \times 10^{-3} \text{ m}^3/\text{s}$  and for air between  $0.8-1.2 \times 10^{-3} \text{ m}^3/\text{s}$ . The void fraction and the two-phase pressure drop have been measured for air-oil two-phase for horizontal cross-flow in a staggered pipe bundle. The obtained data are compared with the results of previous research, and some new correlations for the average bundle of the void fraction are deduced. There were calculated the statistical parameters:

$$\begin{aligned}
 E_1 &= \frac{1}{n} \sum_{i=1}^n r_i & E_2 &= \frac{1}{n} \sum_{i=1}^n |r_i| \\
 E_3 &= \sqrt{\frac{1}{n-1} \sum_{i=1}^n (r_i - E_1)^2} & E_4 &= \sqrt{\frac{1}{n-1} \sum_{i=1}^n (r_i)^2} \\
 E_5 &= \frac{1}{n} \sum_{i=1}^n (e_i) & E_6 &= \frac{1}{n} \sum_{i=1}^n (e_i) \\
 E_7 &= \sqrt{\frac{1}{n-1} \sum_{i=1}^n (e_i - E_5)^2} & E_8 &= \sqrt{\frac{1}{n-1} \sum_{i=1}^n (e_i)^2} \\
 E_9 &= \frac{1}{n} \sum_{i=1}^n (e_i') & E_{10} &= \sqrt{\frac{1}{n-1} \sum_{i=1}^n (e_i' - E_9)^2}
 \end{aligned} \quad (18)$$

Where  $r_i = \frac{f_{TP,pred} - f_{TP,exp}}{f_{TP,exp}} \times 100$  (%)

And  $e_i = \left( \frac{dp}{dL} \right)_{pred} - \left( \frac{dp}{dL} \right)_{exp}$

The model was also tested for a single-phase flow. In these conditions, where the drag factor is deduced, based on Blasius model:

$$\begin{aligned}
 f &= 4.05 \cdot \text{Re}^{-0.55} & \text{Re} &\leq 10^3 \\
 f &= 0.08 \cdot \text{Re}^{0.048} & 10^3 &< \text{Re} \leq 10^4 \\
 f &= 0.774 \cdot \text{Re}^{-0.196} & \text{Re} &> 10^4
 \end{aligned} \quad (19)$$

Where  $f = \rho \frac{\Delta p}{2NG^2}$ ,  $\Delta p$  – pressure drop, N- number of pipelines (for the experimental data  $N=18$ ), G- the mass velocity based on minimum flow area,  $\text{kg}/\text{m}^2\text{s}$ . These equations allow correlating the corresponding data with a standard deviation of  $\pm 3.5\%$ . For the two-phase flow:

$$\frac{v_g}{v_l} = \frac{x}{1-x} \frac{1-\varepsilon}{\varepsilon} \frac{\rho_l}{\rho_g} \quad (20)$$

The void fraction may be calculated, with  $m = 0,3$ :

$$\frac{\varepsilon}{1-\varepsilon} = B^{5-m} \left[ \left( \frac{\mu_g}{\mu_l} \right)^{m/2} \left( \frac{\rho_l}{\rho_g} \right)^{0,5} \left( \frac{x}{1-x} \right)^{\frac{2-m}{2}} \right]^{2.5-0.5m} \quad (21)$$

**The numerical and experimental results**

As mentioned before, there were tested around 3000 cases. In Table 2-a, b has presented the accuracy of the pressure gradient prediction for all 10 statistical parameters, for the homogenous model.

**Table 2a** Statistical parameters E1-E5 for homogenous model

R	E1%	E2%	E3%	E4%	E5(Pa/m)
2.79	-3.47	20.27	41.33	41.44	-102
2.79	-1.46	21.84	43.68	43.79	-111
3.00	-6.16	24.64	47.38	47.82	99.01
3.20	-9.07	29.23	54.66	55.44	-45.4
3.21	-7.39	27.66	51.07	51.63	-24.5
3.26	5.82	32.82	64.51	64.74	-27.3
3.45	21.73	39.42	71.68	75.94	31.58
3.63	-7.62	27.78	63.39	63.84	-109
3.68	63.17	70.67	21.73	22.29	21.06
3.73	22.62	44.02	93.18	90.72	11.31
4.12	8.85	45.92	87.81	92.96	59.36

**Table 2 b** Statistical parameters E6-E10 - homogenous model

E6(Pa/m)	E7(Pa/m)	E8(Pa/m)	E9 <sup>x10<sup>2</sup></sup> (Pa/m)	E10 <sup>x10<sup>2</sup></sup> (Pa/m)
336.00	825.44	832.16	-8.96	39.20
367.36	916.16	922.88	-7.50	39.20
425.60	1109.92	119.84	-13.4	42.56
567.84	1034.88	137.76	-17.9	45.92
465.92	1106.56	122.08	-15.6	47.04
610.40	1291.36	148.96	-3.36	45.70
673.12	1441.44	168.00	8.96	48.16
487.20	1133.44	859.04	-17.9	50.40
220.64	248.64	257.60	35.84	51.52
586.88	164.64	164.64	7.62	52.64
717.92	159.04	170.24	-7.17	56.00

The average percent error  $E_1$  represents a measure of the agreement between the predicted data and the experimental ones. It indicates the degree of the over-prediction (the positive values) and of the under-prediction (negative values). The positive and negative values cancel each other. The average percent error  $E_2$  also represents a measure of the agreement between the predicted data and the experimental ones, but the positive and negative values did not cancel each other, in this case. This is the why the absolute error (%) may represent a characteristic parameter in the evaluation of the availability for the selected estimation model. The standard deviation  $E_3$  represents a measure of estimation of the range of errors, compared with the average values. The root means square percentage error  $E_4$  indicates more accurately of how close the prediction data are, compared to the experimental ones. The statistical parameters  $E_5, E_6, E_7, E_8$  are similar to  $E_1, E_2, E_3, E_4$ , the only difference being that they are not created based on the relative errors of the experimental two-phase drag factor.

The average error  $E_9$  and  $E_{10}$  are the standard deviations, where:

$$e' = \ln \left[ \left( \frac{dp}{dL} \right)_{pred} \left( \frac{dp}{dL} \right)_{exp} \right]$$

The calculation was accomplished, consequently, in the same manner for all the selected models. For the homogenous flow model, was considered the pressure gradient. In Fig.2 is schematically presented the evolution of the parameters based on the statistical correlations, for this model. In literature, there is a large amount of data for evaluation of the drag factor presented in a graphical manner. For this sort of data, the uncertainty information is not generally presented. A supplementary uncertainty appears when we try to convert the graphical data into a numerical one. The numerical program computes the multipliers for each analyzed model and estimates the correlations.

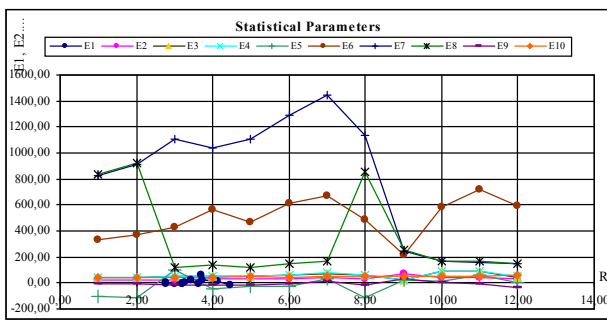


Fig 2 Statistical parameters for the homogeneous model

The difference values  $\epsilon$  are computed for each model. The cumulative mean values and standard deviations are successfully determined for each point.

The saturated gas viscosity is interpolated by a third order Lagrange polynomial form, taking into account that the present paper is dedicated to two-phase flow, the transport of oil in long pipe lines. The remainder is estimated into a reactor code, as to reduce the errors. The statistical parameters between correlations of the tested models and the experimental data are strongly dependent on the selected experimental data. That means that there are some limitations on the range of the applicability of any correlation, function of the concrete

application. The obtained results are influenced by the flow type characteristics, presented further into the Conclusions paragraph. The correlations for gas-liquid drag factor in horizontal pipelines have an average error of 2.4% and an absolute error of 14.7%. In the band of  $\pm 25\%$  are around 84.5% of the considered points and in the band of  $\pm 20\%$  around 72% points. The best results are obtained for dispersed bubbles and slug with an average absolute error of 9.8% and 10.4 respectively. Consequently, the worst agreements are obtained for annular and stratified flow, with an average of absolute error of 32% and 28.7% respectively. In Fig.3 are represented the dependencies between the drag factor  $f$  and Reynolds number,  $Re_{TP}$  for the all three selected models.

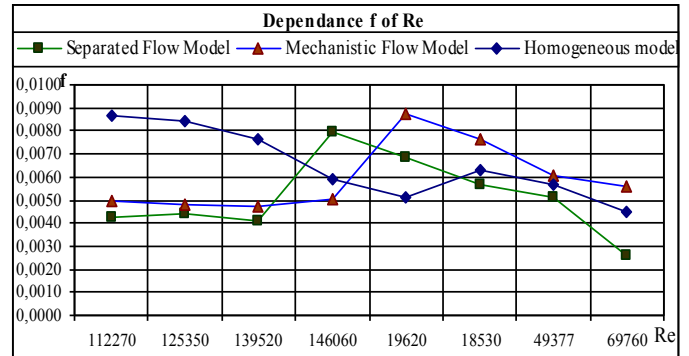


Fig 3 Dependence between  $f$  and  $Re_{TP}$  for the all models

In Fig.4 are represented the dependencies between  $dP_f/dL$  and relative velocity for the selected models. For the experimental data, for the slug flow and annular flow, some correlations are realized, as to improve the average absolute error. After their application, the slug flow has an average error of -1.7% and an average absolute error of 12.4%. In the band of  $\pm 25\%$  are around 94% of the considered points and in the band of  $\pm 20\%$  around 84% points.

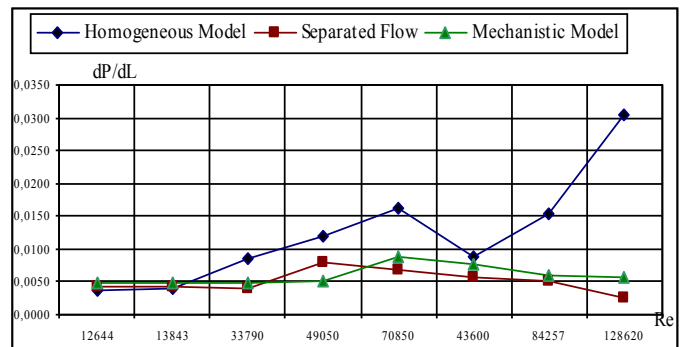


Fig 4 Dependence between  $dP_f/dL$  and  $Re_{TP}$  for the all models

The improved correlation for the stratified flow assures an average error of 5.8% and an average absolute error of 21.2%. In the band of  $\pm 25\%$  are around 68% of the considered points and in the band of  $\pm 20\%$  around 59.5% points. The improved correlation for the annular flow assures an average error of -4.4% and an average absolute error of 18.5%. In the band of  $\pm 25\%$  are around 77.5% of the considered points and in the band of  $\pm 20\%$  around 70.2%.

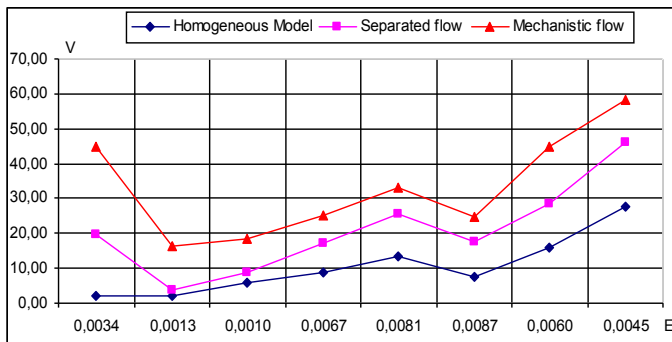
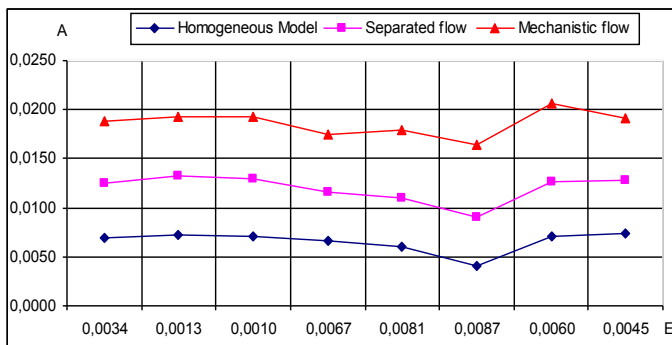
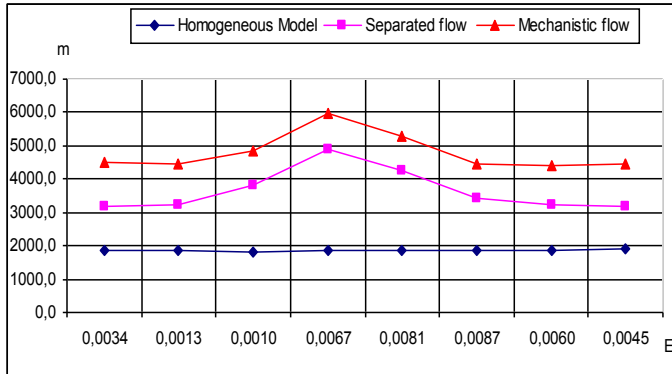
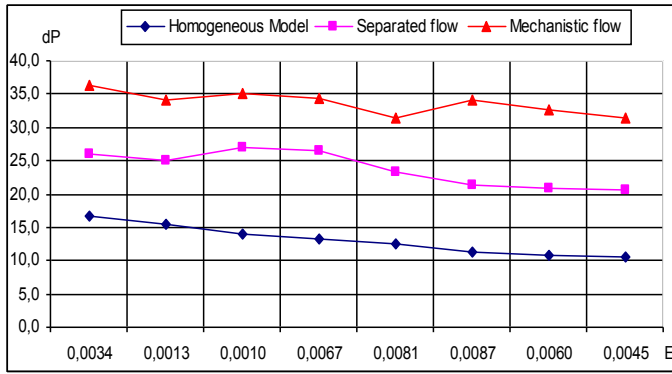


Fig 5 Dependence between mechanical energetic balance for the all models; Fig.5.1  $\Delta p = f(\epsilon)$ , Fig.5.2  $m = f(\epsilon)$ , Fig.5.3  $A = f(\epsilon)$ , Fig.5.4  $V = f(\epsilon)$

The improved correlation for the dispersed bubbles assures an average error of -2.4% and an average absolute error of 11.2%. In the band of  $\pm 25\%$  are around 78% of the considered points and in the band of  $\pm 20\%$  around 72%. In Table 3 are presented the values obtained for the drag factor, the dependence  $dp/dL$  for different values of the Reynolds number,  $Re_{TP}$  and in Table 4 the mechanical energy balance for the homogenous model.

Table 3 Obtained parameters for different Re numbers, for homogenous flow model

$m_{TP}$	$Re_{TP}$	$f$	$dP_f/dL$
0.763	12644	0.0087	0.0038
0.698	13843	0.0084	0.0039
0.273	33790	0.0076	0.0085
0.196	49050	0.0059	0.0120
0.131	70850	0.0051	0.0164
0.218	43600	0.0063	0.0087
0.164	84257	0.0057	0.0153
0.076	128620	0.0045	0.0305

Table 4 The mechanical energy balance for homogenous flow model

$\Delta p$	$m$	$A$	$v$	$\epsilon$
16.8	1848.0	0.0070	2.07	0.0028
15.4	1855.0	0.0073	2.25	0.0014
14.0	1831.2	0.0071	5.91	0.0014
13.3	1857.8	0.0066	8.96	0.0015
12.6	1864.8	0.0060	13.58	0.0017
11.2	1873.2	0.0041	7.70	0.0020
10.9	1867.6	0.0071	15.82	0.0014
10.5	1901.2	0.0074	27.72	0.0028

The mass velocity affects the two-phases drag factor, in correlation with the Reynolds number attached to the flow. Moreover, we must insist that the measurements and correlations made for the two-phase flow considering water and gas are not significant for the two-phase flow between oil and gas (the difference between the viscosity of water and oil). In Table 5, Table 6, Table 7 and Table 8 are presented the same results as in Table 3 and Table 4, but for the separated flow model respectively for the mechanistic model.

Table 5 Obtained parameters for different Re numbers, for separated flow model

$m_{TP}$	$Re_{TP}$	$f$	$dP_f/dL$
0.054	173800	0.0043	0.0414
0.050	199100	0.0044	0.5040
0.043	243100	0.0041	0.0672
0.562	14520	0.0079	0.0090
0.378	21890	0.0068	0.0045
0.151	55110	0.0057	0.0112
0.108	75680	0.0051	0.0123
0.173	52800	0.0026	0.0101

Table 6 The mechanical energy balance for separated flow model

$\Delta P$	$m$	$A$	$v$	$\epsilon$
9.12	1317.6	0.0055	17.70	0.0019
9.58	1382.4	0.0059	1.34	0.0006
13.00	1975.2	0.0058	3.02	0.0010
13.22	3038.4	0.0051	8.06	0.0087
10.83	2397.6	0.0049	12.10	0.0093
10.26	1556.4	0.0049	9.74	0.0014
10.03	1382.4	0.0055	12.77	0.0035
10.15	1296	0.0054	18.26	0.0023

Table 7 Obtained parameters for different Re numbers, for mechanistic flow model

$m_{TP}$	$Re_{TP}$	$f$	$dP_f/dL$
0.0777	112270	0.0049	0.0233
0.0688	125350	0.0048	0.0261
0.0677	139520	0.0047	0.0222
0.0644	146060	0.0050	0.0311
0.5550	19620	0.0087	0.0022
0.3330	18530	0.0076	0.0033
0.1221	49377	0.0060	0.0078
0.0888	69760	0.0056	0.0111

**Table 8** The mechanical energy balance for mechanistic flow model

$\Delta P$	m	A	v	$\epsilon$
10.44	1355	0.0063	25.07	0.0034
9.15	1218	0.0061	12.75	0.0013
8.05	1053	0.0063	9.59	0.0010
7.82	1097	0.0058	7.96	0.0067
7.94	1042	0.0070	7.30	0.0081
12.65	1005	0.0074	7.41	0.0087
11.73	1176	0.0080	16.46	0.0060
10.81	1281	0.0063	12.21	0.0045

**Table 10.d** Statistical parameters – Annular flow

	R	$E_1(\%)$	$E_2(\%)$	$E_3(\%)$	$E_4(\%)$	$E_5(\text{Pa/m})$
HM	3.12	-0.4	12.4	29.3	34.7	11.8
SM	4.79	5.3	35.4	41.8	57.1	-0.7
MM	9.11	24.7	78.3	101.4	124.3	43.8

	$E_6$ (Pa/m)	$E_7$ (Pa/m)	$E_8$ (Pa/m)	$E_9 \times 10^2$ (Pa/m)	$E_{10} \times 10^2$ (Pa/m)
HM	687.1	1124	1483	-4.7	33.7
SM	809.6	974.3	2073	2.4	58.1
MM	1503	2180	3164	-16.3	68.7

In Table 9 are presented the statistical parameters of the mixture drag factor, for each flow pattern (FP) analyzed, where SL – slug, DB-dispersed bubbles, ST- stratified and AN – annular. The obtained results are interesting for high values of the viscosity in pipelines,  $\eta > 400$  m Pa.s. In Table 10 is presented an evaluation of the models HM- homogenous model, SM-Separated model and MM-mechanistic model and correlations for each analyzed flow patterns: a-SL, b-DB, c-ST, d-AN. From the around 3000 tested points: 540 are in dispersed bubble, 310- slug flow, 570- stratified, 360- wavy, 640- slug and 540- annular.

**Table 9** Statistical parameters for each flow patterns

FP	$E_1\%$	$E_2\%$	$E_3\%$	$E_4\%$	$E_5$	$E_6$	$E_7$	$E_8$
SL	-0.72	9.8	14.8	16.4	$-3.7 \times 10^{-4}$	$-1.7 \times 10^{-3}$	$-6.4 \times 10^{-3}$	$7.2 \times 10^{-3}$
DB	-2.4	10.7	12.7	13.5	$-6.6 \times 10^{-4}$	$1.1 \times 10^{-3}$	$2.1 \times 10^{-3}$	$2.1 \times 10^{-3}$
ST	-3.7	19.7	27.8	27.5	$-2.8 \times 10^{-4}$	$-5.8 \times 10^{-3}$	$4.7 \times 10^{-3}$	$5.8 \times 10^{-3}$
AN	-3.2	20.2	24.3	25.4	$-5.4 \times 10^{-4}$	$-2.8 \times 10^{-3}$	$5.4 \times 10^{-3}$	$4.2 \times 10^{-3}$

**Table 10.a** – Statistical parameters – Slug flow

	R	$E_1(\%)$	$E_2(\%)$	$E_3(\%)$	$E_4(\%)$	$E_5(\text{Pa/m})$
HM	2.45	-2.1	17.4	24.3	29.4	212.4
SM	1.74	-5.8	14.3	18.7	21.9	120.4
MM	3.94	4.2	22.9	52.4	54.7	-12.8

	$E_6$ (Pa/m)	$E_7$ (Pa/m)	$E_8$ (Pa/m)	$E_9 \times 10^2$ (Pa/m)	$E_{10} \times 10^2$ (Pa/m)
HM	540.2	1280	1380	-9.7	33.8
SM	658.6	954.8	971.7	-8.2	21.9
MM	530.4	980.3	1104	-2.3	27.2

**Table 10.b** Statistical parameters – Dispersed bubbles

	R	$E_1(\%)$	$E_2(\%)$	$E_3(\%)$	$E_4(\%)$	$E_5(\text{Pa/m})$
HM	1.74	-3.3	10.4	12.4	17.7	-212.4
SM	1.52	-4.5	12.7	17.1	19.1	-120.4
MM	3.12	2.7	16.9	21.9	31.9	-401.8

	$E_6$ (Pa/m)	$E_7$ (Pa/m)	$E_8$ (Pa/m)	$E_9 \times 10^2$ (Pa/m)	$E_{10} \times 10^2$ (Pa/m)
HM	603.4	880.5	924.2	-4.8	16.4
SM	711.4	814.7	971.7	-11.1	17.6
MM	804	1124	1289	-8.3	19.4

**Table 10.c** Statistical parameters – Stratified flow

	R	$E_1(\%)$	$E_2(\%)$	$E_3(\%)$	$E_4(\%)$	$E_5(\text{Pa/m})$
HM	4.84	11.2	19.7	83.4	91.3	18.7
SM	5.19	2.9	37.1	54.7	59.3	-3.2
MM	7.52	14.3	53.1	91.7	101.5	16.7

	$E_6$ (Pa/m)	$E_7$ (Pa/m)	$E_8$ (Pa/m)	$E_9 \times 10^2$ (Pa/m)	$E_{10} \times 10^2$ (Pa/m)
HM	89.3	114.7	134.5	81.7	74.2
SM	104.3	201.2	371.6	-18.4	32.4
MM	91.2	173.6	202.5	-32.3	85.7

## DISCUSSION

The main purpose of this paper is to analyze, to estimate and to correlate the personal measured data with some other data from the literature, as an application for the two-phase flow in long pipelines for oil transportation. For this particular type of flow were analyzed all types of possible aspects of fluid flow (6), taking into account that the aspects of the two-phase fluid flow are different in horizontal and in vertical pipes. The selected methods assure a more accurate correlation and better values for average error and for the absolute average error. It was also considered the effect of the relative velocity between the two-phase of the fluid flow. In the literature when it is analyzed the two-phase flow in pipelines the selected points chosen for correlation are both for the water-air and for the oil-air. Because water and oil have different values of viscosity, the obtained results cannot be put together and interpreted in the same manner. There are some other papers where the two-fluid flow is not treated separately for the horizontal or for the vertical pipes. As I mentioned before the aspects of flow are different, depending on the flow direction. In literature are known more than 20 methods of correlation for the two-phase fluid flow, [17], [18], [19]. From these, each one has a specific difference: the Lockhart-Martinelli, Armand, and Sze-Foo Chien and Ibele correlations are dedicated to flows with low pressure; the Martinelli-Nelson correlation is used especially when water is the considered fluid; the Thom correlation is based on data with high mass velocity. In the present paper, there were selected for testing three methods of correlation, the homogeneous flow model, the separated flow model and the mechanistic flow model, considered the most suitable for this specific type of application. Indifferent of the selected correlation, may appear differences more or less favorable, depending on data selected for analysis, how the selected dimensionless parameters were reduced, the geometry, and the environment of the test. For a specific data selection, for every correlation, may be considered a favorable set domain. The closest correlations used here are the Thom correlations, the Baroczy correlations, and the Ortega correlations, but in all cases, the average error and the absolute average error are a little bit higher than expected. In comparison, with the predictions of the homogeneous two-phase model, the measured void fractions were significantly smaller in the slug and annular flow. For  $x < 0.15$  a strong mass velocity appears for stratified and plug flows. For the same quality, a higher void fraction was obtained if the liquid velocity was increased. The effect of buoyancy is significant, especially for the wavy aspect of flow. At low mass velocity, and at low quality, the flow pattern appears stratified. At a high mass velocity and quality  $x < 0,1$ , the turbulence of the liquid phase help in mixing the two-phases and the flow aspect appears as a

uniform, discrete bubbles; a more homogeneous mixture is obtained. For  $x > 0.15$  the mass velocity has a smaller effect on the void fraction. Appear differences between predictions and the measured data, even if the quality increases. In this case, the flow pattern is annular, with an annular liquid phase, with some droplets of liquid entrained into the gas phase. At higher quality, the void fraction increases if the slip is reduced. For values,  $x > 0.2$  the drag coefficient decreases when the mass velocity decreases and may appear differences in the flow patterns. The flow patterns are well estimated by using the all ten parameters of correlation. When the flow pattern is annular, bubbly flow, intermittent flow, stratified-annular flow, smooth-stratified flow, or wavy-stratified flow, the estimations are appropriate with the experimental data. For the homogeneous model, because there are some discrepancies and a poor agreement between the measured data and estimations based on numerical calculations, this model cannot be a good solution for oil transportation in long pipes. On the future, some specific measurements, dedicated to other applications as the food processing, the polymer processing industry, the pharmaceutical domains, may assure supplementary information in two-phase fluid flow and transportation.

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